Exercises Part II

Exercise 2.1

Exercise 9.6.2 in Ch9. The file danish.txt can be downloaded from the 2005 website of STK4520. It is located under “oblig og datafiler” on the left.

Exercise 2.2

Exercise 1.7,1.8 and 1.9 (in Norwegian) from STK4520 for the year 2007[1].

Exercise 2.3

From Ch 9: 9.4.1, 9.4.3, 9.4.4, 9.5.2, 9.5.4

Exercise 2.4

We are in this exercise going to work with reinsurance problems with much historical data for claim size modelling. We are going to use the so-called Danish fire data, a pool of the damages of more than 2000 industrial fires in Denmark. Their measuring unit is in million DKK, the largest is several hundred. You may download the data from the file **danish.txt**[2] . The portfolio size is $J = 5000$ and there is a constant claim frequency of $\lambda = 0.02$.

a) Suggest a model for the claim size (all sensible possibilites open, always remember for issues like this that log-transformation is a possibility, but that does not mean that it necessarily is a good idea here)

b) Compute the 1% value-at-risk for unlimited responsibility

c) Repeat b) when no claim may exceed 100 million.

The portfolio is reinsured on a claim to claim basis using a layer b xs a contract with $a = 5$ million and $b = 50$

d) Compute the pure premium of the reinsurnce.

e) How much must the reinsurer reserve to carry his responsibility wih probability 99%

f) How much does the reserve of the cedent change by taking out the reinsurance?

Another form of reinsurance is to define in in terms of the total loss X of the portfolio. Suppose the reinsurance again is of the b xs a type, but now with $a = 300$ million and $b = 3000$ million.

[1] You find them on http://kurs.nr.no/stk4520 (which you enter via the currrent home page of STK4520), click into ‘the “oppgaver” section. You can download the necessary data from the “data”-section.

[2] Enter the 2005 website of STK4520 and look for “oblig og data” on the left.

g) Compute the pure premium of the reinsurance and the required reserve of reinsurer and cedent now.

Exercise 2.5

The table shows pay-outs (1980 to 1999 in million NOK) from the the Norwegian fund set up by the insurance industry in Norway to cover damage caused by natural disasters. These are claims caused by a *single event* such as a storm or flood with accumulated effect that could be huge. Prior to 1980 such events were not insurable and responsibility was carried by home-owners and companies themselves. There were 21 claims against the fund in 20 years.

21.1	25.6	25.9	30.3	30.8	30.8	30.8	32.8	51.1	55.6	57.2
65.0	69.5	82.5	103.9	119.3	174.1	175.6	514.8	855.0	1210.6	

A natural model for the data in the Table is the Pareto. If you fit it, you will discover that the estimate of the shape α is $\hat{\alpha} = 1.72$ and that the estimated mean under the fitted Pareto distribution is exactly 200. This means that the scale parameter estimate $\hat{\beta} = 144$.

Suppose the fund has placed a reinsurance treaty in the international market which is of the layer b xs a type with $b = 6000$ and $a = 0$[3]. This means that the contract covers everything up to 6000 and above that limit nothing.

a) Compute pure premium of the treaty using both the mean claim and the exectation within the reinsurance layer under the Pareto. [Hint: For the latter you must compute the integral

$$\int_a^b (z-a)f(z)dz + \int_b^\infty (b-a)f(z)dz$$

which, through integration by parts equals

$$\int_a^b (1-F(z))dz$$

where F is the ordinary distribution function.]

b) Examine the *uncertainty* of the re-insurance premium through simulation. **Procedure**: A simple way is to draw the number of claims N from the Poisson distribution and the data from the estimated Pareto distribution. Claim frequency and expected claim size are then re-estimated for the Monte Carlo data and the pure premium found by multiplication. Since Pareto fitting is slightly troublesome technically, you may (as an alternative) carry out the following alternative: Estimate mean claim size as the ordinary mean (called $\hat{\xi}$), draw $n = 21$ claims (*with replacement*) from the orginal data and re-estimate $\hat{\xi}$. Results of both methods will be shown and discussed.

c) Estimate uncertainty through mathematics. If $\hat{\lambda}$ and $\hat{\xi}$ are estimates of claim frequency and expected claim size (within the layer) then

$$E(\hat{\lambda}\hat{\xi} - \lambda\xi)^2 = E(\hat{\lambda})^2 E(\hat{\xi})^2 - \lambda^2\xi^2$$

[3] Almost as it is done in practice

if we ignore that $\hat{\xi}$ may not be unbiased (this inaccuracy can be rectified). From this deduce that

$$E(\hat{\lambda}\hat{\xi} - \lambda\xi)^2 = (\text{var}(\hat{\lambda}) + \lambda^2)(\text{var}(\hat{\xi}) + \xi^2) - \lambda^2\xi^2 = \text{var}(\hat{\lambda})\text{var}(\hat{\xi}) + \text{var}(\hat{\lambda})\xi^2 + \text{var}(\hat{\xi})\lambda^2.$$

How will you find estimates for the two variances?

