Exercises Part III

Exercise 3.1

We shall in this exercise examine how pension obligations and present values might depend on the age distribution of the portfolio by studying two examples:

Portfolio I:	$\mathcal{N}_{l_0} = ce^{-0.05\vert l_0-40\vert}$	$l_0 = 30, \dots, 89$
Portfolio II:	$\mathcal{N}_{l_0} = ce^{-0.05\vert l_0-60\vert}$,	$l_0 = 30, \dots, 89$

The first portfolio has the highest membership at 40 years and the second at 60. Furure premium into the pension scheme is *not* taken into account. All pensions are one money unit per year in advance and starts to run at age $l_r = 65$ years. The mortalities are

$$q_l = 1 - \exp(-\theta_0 - \theta_1 e^{\theta_2 l})$$

where θ_0 =0.000309, θ_1 =0.0000219 and θ_2 =0.100047 (estimates for Norwegian men in year 2004).

a) Determine the constant c so that both portfolios have 100000 members.

b) Write a computer program that calculates the obligations $\mathcal{X}_k$ in year k for $k = 0, 1, \dots, K$.

c) Compute $\mathcal{X}_k$ for both portfolios and plot them jointly against k up to half a century ahead (i.e. when $K = 50$).

d) Calculate the present value for both portfolios using $r = 4\%$ as technical rate of interest.

Stress testing (key assumptions varied) is often undertaken by cutting all mortalities q_l by a flat rate.

e) Redo d) with new mortalities $q_l' = 0.85 \cdot q_l$. Commment on how the present value of the two portfolios is altered.

Exercise 3.2

This is a continuation of the previous exercise with the same assumptions and conditions as there.

a) Compute the equivalence premium for a person who enter the pension scheme at age 35.

b) Redo b)-e) of Exercise 3.1 when future premium is subtracted the obligations (giving us *net* obligations. Assume that all members of the scheme entered at 35 so that their premium is the same.

Exercise 3.3

Mortalities have since the Second World War been in steady decline in all developed countries (and during the last decades also in under-developed ones). This trend (if it continues) is going to have consequences for pension obligations. We shall in the next exercise examine the issue through the Lee-Carter model that links the mortalities q_l of today to future ones q_{lk} that applies k periods ahead. Lee-Carter (which goes back to 1992) is arguably the most commonly used dynamic model for mortalities. The form adpoted here is

$$q_{lk} = \omega_l^k q_l, \qquad k = 0, 1, 2, \ldots$$

where ω_l makes the the mortalities change with k. In practice ω_l is a little less than 1, and survival increases with k. The following version have been derived from studies of the Norwegian population since 1950:

$$\log(\omega_l) = -b_0 \frac{e^{h_l}}{(1+e^{h_l})^2} \qquad \text{where} \qquad h_l = a_0 + a_1 l + a_2 l^2$$

with $b_0 = 0.0632$, $a_0 = -1.3897$, $a_1 = -0.0172$ and $a_2 = 0.0007$ as parameters. The model which is for men, is on annual time scale. For q_l use the model in Exercise 3.1.

a) Plot $\log(q_{lk})$ against l for $k = 0$, $k = 20$ and $k = 50$ in a joint plot. Comment on how the mortalities are influenced by the period ahead.

b) Fix k and use Algorithm 12.1 to compute how much longer a 20-year old lives on average according to the model q_{lk}. Plot against time for k up to 50.

c) Again fix k. How do you determine the 25% and the 75% percentiles for the remaining life of a twenty-year old under this model? Plot each of them against time for k up to 50.

Exercise 3.4

If the upsurge in longevity persists, pension obligations will be under-estimated if the mortalities used are those of today. The purpose of this exercise is to analyse the issue by means of the dynamic model of the preceding exercise.

a) Explain by dipping into Section 15.2 how the survival probabilities ${}_kp_l$ are calculated from the mortalities. Write a computer program.

b) Compute the present values for the obligations (future premium not counted) for the pension portfolios I and II in Exercise 3.1 when $r = 4\%$, $l_r = 65$ and the pension is one money unit and in advance; i.e redo d) in Exercise 3.1 with the dynamic model. Compare with the earlier results.

The impact of dynamic mortalities can be illustrated by means of the present value of pension costs for a given individual. In mathematical terms we are then considering

$$a_{l_0} = \sum_{k=k_0}^{\infty} \frac{{}_kp_{l_0}}{(1+r)^k} \qquad \text{where} \qquad k_0 = \max(l_r - l_0, 0)$$

c) Compute a_{l_0} for the dynamic model and the model in e) in Exercise 9.3 (i.e which took flat 15% cuts of the current mortality) under the ame conditions as before.

d) Plot both assesments in c) jointly against l when l_0 is varies between 20 and 100. Comment and interprete the results.