## Oblig 8: Earthquake risk in Solvency II

**Background** The Solvency II evaluation of earthquake risk in a country is based on a subdivision into so-called CRESTA zones[1] with the total gross loss calculated by a simplistic formula using the sums insured in all zones; see below for details. The purpose of this oblig is to compare the Solvency II evaluation of the capital requirement for earthquake risk in an imaginary CRESTA country with what one obtains from a comparable stochastic model. You have to use Monte Carlo, and an R program will be put at your disposal; you may have provide some additional commands yourself.

**Cooperation** You are allowed to and encouraged to cooperate with the student analysing NatCat disaggregation (oblig 7).

**Objective:** Determine whether the Solvency II approach yields the 99.5% certainty that is the target.

**Material:** Chapters on Solvency modelling offered the STK4520 students as handouts, especially Chapter 8 on non-life insurance risk. You may benefit from Section 5.4 in Bølviken, E. (2014). *Computation and modelling in Insurance and Finance*, Cambridge University Press.

**Details and simplifications** Suppose the country consists of $n$ CRESTA zones with total sums insured $\mathrm{SI}_1, \ldots, \mathrm{SI}_n$. Here each $\mathrm{SI}_i$ is the sum of all limits of responsibility over all policies for zone $i$. The Solvency II specification of the 99.5% annual gross loss of eaurthquake risk is

$$G = Q \times \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \times (w_i \times \mathrm{SI}_i) \times (w_j \times \mathrm{SI}_j)$$

where $Q$ is a so-called risk coefficient specified in Solvency II, and $w_1 \ldots, w_n$ are risk weights taken from Solvency II documentation whereas $\rho_{ij}$ are correlations between zones $i$ and $j$ offered by the European insurance regulators. To include reinsurance (which is assumed to apply per zone) Solvency II specifies a pro rata disaggregation method so that

$$\mathrm{SCR} = \sum_{i=1}^{n} H_i(G_i) \qquad \text{where} \qquad G_i = \frac{\mathrm{SI}_i}{\mathrm{SI}_1 + \ldots + \mathrm{SI}_n} \times G$$

and where $H_i(\cdot)$ is the responsibility of the cedent company for zone $i$ after the reinsurance reimbursements have been subtracted. Use $a_i \times b_i$ contracts with $a_i = \mathrm{SI}_i/2$ and $b_i = \mathrm{SI}_i$. For your numerical study take $Q = 0.10\%$ (the Solvency II value for Germany), $n = 10$, equal weights $w_i = 1$ and all sums insured $\mathrm{SI}_i = 1$ too.

Then there is the stochastic model for the gross losses $G_1, \ldots, G_n$ of the $n$ zones. Instead of computing them in a mechanistic manner as above, let

$$G_i = I \times Z_i \times Y, \qquad i = 1, \ldots, n$$

[1] CRESTA is acronym for *Catastrophic Risk Evaluation and Standardizing Target Accumulation*, and the system is maintained and developed by a group lead by Munich Re and Swiss Re.

where $I$ is an indicator variable signalling whether there is an earthquake or not that year, $Z_1, \ldots, Z_n$ define how losses distribute between the $n$ zones and $Y$ is the gross loss for the entire country. The three components (all independent) are modelled as follows. Firstly

$$\Pr(I = 1) = 1 - \Pr(I = 0) = Q,$$

then

$$Z_i = \frac{\mathrm{SI}_i \times e^{\sigma\varepsilon_i}}{\mathrm{SI}_1 \times e^{\sigma\varepsilon_1} + \ldots + \mathrm{SI}_n \times e^{\sigma\varepsilon_n}}$$

where $\varepsilon_1, \ldots, \varepsilon_n$ are dependent standard Gaussian variables with correlations

$$\mathrm{cor}(\varepsilon_i, \varepsilon_j) = e^{-\theta d_{ij}}.$$

Here $\theta > 0$ is a parameter and $d_{ij}$ the geographical distance between the centres of zone $i$ and $j$. You take the distances from the R program that will be put at your disposal. Finally, for $Y$ a Pareto model might be suitable which means that its density function is

$$f(y) = \frac{\alpha/\beta}{(1 + y/\beta)^{1+\alpha}}, \qquad y > 0$$

where $\alpha > 0$ and $\beta > 0$ are parameters.

The model is partially calibrated against the Solvency II set-up, but a couple of things remain. Use $\rho_{ij} = \mathrm{cor}(\varepsilon_i, \varepsilon_j)$ for the correlations in the Solvency II specification of $G$ and determine $\beta$ in the Pareto model so that

$$\Pr\left(Y > \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij} \times (w_i \times \mathrm{SI}_i) \times (w_j \times \mathrm{SI}_j)\right) = 0.5\%.$$

**Main points:** The presentation (45 minutes) should cover

1 How Solvency II calculates the Solvency Capital Requirement for earthquake risk.

2 The stochastic model that imitates losses caused by an eathquake, explain the elements of the model and how they should be interpreted.

3 Discuss the calibration against Solvency II.

4 Explain how the model can be simualted with insurance specifics added.

5 Comparisons between the 99.5% percentiles under the simulations and those returned by Solvency II. Use the two sequences of the sums insured introduced above and vary $\theta$. $\sigma$ and $\alpha$.