

#### Oblig 4: Asset/liability management in Solvency II

**Background** The Solvency II evaluation of longevity and asset risk is handled in the Life insurance risk and Market risk modules with their combined impact taken care of by the Solvency II correlation method. It is folklore in the insurance industry that this approach fails to take into account properly the risk-reducing effect of tailoring financial investments to the time profile of the liabilities. That is what you in this oblig shall investigate through fixed-income securities like bonds adapted a given pension portfolio.

**Cooperation** You are allowed to and encouraged to cooperate with the students analysing Solvency II longevity and discount risk (Oblig 3) and longevity and currency risk (Oblig 5).

**Objective:** Determine whether the Solvency II approach yields the 99.5% certainty that is the target.

**Material:** Chapters on Solvency modelling offered the STK4520 students as handouts, especially Chapter 5 on Market Risk and Chapter 6 on Life insurance risk. You may like to take inspiration from Sections 15.2 and 15.6 in Bølviken, E. (2014). *Computation and modelling in Insurance and Finance*, Cambridge University Press.

**Details and simplifications** The first point is to define an investment strategy. Assume bonds to be available up to maturity  $J$  and that these opportunities have been utilized, but that  $J$  is not large enough to cover the liabilities which last to year  $K > J$ .

We need models that describe random variation in mortalities and the bond market. The former is in Solvency II defined by a downwards, flat shock  $s_0 = 20\%$  that is seen as a 99.5% percentile. One possibility that accords with this condition is to assume that the relationship between current mortalities  $q_j$  (for individual  $j$ ) and the stressed ones  $q_j^s$  is of the form

$$q_j^s = (1 - S)q_j \quad \text{where} \quad S \sim \xi \times \text{Gamma}(\alpha).$$

Here  $\text{Gamma}(\alpha)$  stands for a Gamma distribution with mean 1 and shape  $\alpha$ . Determine  $\xi$  for given  $\alpha$  so that the 99.5% percentile becomes  $s_0$ .

For the bond market you may use a similar model adapted the shocks in Solvency II documentation. Look that up and invent a simple portfolio of fixed-income securities of a given credit quality.

You also have to invent a pension portfolio with a given age distribution and introduce baseline mortalities  $q_j$ . If you like, take ideas from Section 15.2 in Bølviken (2014).

**Main points:** The presentation (45 minutes) should cover

- 1 How Solvency II calculates the Solvency Capital Requirement for longevity and fixed-income asset risk which are in two different main modules.
- 2 The pension portfolio with age distribution and the default mortalities assumed. Use two different age distributions, one for a young portfolio and one for an old one.
- 3 How the Best Estimate is calculated under default conditions

- 4 The stochastic models that adapt to the Solvency II specifications.
- 5 How the stochastic model and the results it yields is simulated.
- 6 Comparisons between the 99.5% percentiles under the simulations and those returned by Solvency II. Use both the young and old portfolio in combination with  $\alpha = 1$  for the mortality model. Vary  $J$  and examine how the difference between the Solvency II assessments and your more nuanced ones depends on it.