Non-life insurance mathematics

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About the lecturer

- PhD in risk analysis, Institute for Mathematics, University of Oslo
- 10 years experience from insurance
  - Actuary in DNB Skadeforsikring (current position)
  - Actuary in Gjensidige
  - Actuary in KLP
  - Actuarial consultant in Avenir
- 6 years experience from other sectors (energy, research)
  - Quantitative economic risk assessments (consulting, research)
- Member of Norwegian Actuarial Association
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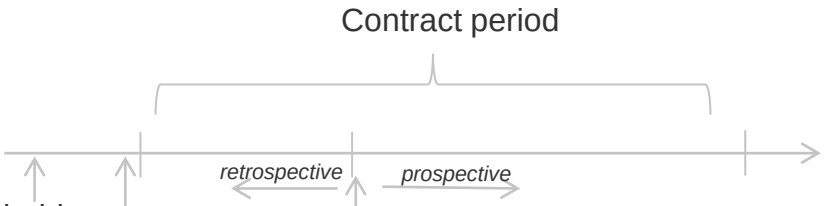
Overview of this session

- Non-life insurance from a financial perspective – result elements and result drivers
- The balance sheet of a non-life insurance company
- Premium Income
- Losses
- Loss ratio
- Costs, cost ratio and combined ratio

Overview

Non-life insurance from a financial perspective:
for a premium an insurance company commits itself to pay a sum if an event has occured



Policy holder signs up for an insurance

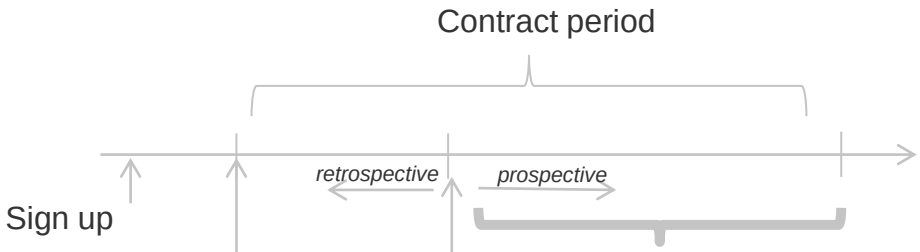
Policy holder pays premium. Insurance company starts to earn premium

During the duration of the policy, some of the premium is earned, some is unearned

- How much premium is earned?
- How much premium is unearned?
- Is the unearned premium sufficient?

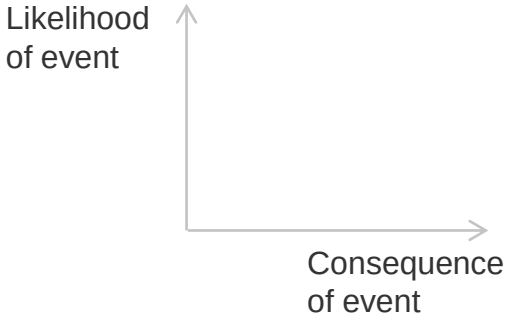
Premium reserve, *prospective*

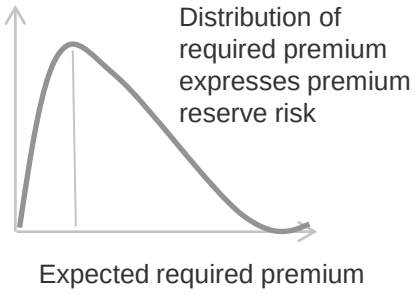
Overview

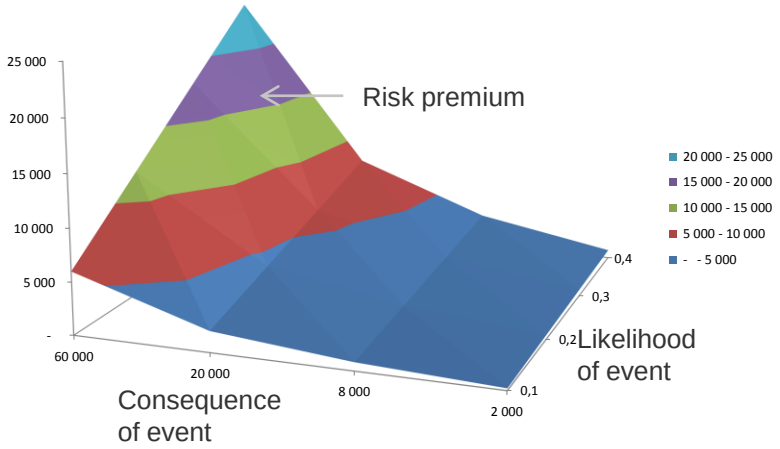


- Is the unearned premium sufficient? } Premium reserve, *prospective*

- Risk premium
  = Likelihood of event * Economic consequence of event
- Risk premium expresses claims cost per policy
- Risk premium is used to price the portfolio
- Since pricing is prospective, we typically want to know how well did we do? (i.e., how uncertain are our risk premium estimates)
- The uncertainty of the risk premium is closely related to the premium reserve risk







