Selection and Dynamics in Mortality Modeling

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Overview

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3. The Lee-Carter model for time-varying mortalities and plots illustrat- ing it.
4. How time-varying mortalities can be included in ordinary life-insurance calculations.
5. Plots of one-time premia under the dynamic model compared to a flat 15 percent reduction of the mortalities that are at work today.
6. Presentation of the selection model and its impact on the one-time premia.

Introduction

The Lee-Carter model was designed in 1992 to predict the future mortality probability of the US population.It capture age specific trends from an observed period and extrapolate these trends in the future. It is widely used by many institutions today. We will use it here to present selection effects and dynamic changes of mortalities and illustrate their economic impact.

Mortalities since the Second World War

- Life table or Actuarial table.
  - Period life table.
  - Cohort life table or generation life table.
- The force of mortality μ_{xt}.

$$\mu_{xt} = \frac{D_{xt}}{E_{xt}}$$

Where D_{xt} is the number of deaths of people aged x in year t and E_{xt} the exposure of age x in year t.

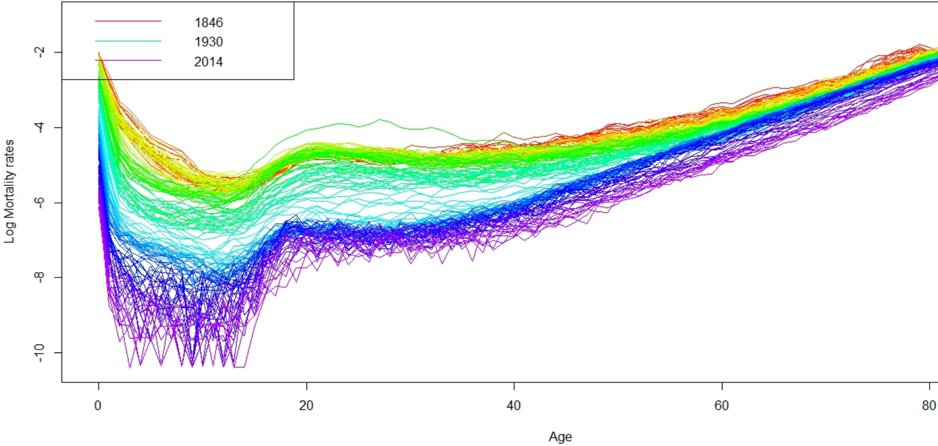


Figure: **Force of Mortality male**

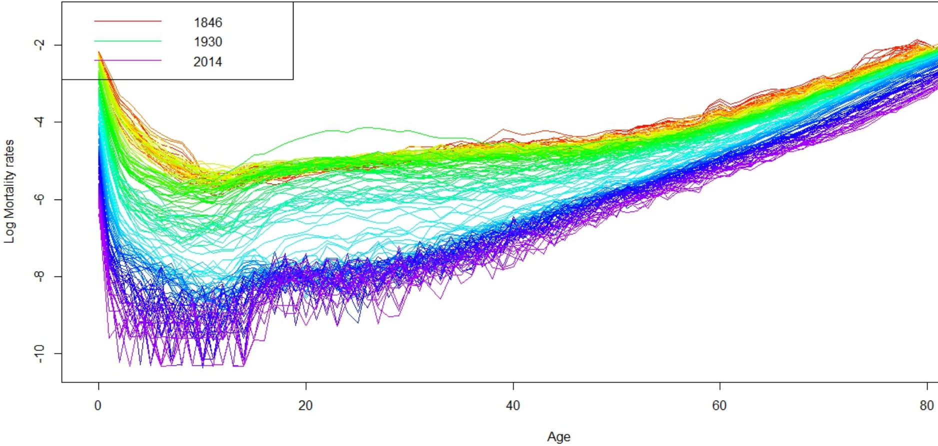


Figure: **Force of Mortality female**

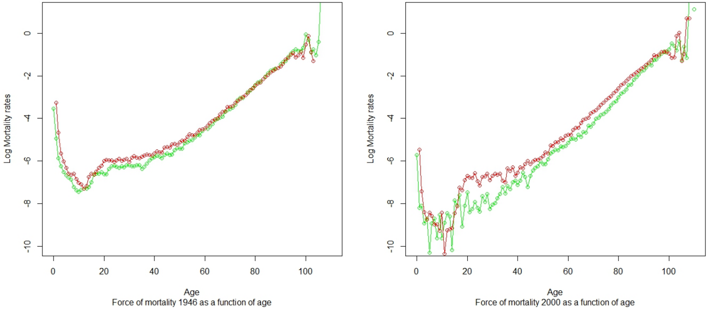


Figure: **Force of Mortality for the years 1946 and 2000**

Some notations in life Insurance

- l_0 is the age at the beginning of the contract so $l_0 + k$ will be the age at time k; l_r is the age when retirement start.
- s is the pension benefit until the maximum realistic age l_e.
- ${}_k p_{l_0}$ is the probability that and individual of age l_0 is alive at age $l + k$
- d $= \frac{1}{1+r}$
- The equivalence premium π is the solution of :

$$\pi \sum_{k=0}^{l_r - l_0 - 1} d^k \ {}_k p_{l_0} = s \sum_{k=l_r - l_0}^{l_e - l_0} d^k \ {}_k p_{l_0}$$

Lee-Carter model for time-varying mortalities

- The main goal of this model is to predict future mortality for a given population.
- We supposed for the model that the life expectancy is growing. We take the mortality of an individual of age l in year k.

