Oblig 6: Solvency II non-life premium risk

Background Premium risk in Solvency II applies to new claims from existing and new contracts during the coming year and the purpose of this oblig is to evaluate the Solvency Capital Requirement for such risk by comparing with the 99.5% percentile under a reasonable and carefully calibrated stochastic model. Reinsurance (which has a Solvency II treatment that is very simplistic indeed) is to be taken into account. We are not going to distinguish between old and new contracts; the portfolio size is a fixed number J, and all contracts are assumed identivcal and independent given a random, annual claim frequency μ. The portfolio is so large that the portfolio loss becomes Gaussian when the random claim frequency μ is given. Only a single line of Solvency II business (motor with personal injuries excluded) will be considered.

Objective: Determine to what extent the Solvency II approach seems to yield the 99.5% certainty that is the target.

Material: Chapters on Solvency modelling offered the STK4520 students as handouts, especially Chapter 8 on non-life insurance risk. You find relevant material in Section 6.3 in Bølviken, E. (2014). *Computation and modelling in Insurance and Finance*, Cambridge University Press.

Details and simplifications You must start by looking up the Solvency II SCR for motor which is based on a portfolio standard deviation $\sigma \times V$ where σ is a standard deviation factor for motor (look it up) and V is the volume defined as the expected premium P. If $\xi_\mu = E(\mu)$ and $\xi_z = E(Z)$ where Z is a claim, then

$$V = (1+\gamma)J\xi_\mu\xi_z$$

where γ is the loading (for example 20%). Let $\mathcal{X}$ be the total loss under premium risk. Then

$$E(\mathcal{X}) = J\xi_\mu\xi_z \qquad \text{and} \qquad \text{var}(\mathcal{X}) = J^2\tau_\mu^2\xi_z^2 + J\xi_\mu(\xi_z^2+\tau_z^2)$$

where τ_μ and τ_z are the standard deviations of μ and Z respectively. Choose $\xi_\mu = 5\%$ and $\xi_z = 1$, allow τ_z to vary and determine τ_μ so that $\text{sd}(\mathcal{X})$ coincides with the standard deviation σV assumed in Solvency II. You can now simulate the 99.5% percentile under the model. Recall here that we are assuming Gaussian losses given μ. You have to introduce a model for μ; a gamma distribution is a reasonable choice. Remember that the percentile for $\mathcal{X}$ does not include its expectation (which has been taken out and included in the Best Estimate BE). Make a table with a few values of J and τ_z.

As a second part intrduce an $a \times b$ reinsurance contract on $\mathcal{X}$ with $a = 0$ and repeat both the Solvency II calculations and the simulated ones.

Main points: The presentation (45 minutes) should cover

1 How Solvency II calculates the Solvency Capital Requirement for premium risk.

2 The stochastic model and how its is calibrated to match the Solvency II assumptions.

3 How the Best Estimate is calculated.

4 The stochastic models that adapt to the Solvency II specifications.

5 How the stochastic model and the results it yields is simulated.

6 Comparisons between the 99.5% percentiles under the simulations and those returned by Solvency II.