Copulae and their impact on equity risk

Background

The concept of copulae goes back to the work of Hoeffding from the 1940's, whereas Sklar was the one to introduce the name "copula" in 1959. However, it was not until the turn of the century that these models really received interest. Parts of the academic world may exaggerated the importance of copulae; yet they are undoubtedly here to stay as a useful modelling tool.

Objective:

You are to present the Student's t and the Clayton copulae and analyse their impact on equity risk.

Material:

Introductory note on copulae and Section 7 from Chapter 6: "Modelling II: Conditional and non-linear".

Main points:

The presentation (45 minutes) should cover

- An outline of the models:
 - Definitions, how marginal distributions can be accommodated, independence as a special case, comparison with ordinary Gaussian models, how Monte Carlo is carried out.
- A case study of daily and annual equity risk under the pure Gaussian model and under the models based on the Student's t and the Clayton copulae:
 - Model the log-returns for 2 assets. The marginal distributions are Gaussian with mean 7% and volatility 25% for both.
 - Use Kendall's $\tau \rho_{\tau} = 0$ and $\rho_{\tau} = 0.9$. Find the corresponding parameter of the Clayton copula and the correlations of the Student's t and the Gaussian model. Moreover, let $\nu = 6$ for the Student's t copula.

 Use portfolio weights 0.5 for both assets, and compare daily and yearly portfolio returns from the three models by plotting density functions jointly and by comparing percentiles.