Overview

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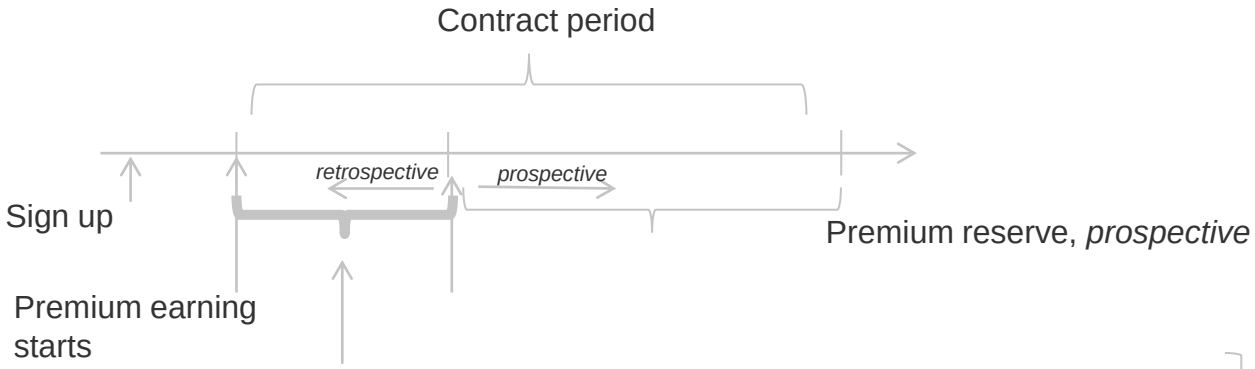
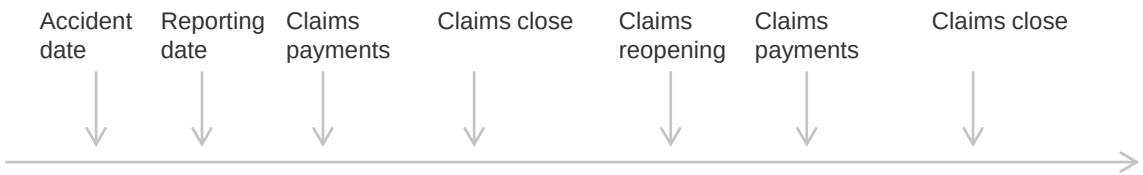
for a premium an insurance company commits itself to pay a sum if an event has occured

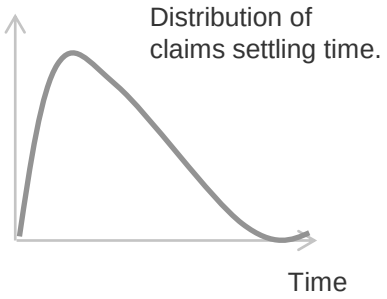


During the duration of the policy, claims might or might not occur:

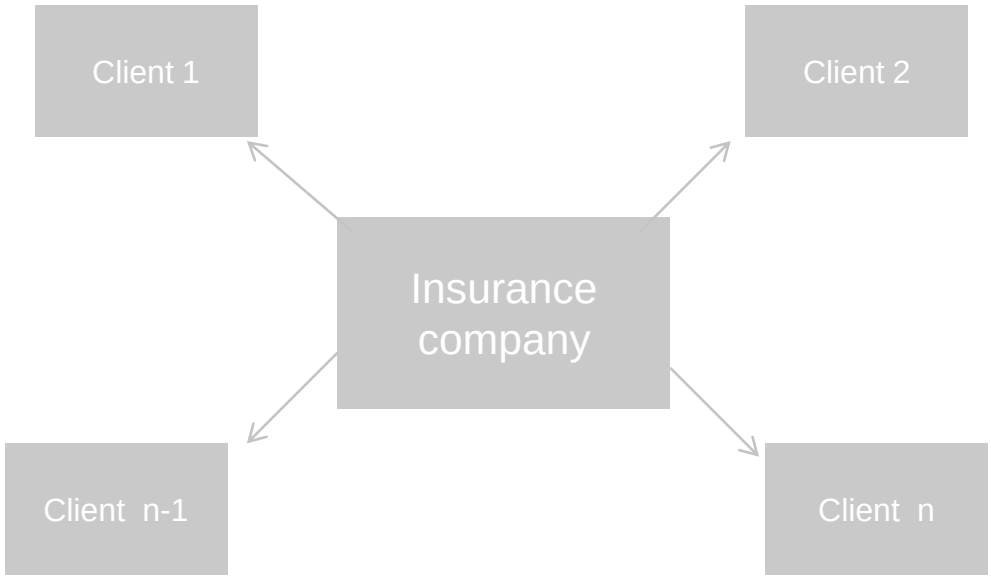
- How do we measure the number and size of unknown claims?
- How do we know if the reserves on known claims are sufficient?

