Oblig 7: NatCat disaggregation in Solvency II

Background Natural catastrophes (NatCat) are in Solvency II typically described as hitting a certain region with a certain degree of severity whereas reinsurance contracts often are defined in terms of underlying units. The issue may be called the **granularity** problem. Total gross losses (denoted G) apply to the region while we need them disaggregated in order to be able to include the effect of reinsurance properly. In this oblig the underlying units will be referred to as policies.

The Solvency II approach to disaggregation is a pro rata technique. Suppose n policies hit by the catastrophe have insured sums $\mathrm{SI}_1, \ldots, \mathrm{SI}_n$ and pay premia $P_1, \ldots, P_n$. It is then assumed that the gross losses $G_1, \ldots, G_n$ allocated the n policy holders are

$$G_i = \frac{\mathrm{SI}_i}{\mathrm{SI}_1 + \ldots + \mathrm{SI}_n} \times G \qquad \text{or} \qquad G_i = \frac{P_i}{P_1 + \ldots + P_n} \times G$$

for $i = 1, \ldots, n$. After specifying G as a 99.5% percentile, the indvidual gross losses $G_1, \ldots, G_n$ can be calculated and the reinsurance programs applied for a NatCat Solvency Capital Requirement.

The procedure is known as the **spread** method. Is it sound? One might surmise that it underestimates risk since there will be randomness in how G distributes on the n units, and that is what this oblig shall investigate.

Cooperation You are allowed to and encouraged to cooperate with the student analysing earthquake risk in (oblig 8).

Objective: Determine whether the Solvency II approach through the spread method yields the 99.5% certainty that is the target.

Material: Chapters on Solvency modelling offered the STK4520 students as handouts, especially Chapter 8 on non-Life insurance risk.

Details and simplifications Use reinsurance programs of the $a \times b$ type with $a = 0$ and recall that $b = \mathrm{SI}$ (in Solvency II jargon). We must construct a model that adapts to the spread method. A simple possibility is

$$G_i = Y_i \times G, \qquad i = 1, \ldots, n$$

where G and the vector $(Y_1, \ldots, Y_n)$ are independent and $Y_1 + \ldots + Y_n = 1$. A natural model for the latter is the so-called **Dirichlet** distribution; look it up in Wikipedia if you haven't heard of it. It has the stochastic representation

$$Y_i = \frac{Z_i}{Z_1 + \ldots + Z_n}, \qquad i = 1, \ldots, n$$

where $Z_i \sim \mathrm{gamma}(\alpha_i)$ is Gamma-distributed with mean one. You can calibrate the model to the Solvency II specification by utlizing that

$$E(Y_i) = \frac{\alpha_i}{\alpha_s} \qquad \text{where} \qquad \alpha_s = \alpha_1 + \ldots + \alpha_n.$$

Use a Pareto model for G.

Main points: The presentation (45 minutes) should cover

1 Explain how Solvency II calculates the Solvency Capital Requirement for NatCat in this situation. Choose a suitable reinsurance contract yourself.

2 Outline and motivate the stochastic model and how it is adapted the Solvency II calculations.

3 Explain how the stochastic model and the results it yields is simulated.

4 Compare the 99.5% percentiles under the simulations with those returned by Solvency II. Use two different Pareto distributions for G and vary α_s (three values, for example $\alpha_s = 1$, $\alpha_s = 10$ and $\alpha_s = 100$).