

Stochastic volatility and its impact on risk

You are to give a 20 minute presentation where you present present and analyse the impact on equity risk when volatilities follow a GARCH(1,1) model.

Assume that Y_k is the daily log-return from a stock with price S_k at time k , i.e.

$$Y_k = \log \left(\frac{S_k}{S_{k-1}} \right).$$

Further, assume that Y_k follows the model

$$Y_k = \xi_y + \sigma_k \varepsilon_k,$$

with σ_k from a GARCH(1,1) model.

1. Give a short introduction to the GARCH(1,1) model and its mathematical formulation.
2. Explain the recursion for computing the K-day return $R_{0:K}$.
3. Define an example model on daily time scale with plausible parameters.
4. Show numerical examples on both daily and annual ($K = 250$) time scale, illustrating the importance of adding the GARCH components to a model with fixed volatilities.