Claims reserve, *retrospective*





Why does it work??



- Economic risk is transferred from the policyholder to the insurer
- Due to the law of large numbers (many almost independent clients), the loss of the insurance company is much more predictable than that of an individual
- Therefore the premium should be based on the expected loss that is transferred from the policyholder to the insurer

Much of the course is about computing this expected loss

...but first some insurance economics

How can the result of an insurance company be decomposed?

Insurance economics in its most basic form:

Result elements:
+ Insurance premium paid by the clients
+ financial income generated by the premium from the clients
- claims paid to the clients
- operational costs of the Insurance company
= result to be distributed among the owners and the authorities

Insurance mathematics is fundamental in insurance economics

The result drivers of insurance economics:

Result elements:	**Result drivers:**
+ Insurance premium	Risk based pricing, reinsurance
+ financial income	International economy for example interest rate level, risk profile for example stocks/no stocks
- claims	risk reducing measures (for example installing burglar alarm), risk selection (client behaviour), change in legislation, weather phenomenons, demographic factors, reinsurance
- operational costs	measures to increase operational efficiency, IT-systems, wage development
= result to be distributed among the owners and the	Tax politics

Insurance economics

- Risk selection: Object risk

……which house is most likely to burn down??

Insurance economics

- Risk selection: subject risk

……..”sloppy” client who is always unlucky….

What is a balance sheet?

- The balance sheet contains result and balance:

The result presents the development in balance between two point in time. The result 2012 shows the value creation in the period between 31.12.11 and 31.12.12.

Result element	**2012**
+ Premium income	100
- Claims	-75
-Operational costs	-20
+ Financial income	5
= Result before tax	10
- tax	-3
=Result after tax	7

What is a balance sheet?

- The balance sheet contains result and balance:

The balance presents the financial state of the company at a given point in time, for example 31.12.12. Equity = the residual between assets and debt. The change in equity between two balance periods = the annual result.

Assets	**31.12.2012**	**Debt and equity**	**31.12.2012**
Financial assets	200	Liabilities	150
Receivables	100	Equity	150
Sum	300	Sum	300

- Questions:
  - How large was the equity 31.12.11?
  - What is there to say about the financial yield?
  - Is the company financially solid?

The result as presented in the annual report

Amounts in 1 000 000 NOK	2012
Written gross premium	1 450
- ceded reinsurance premium	-270
Change in reserve for unearned gross premium	-110
-change in reinsurance share of unearned premium	25
Net premium income	**1 095**
Allocated investment revenue transferred from non-technical balance sheet	**25**
Other insurance related income	**20**
Loss disbursements gross	-870
- Reinsurance share of gross loss disbursements	120
change in gross loss reserve	-200
-change in reinsurance part of gross loss reserve	100
Net loss costs	**-850**
Sales costs	-85
Insurance related operational costs	-200
received provision for ceded reinsurance	45
Insurance related operational costs	**-240**
technical result before additional reserve	**50**
Change in additional reserve	-10
technical result for non-life insurance	**40**
Interest rate income and dvidends on financial assets	40
change in market capitalisation of investments	5
Realized gain and loss of investments	-20
Administration costs associated with investments	-2
Net income from investments	**23**
Allocated investment yield transferred to technical balance sheet	**-25**
Other costs	**-8**
Result non-technical balance sheet	**-10**
result before tax	**30**
tax	-7
result before other result components	**23**
other result components	0
total result	**23**

The annual report presentation (left) and the summarized result (below) are connected:

MNOK	**2 011**
Net premium income	1 095
net claims costs	-850
Net insurance related operational costs	-240
technical result	5
net income from investments	23
additional reserve	-10
other income and costs	12
result before tax	30
tax cost	-7
total result	**30**

The following slides are based on the summarized result

Premium income

Amounts in 1 000 000 NOK	2012
Written gross premium	1 450
- ceded reinsurance premium	-270
Change in reserve for unearned gross premium	-110
-change in reinsurance share of unearned premium	25
Net premium income	**1 095**

• Written premium : Sum of premium for policies commenced (new business or renewals) in the period. The entire premium for the agreement period is included. Thus it is possible to account premium belonging to the ”next” reporting year.

• Change in reserve for unearned premium: This includes accounted written premium in which the agreement period spans the next reporting year.

