The Student's t copula and property insurance

You are to give a 20 minute presentation on the Student's t copula and an application to property insurance .

Consider the property insurance portfolio payout

$$\mathcal{X} = Z_1 + \ldots + Z_{\mathcal{N}}$$

where $\mathcal{N}$ is the number of claims (Poisson-distributed with parameter λ) and $Z_1, Z_2, \ldots$ are stochastically independent and identically distributed payouts for these incidents. Usually $\mathcal{N}$ and $Z_1, Z_2, \ldots$ are assumed independent, but there are cases where sizes of claims depend on how many they are. To investigate the impact on portfolio risk an obvious way is to assume that $Z_1, \ldots, Z_n$ are conditionally independent given $\mathcal{N} = n$ with $(\mathcal{N}, Z_i)$ Student's t-dependent. Note that this does not change the marginal distributions of $\mathcal{N}$ and Z_i. One way to carry out such simulations is to generate $\mathcal{N}^*$ as a Monte Carlo realization of $\mathcal{N}$. If $F(n)$ is the distribution function of $\mathcal{N}$ and U^* is uniform, then it may be shown that

$$U_1^* = (1 - U^*)F(\mathcal{N}^* - 1) + U^* F(\mathcal{N}^*) = G(N^*, U^*) \tag{1}$$

is uniform $U(0, 1)$ and that

$$\mathcal{N}^* = G^{-1}(U_1^*). \tag{2}$$

Moreover, the conditional copula $C(u_2|u_1)$ when $C(u_1, u_2)$ is a Student's t copula with parameters (ρ, ν) is given by

$$C(u_2|u_1) = t_{\nu+1}\left(\frac{t_\nu^{-1}(u_2) - \rho t_\nu^{-1}(u_1)}{\sqrt{\frac{\left(\nu+(t_\nu^{-1}(u_1))^2\right)(1-\rho^2)}{\nu+1}}}\right), \tag{3}$$

where t_ν is the cdf of the univariate t distribution with ν degrees of freedom.

1. Introduce the Student's t copula and its basic properties including the equity return patterns you can create with it.

2. Explain how the conditional simulation algorithm for the Student's t copula can be used to generate conditionally independent losses $Z_1, \ldots, Z_{\mathcal{N}}$ given $\mathcal{N}$, using (1) and (2).

3. Report on how sensitively the 99% reserve depends on the parameter ρ of the Student's t copula when $\nu = 3$, $\lambda = 10$ and the marginal distribution of Z_i is the standard Gamma with shape $\alpha = 2$. The R-function below draws simulations of $\mathcal{X}$, and you may follow the commands there.

```
reserve <- function(lambda=10,xi=1,alpha=2,nu=3,rho=0.5,m=10000)
 {
   N <- rpois(m,lambda)
   U <- runif(m)
   P <- ppois(N,lambda)
   U_1 <- (1-U)*(P-dpois(N,lambda))+U*P
   X <- rep(0,m)
   for (i in 1:m)
    {
       V <- runif(N[i])
       U_rest <- pt((qt(V,df=nu)-rho*qt(U_1[i],df=nu))/sqrt((nu+qt(U_1[i],df=nu)^2
       X[i] <- xi*sum(qgamma(U_rest,alpha))/alpha
     }
   X
 }
```