5th October, 2022

STK4530

Mandatory assignment 1 of 1

Submission deadline

Thursday 27th October 2022, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) no later than the same day as the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

Problem 1. In this task you are going to develop a Monte Carlo simulator for the Vasicek interest rate model, and eventually use this to price an option on a bond by numerically finding the expected value of the payoff.

Suppose that the short rate $r(t)$ is modelled by the Vasicek dynamics.

a) Show that the limiting probability distribution of r when t goes to infinity is normal, and find its expectation and variance.

b) Express $r(t+1)$ in terms of $r(t)$, and use this to create a simulation algorithm. Choose reasonable parameters in the Vasicek model, and simulate the interest rate for the next 100 days (do at least 10 such simulated trajectories, and time t is measured in days).

c) Based on your algorithm for simulating r, simulate the zero-coupon bond prices $P(t,T)$ for various T. Simulate also the forward rates given by

$$f(t,T) := -\frac{\partial}{\partial T} \ln P(t,T)$$

d) Create a plot in t and x where $T = t + x$, $x \geq 0$, of the bond prices and forward rates. Use your algorithm to create a plot of the evolution of the yield and forward term structure curves.

e) Use your simulator to price a call option on a zero-coupon bond. The bond matures at time T being one month from now, while the option has exercise in two weeks from now. Analyse how sensitive the price is with the number of Monte Carlo simulations in your pricer.

Problem 2. Suppose that the short rate $r(t)$ is modelled by the Vasicek dynamics and let $P(t,T)$ be the price at time $t \leq T$ of a T-bond.

a) Find the dynamics (under Q) of $P(t,T)$, that is, find $dP(t,T)$, for $t \leq T$.

b) Find the dynamics (under Q) of the forward rates, $df(t,T)$, for $t \leq T$.

c) Find the dynamics (under Q) of the forward rates $t \mapsto \bar{f}(t,x)$, $t \geq 0, x \geq 0$, where $\bar{f}(t,x) := f(t,t+x)$. We say that $\bar{f}(t,x)$ is the forward rate at time t with *time to maturity* x, and is the forward rate in the Musiela parametrization.