• Written premium +- change in reserve for unearned premium = gross earned premium

• Ceded reinsurance premium and change in reinsurance share of unearned premium are the reinsurers share of the entries above

Premium income

- The relationship between written premium, unearned premium and paid premium:

	September 1st	November 1st	December 31st	August 31st
Written premium	12 000			
Unearned premium	12 000	10 000	8 000	0
Earned premium	0	2 000	4 000	12 000
Paid premium	0	12 000		
Premium arrears	12 000	0		

- The yearly premium for the entire agreement period is 12 000. The maturity is September 1st. Number of installments is 1.
- The premium is earned with 1/12 per month, i.e., 1000 NOK in the example
- Premium arrears = client receivable

Premium income

- Ceded reinsurance premium – premium to the reinsurer:

Amounts in 1 000 000 NOK	2012
Written gross premium	1 450
- ceded reinsurance premium	-270
Change in reserve for unearned gross premium	-110
-change in reinsurance share of unearned premium	25
Net premium income	**1 095**

- 19% of the written gross premium is ceded to the reinsurer in the example. Why pay reinsurance premium?

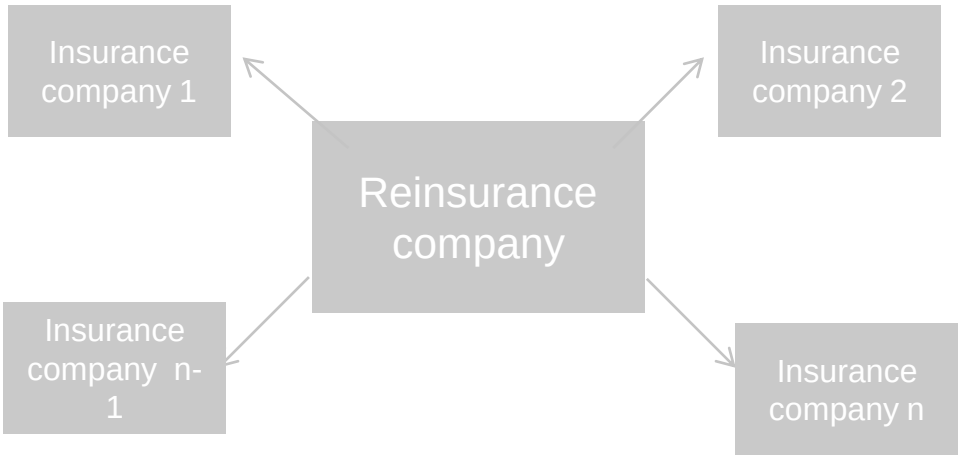
Premium income

- Catastrophe events:
  - The Kielland-accident (1980)
  - Scandinavian Star (1990)
  - The tsunami (2004)
- Natural catastrophes:
  - Dagmar and Berit (2011)
  - 23.000 claims – 1,7 billion
- Large singular claims or events that yield high claims frequency (frost 2010)

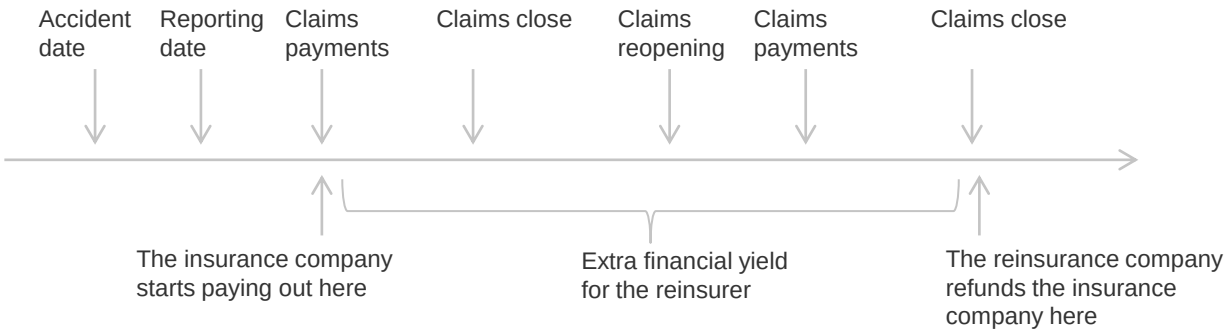
Picture above (text in Norwegian):
Last year (2010) was the coldest since 1941.
Only two years have been colder than 2010 and only parts of Northern Norway escaped the cold.

”Record high losses from villa fires
The insurance companies’ losses from villa fire were sky high at 3.3 billion NOK in the first nine months of the year (2010). This is a 31% increase compared with the same period last year, according to statistics from FNO”

Why does it work??