Simplified version of the Lee-Carter with parameter estimates for Norway.

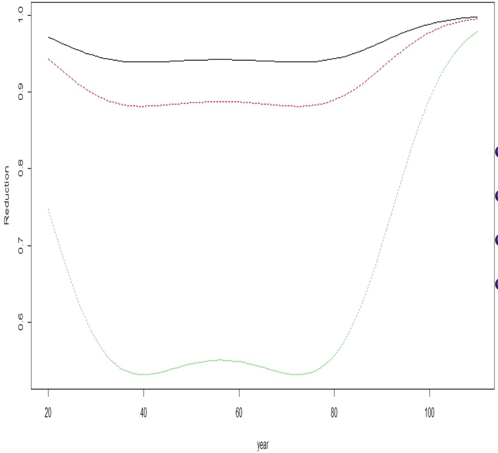
$$q_{lk} = w_l^k q_{lo}$$

Where

$$\log(w_l) = -\alpha \frac{e^{g_l}}{(1+e^{g_l})^2}$$

and

$$g_l = \beta_0 + \beta_1 l + \beta_2 l^2$$



Lee-carter Reduction factor over 5, 10 and 50 years,

- Red line k=5
- Green line k=10
- Blue line k=50
- w_{20}^{80} = reduction factor for individual aged 20 in 80 years

Dealing with longer lives

- How Mortalities are enter in actuarial calculation.

$$p_l \rightarrow \mathbf{Z} \rightarrow \hat{p}_l \rightarrow {}_k\hat{p}_l \rightarrow \hat{\pi}, \hat{\chi}_k, \widehat{PV_0}$$

- Now people live longer than expected.This can be analyse by introducing a default sequence q_{l0} of mortality. Two approaches can be use her, the cohort version and the time-dynamic version.

Simple model

$$q_l(i) = q_{l0}e^{-\gamma(i)} \text{, cohort version}$$
$$q_{lk} = q_{l0}e^{-\gamma_k} \text{, time dynamic-version}$$

- $\gamma(i)$ and γ_k are parameters that make the mortalities deviate from the default sequence q_{l0}.

k-step survival probabilities required for actuarial calculations.

$${}_{k+1}p_l = 1 - q_{l+k}(l) . {}_kp_l, \text{ Cohort version.}$$
$${}_{k+1}p_l = (1 - q_{l+k,k}) . {}_kp_l, \text{ time-dynamic version.}$$

- For k = 0,1,2,..., and people of age l born at -l, the probability that they survive the periode from k to k+1 is then $1 - q_{l+k}(l)$ and $(1 - q_{l+k,k})$
- By inserting the simple model into the k-step survival probabilities we obtain:

Life table.

$${}_{k+1}p_l = (1 - q_{l0}^{-\gamma(l)}) . {}_kp_l, \text{ Cohort version.}$$
$${}_{k+1}p_l = (1 - q_{l0}^{-\gamma_k}) . {}_kp_l, \text{ time-dynamic version.}$$

Present value of pension portfolios

- With the appropriate life table ${}_kp_l$ it is now possible to compute the estimated equivalence premium,liability and present value for k = 1,2,3,...

$$\widehat{PV_0} = \sum_{k=0}^{l_e-l_0} d^k \hat{\chi}_k$$

Where

$$\hat{\chi}_k = -\hat{\pi} \sum_{k=0}^{l_r-l_0-1} J_l \; {}_k\hat{P}_l + s \sum_{l=l_r-k}^{l_e-k} J_l \; {}_k\hat{P}_l$$

Table: Net reserve and mortality Parameter γ when people (on average) live longer than we expect.

Added years	0	1	2	3	4	5
Parameter	0	0.077	0.155	0.233	0.312	0.391
Net reserve(billion)	30.7	32.1	33.5	34.9	36.3	37.8

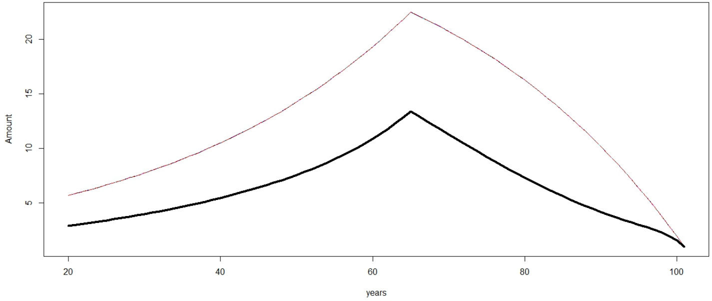


Figure: **One time Premia**

Tick line true model and dynamique model in red.

- One-time premia under the dynamic model compared to a at 15% reduction of the mortalities.
- A flat cut of 15% will give $\zeta_l = 0.85$ for all l.

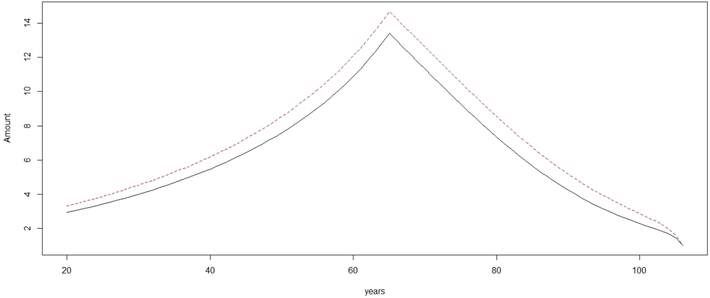


Figure: **Flat Cut versus Dynamic model**

Thank You For your attention !!!