Exercise 2.6

We are in this exercise going to examine how hetereogenity in claim frequency influences the reserve when the claim size is the same for the entire portfolio. Let

$$\mathcal{N} = \sum_{j=1}^{J} N_j$$

be the total number of claims against the portfolio. Here $N_1, \ldots, N_J$ are the claims against the individual policies. It is assumed that each N_j is Poisson distributed with parameters $T\mu_1, T \ldots, T\mu_J$ where T is the period of time in question. The intensites $\mu_1, \ldots, \mu_J$ may be both fixed or stochastic. The total claim against the portfolio is then

$$\mathcal{X} = \sum_{i=1}^{\mathcal{N}} Z_i$$

where Z_i is the size of the i'th claim. It is assumed that all claims are log-normal distributed.

a) Implement a computer program that samples the portfolio claim when the intensities $\mu_j = \xi$, a fixed parameter that applies to all policies.

b) Repeat a) when all μ_j are drawn independently of each other from a log-normal distribution with mean ξ and standard deviation τ.

c) Repeat a) when again all $\mu_1 = \ldots = \mu_J = \mu$ are equal, but when no the *common value* μ is generated from a log-normal with mean ξ and standard deviation τ.

Numerical experiement are run when

- $J = 1000$ and $J = 100000$
- $T = 1$, $\xi = 0.05$ and $\tau = 0.03$
- The log-normal for X_i has mean 1 and standard deviation 1

d) Compute the upper 1% and 5% percentiles for the total portfolio claim under the three models described in a), b), and c) and the parameters listed above.

d) Draw your conclusions: What have you learnt through these experiements.

You may need the following: If $\log(Z)$ is normal (η, σ),then

$$E(Z) = \exp(\eta + \sigma^2/2), \qquad \text{var}(Z) = \exp(2\eta + \sigma^2)\{\exp(\sigma^2) - 1\}$$

You adapt the parameters η and σ corresponding to given mean and standard deviations of Z trough these formulas.

Exercise 2.7

This exercise deals with an automobile portfolio and is, essentially, a real one[4] The statistical record is of $n = 183999$ rows and is stored as **automobile.txt**[5] Each row consists of the following 15 columns

- Column 1: Age
  coding: 0 for 'below 26', 1 =for '26 or above'
- Column 2 Sex
  coding: 0 for ' male', 1 for 'female'
- Columns 3-7: Traffic density
  coding: 00000 for 'highest traffic density'
  10000 for 'next highest'
  01000 for 'third highest
  00100 for 'fourth highest
  00010 for 'next lowest
  00001 for 'lowest'
- Columns 8-12: Driving limit
  coding: 00000 for 'less than 8000km'
  10000 for 'less than 12000 km'
  01000 for 'less than 16000 km'
  00100 for 'less than 20000 km'
  00010 for 'less 25-30000 km'
  00001 for 'unlimited'
- Column 13: Number of claims
- Column 14: Exposure time
- Column 15: Claim size (in NOK)

We are going to assume that there is no connection between the the claim size and the explanatory variables

a) Use Poisson regression to understand the importance of the explanatory variables for claim intensity.

b) Suggest a model under a) for differentiated pricing. Outline carefully the interpretation of the regression coefficients

c) Carry out differentiated pricing and report the pure premium for the groups you find relevant to define (if there are many such groups due to cross-classifications, write a program; at least give som examples of dfiiferences in prices)

You work with Poisson models for various groups. Think through the following question:

---

[4]Some steps have been taken to change the data slightly from the real example on which this exercise is patterned.

[5]To download enter the website of STK4520 of 2005, and look for the file on the left.

d) What is the significance of the hetereogenous claim intensity model for the aggregated claim distribution for the portfolio? What would have been the result had we ignored hetereogenity completely?