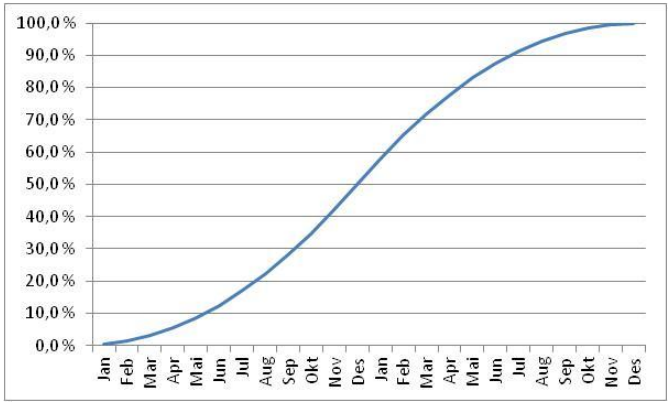
- Economic risk is transferred from the insurance company to the reinsurer
- Due to the law of large numbers, the loss of the reinsurance company is much more predictable than that of a smaller insurance company
- Therefore the premium should be based on the expected loss that
- is transferred from the policyholder to the insurer
- Note that the financial yield of the reinsurer will exceed the financial yield of the insurance company, since the reinsurer does not pay out until the claim is settled



Premium income

- Earning of premium adjustments take 2 years in non-life insurance:

	Maturity pattern	Year 1 Jan	Year 1 Feb	Year 1 Mar	Year 1 Apr	Year 1 May	Year 1 Jun	Year 1 Jul	Year 1 Aug	Year 1 Sep	Year 1 Oct	Year 1 Nov	Year 1 Dec	Year 2 Jan	Year 2 Feb	Year 2 Mar	Year 2 Apr	Year 2 May	Year 2 Jun	Year 2 Jul	Year 2 Aug	Year 2 Sep	Year 2 Oct	Year 2 Nov	Year 2 Dec
January	8 %	0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %											
February	8 %		0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %										
March	8 %			0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %									
April	8 %				0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %								
May	8 %					0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %							
June	8 %						0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %						
July	8 %							0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %					
August	8 %								0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %				
September	8 %									0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %			
October	8 %										0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %		
November	8 %											0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %	
December	8 %												0,3 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,7 %	0,3 %
Sum		0 %	1 %	3 %	6 %	9 %	13 %	17 %	22 %	28 %	35 %	42 %	50 %	58 %	65 %	72 %	78 %	83 %	88 %	91 %	94 %	97 %	99 %	100 %	100 %



- Assumes that premium adjustment is implemented January 1st.
- Assumes that the portfolio's maturity pattern is evenly distributed during the year

Losses (claim incidents)

Amounts in 1 000 000 NOK	2012
Loss disbursements gross	-870
- Reinsurance share of gross loss disbursements	120
change in gross loss reserve	-200
-change in reinsurance part of gross loss reserve	100
Net loss costs	**-850**

• Loss disbursements: payments in the reporting period. Can be incurred losses in the reporting year and incurred losses from previous years.

• Change in gross loss reserve. Consists of reserves for reported losses and incurred losses that are not reported (yet)

• Loss disbursements +- change in loss reserve = gross accrued losses

• Reinsurance share of loss disbursements and change in reinsurance share of gross loss reserve are the reinsurer's share of the entries above

Losses

• The loss reserve is an important measure in a non-life insurance company – 30-40% of the balance

• The company accounts as income the share of the premium income that covers risks during the period (earned premium). The incurred losses are accounted as costs in the same period. Incurred losses that are not disbursed are included in the loss reserve

• Which claims are incurred but not disbursed?

- claims that are reported to the company but not settled (RBNS- reported but not settled)

- claims that are incurred but not reported to the company (IBNR – incured but not reported)

• Examples of what RBNS and IBNR can be?

Losses

- The loss reserve is important in several contexts:

  - it is used to periodize the balance sheet, ie., to assess a ”correct” result

  - it is used by the product managers in pricing and when working with terms

  - it is reported to the reinsurers

  - it is used by the fund managers

  - last but not least important the claims settling unit

.....but the loss reserve is based on judgement and computations. ”Deviations” are called development result (*avviklingsresultat* in Norwegian)

Losses

- Development result:

Cost (-)	-Disbursements for claims incurred previous years
Revenue (+)	+ Reserve from last year for claims incurred previous years (IBNR/RBNS)
Cost (-)	-Reserve from this year for claims incurred previous years (IBNR/RBNS)
Sum =	windup profit (+) / windup loss (-)

Losses

- Development result: can be delopment profit or development loss.

Computation 31.12.12:

Result elements	Incurred 2012	Incurred previous years	Sum
Loss disbursements	-500	-370	-870
Loss reserve 1/1-12		480	480
Loss reserve 31/12-12	-560	-120	-680
Change in loss reserve in 2012	-560	360	-200
Gross accrued losses	-1060	-10	-1070
		windup loss	

Development result example

Car claim: personal injury occurs in 2011

Development	Year	Claims cost estimate
Personal injury occurs	2011	100
Doctors declaration provided	2012	110
Claim is settled and closed	2013	90

	2011	2012	2013
Change in loss reserve	-100	-10	110
Payment			-90
Development result		-10	20

Loss ratio

- Shows how much of the premium income is spent to cover losses

Amounts in 1 000 000 NOK	2012
Written gross premium	1 450
- ceded reinsurance premium	-270
Change in reserve for unearned gross premium	-110
-change in reinsurance share of unearned premium	25
Net premium income	**1 095**

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- Reinsurance share of paid claims gross	120
change in gross claims reserve	-200
-change in reinsurance part of gross claims reserve	100
Net claims costs	**-850**

	Gross	Net
Incurred losses	1070 (-870-200)	850
Earned premium	1340 (1450-110)	1095
Loss ratio	79.9%	77.6%

- What does the difference in loss ratio gross and net tell us?

Loss ratio

- The claim severity and the claim frequency – two key drivers for the loss ratio

$$Claim\ severity = \frac{total\ claim\ amount}{number\ of\ claims} \qquad Claim\ frequency = \frac{number\ of\ claims}{number\ of\ policy\ years}$$

•Number of claims:

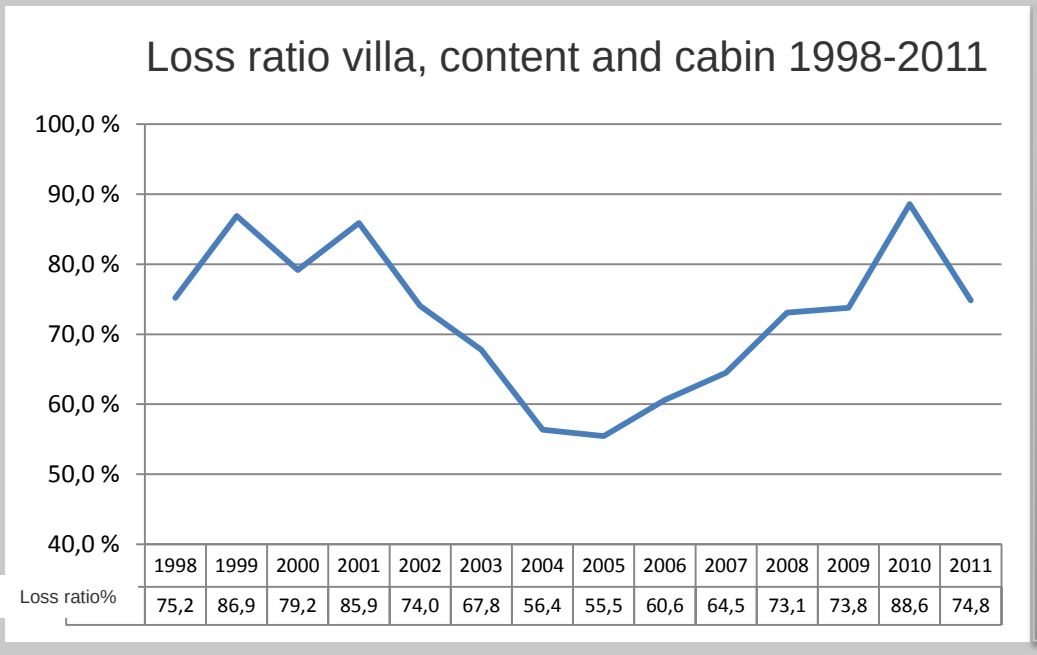
- 1 claim incident may hit several covers. Ex. The incident fire may hit both villa and contents
- We may count the number of incurred claims or the number of reported claims
- 0-claims may or may not be included

•Number of policy years:

- The total amount of time all active policies have been in force in the period
- A company has two clients. 1 was active 1/1 and 1 entered 1/9.
  Number of policy years = 1+8/12 = 1,67

Pure premium = Claim frequency x claim severity

Loss ratio



	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Loss ratio%	75,2	86,9	79,2	85,9	74,0	67,8	56,4	55,5	60,6	64,5	73,1	73,8	88,6	74,8

- Difference of 33 pp in 6 years !?!
- Yearly premium of 9 billion

- The graph presents loss ratio for all insurance companies for villa, content and cabin
- The graph illustrates the delay in the price adjustments and the need for reinsurance
- The graph illustrates the effect of claims frequency (frost 2010)
- Source: FNO.no

Costs

Amounts in 1 000 000 NOK	2012
Sales costs	-85
Insurance related operational costs	-200
received provision for ceded reinsurance	45
Insurance related operational costs	**-240**

- Sales costs: Provisions, sales offices, marketing, back-office sale

- Insurance related operation costs: management, accounting, actuary, house rent, HR, IT etc.Up to 2012 also claims settling costs – NB:these were transferred to claims in 2012

- Received provision reinsurance:
  - Normally it constitutes 20% to 25% of ceded premium..
  - NB: ”Cost income” in the table – why?
  - Why do the companies receive this provision?

Cost ratio (percent)

- Shows how much of the premium income is spent to cover operational costs

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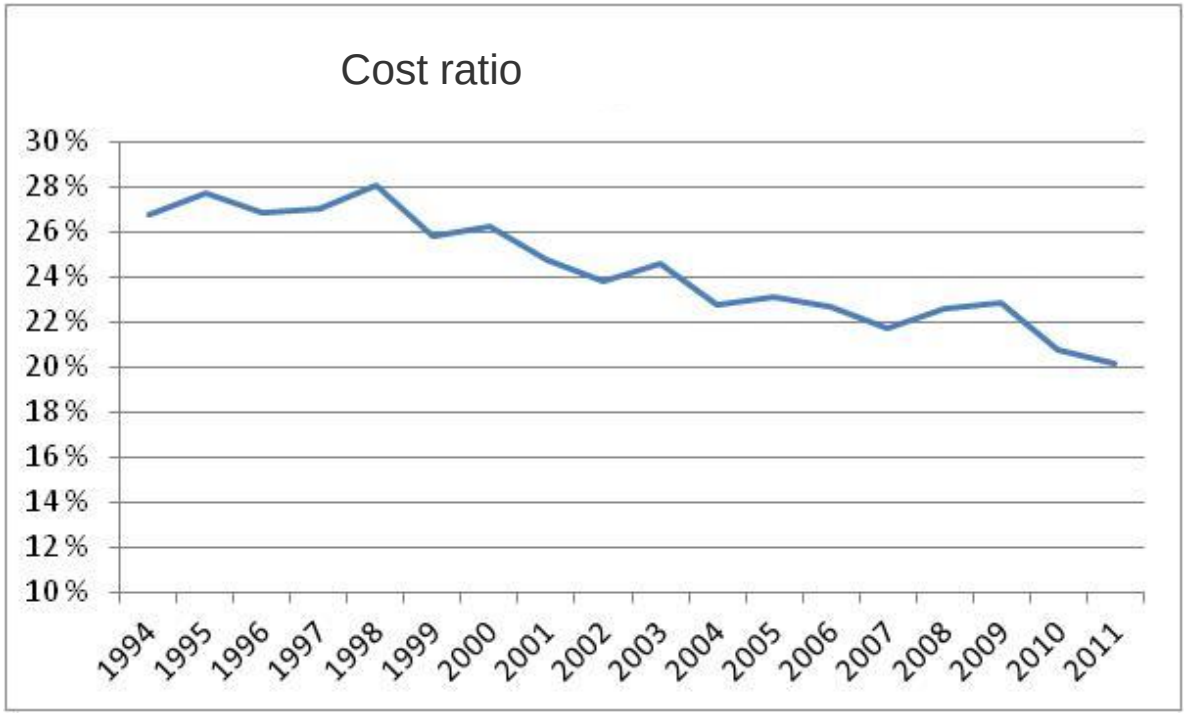
Sales costs	-85
Insurance related operational costs	-200
received provision for ceded reinsurance	45
Insurance related operational costs	**-240**

	Gross	Net
Operational costs	285 (-85-200)	240
Earned premium	1340 (1450-110)	1095
Cost ratio (percent)	21.3%	21.9%

- What does the difference in cost ratio gross and net tell?

Cost ratio (percent)



- What is causing the reduction in cost ratio to 20%?Where are the companies heading?
- Source: fno.no – Results in non-life insurance: includes all non-life insurance companies in Norway

Combined ratio

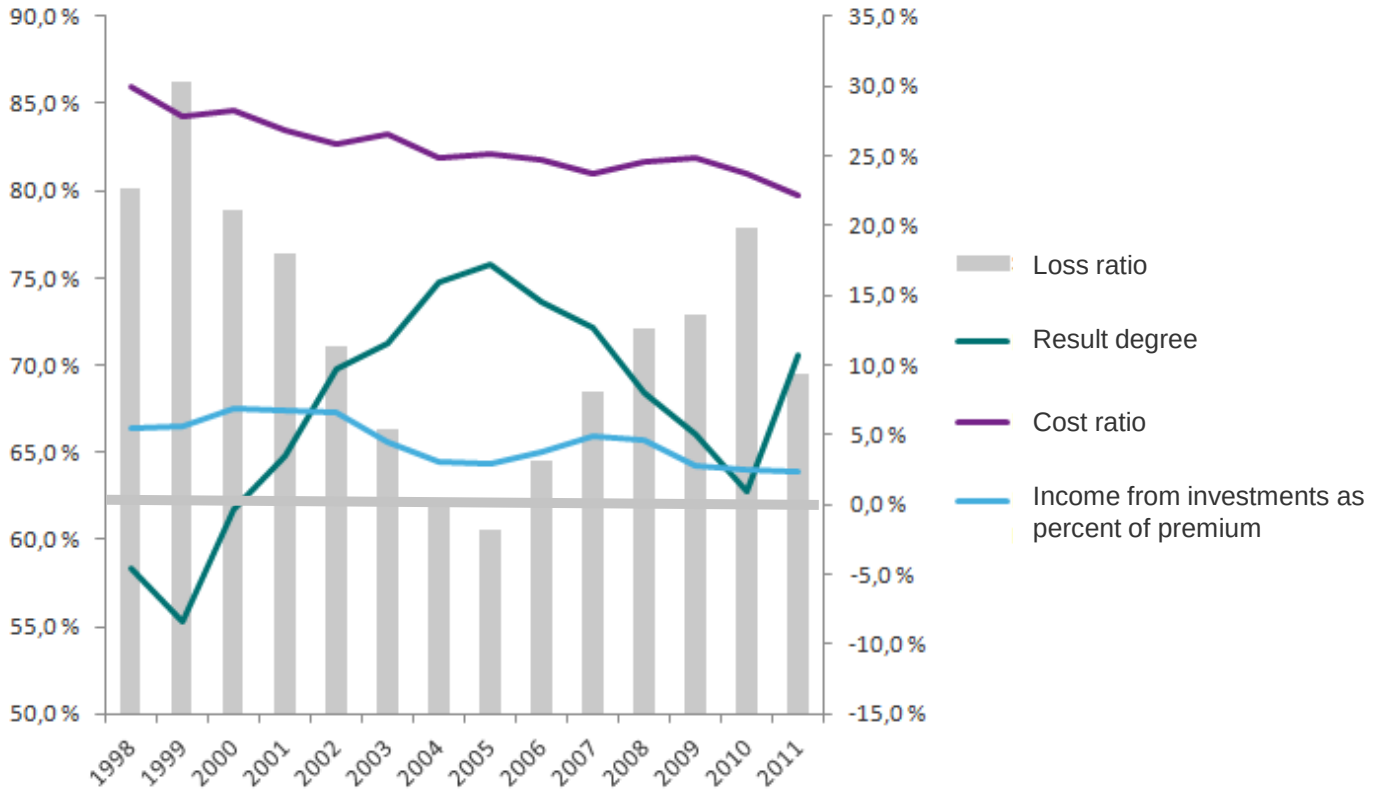
- Shows how much of the premium income that is spent to cover claims and operational costs

	Gross	Net
Loss ratio	79.9%	77.6%
Cost ratio	21.3%	21.9%
Combined ratio	101.1%	99.5%

- Combined ratio above 100 % implies that the insurance operations are not profitable
- What do the combined ratio gross and net express for the example company?
- Long term CR for insurance companies in Norway are between 90% and 95%
- What key ratio is most problematic for the example company?

Key parameters for non-life insurance in Norway

- The graph shows loss ratio (left axis), result degree (total revenue minus total costs, right axis), cost ratio (right axis) andincome from investments in percent of premium (right axis) for the period 1998-2011
- The result degree and the loss ratio vary a lot.
- The loss ratio seems to be the most important driver for profitability in non-life insurance
- The cost ratio and income from investments in percent of premium are decreasing during the period.



Outline of the course

Important issues	Models treated	Curriculum	Duration (in lectures)
What is driving the result of a non-life insurance company?	insurance economics models	Lecture notes	1
How is claim frequency modelled?	Poisson, Compound Poisson and Poisson regression	Section 8.2-4 EB	2
How can claims reserving be modelled?	Chain ladder, Bernhuetter Ferguson, Cape Cod,	Note by Patrick Dahl	2
How can claim size be modelled?	Gamma distribution, log-normal distribution	Chapter 9 EB	2
How are insurance policies priced?	Generalized Linear models, estimation, testing and modelling. CRM models.	Chapter 10 EB	2
Credibility theory	Buhlmann Straub	Chapter 10 EB	1
Reinsurance		Chapter 10 EB	1
Solvency		Chapter 10 EB	1
Repetition			1

Course literature

Curriculum:

Chapter 8(except 8.5),9,10 by Professor Erik Bølviken(UIO)
Note by Patrick Dahl (Stockholm University, Sweden)
Lecture notes by NFH

Additional literature (for deeper understanding and personal development):

Non-life insurance pricing with generalized linear models (2010)
Esbjörn Ohlsson and Björn Johansson

Stochastic claims reserving methods in insurance (2008)
Mario Wüthrich and Michael Merz

Generalized Linear models (1989)
John Nelder and Peter McCullagh