# Non-life insurance mathematics

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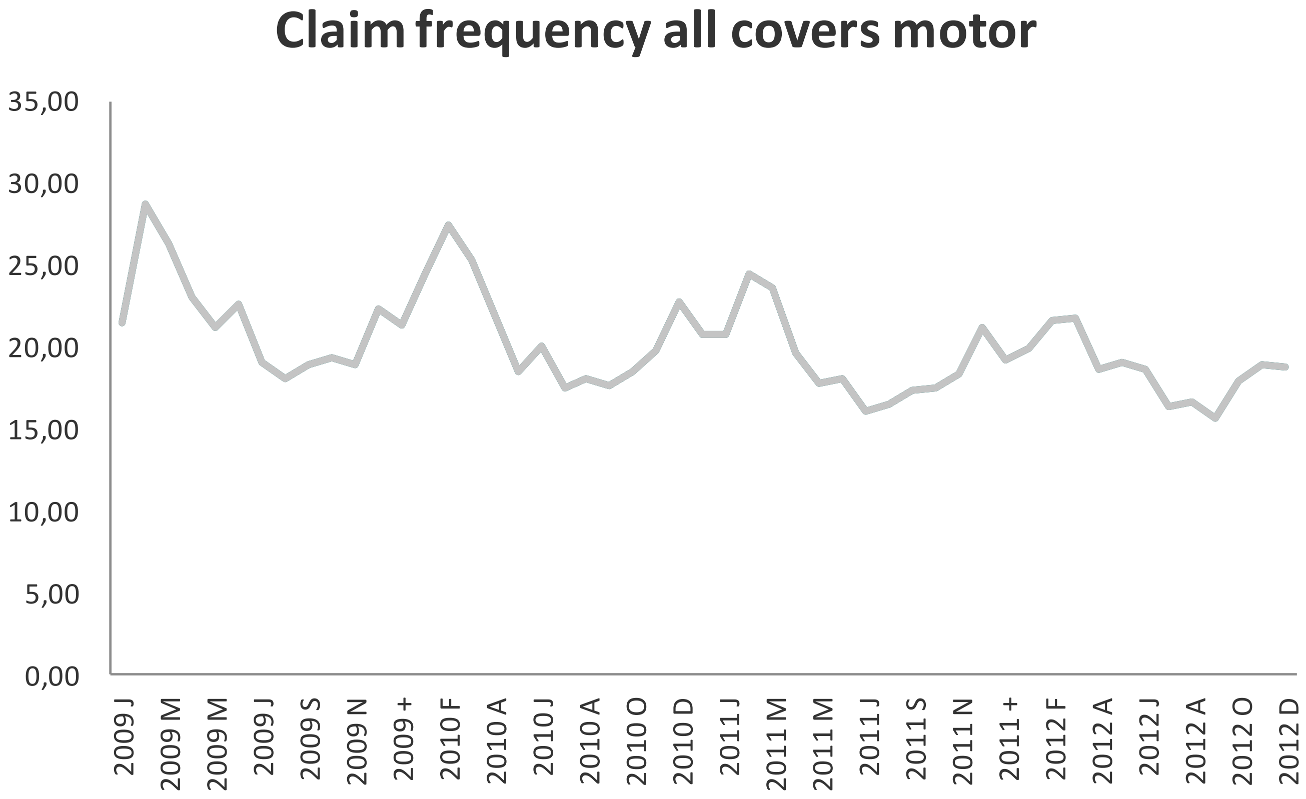
# Key ratios – claim frequency

- The graph shows claim frequency for all covers for motor insurance
- Notice seasonal variations, due to changing weather condition throughout the years



# The model (Section 8.4)

- The idea is to attribute variation in $\mu$ to variations in a set of observable variables $x_1,\ldots,x_v$. Poisson regressjon makes use of relationships of the form

$$\log(\mu) = b_0 + b_1 x_1 + \ldots + b_v x_v \qquad (1.12)$$

- Why $\log(\mu)$ and not $\mu$ itself?
- The expected number of claims is non-negative, where as the predictor on the right of (1.12) can be anything on the real line
- It makes more sense to transform $\mu$ so that the left and right side of (1.12) are more in line with each other.

- Historical data are of the following form

| | | |
|---|---|---|
| $n_1$ | $T_1$ | $x_{11}\ldots x_{1x}$ |
| $n_2$ | $T_2$ | $x_{21}\ldots x_{2x}$ |
| | | |
| $n_n$ | $T_n$ | $x_{n1}\ldots x_{nv}$ |
| Claims | exposure | covariates |

- The coefficients $b_0,\ldots,b_v$ are usually determined by likelihood estimation

# Introduction to reserving

Non-life insurance from a financial perspective:

for a premium an insurance company commits itself to pay a sum if an event has occured

Contract period

*retrospective*

*prospective*

Policy holder signs up for an insurance

Policy holder pays premium. Insurance company starts to earn premium

During the duration of the policy, some of the premium is earned, some is unearned

- How much premium is earned?
- How much premium is unearned?
- Is the unearned premium sufficient?

Premium reserve, *prospective*

During the duration of the policy, claims might or might not occur:

- How do we measure the number and size of unknown claims?
- How do we know if the reserves on known claims are sufficient?

Claims reserve, *retrospective*

Accident date

Reporting date

Claims payments

Claims close

Claims reopening

Claims payments

Claims close

# Imagine you want to build a reserve risk model

There are three effects that influence the best estimat and the uncertainty:

- Payment pattern
- RBNS movements
- Reporting pattern

Up to recently the industry has based model on payment triangles:

| Year | Period + 0 | Period + 1 | Period + 2 | Period + 3 | Period + 4 |
|---|---|---|---|---|---|
| 2008 | 7 008 148 | 25 877 313 | 31 723 256 | 32 718 766 | 33 019 648 |
| 2009 | 30 105 220 | 65 758 082 | 76 744 305 | 79 560 296 | |
| 2010 | 89 181 138 | 171 787 015 | 201 380 709 | | ? |
| 2011 | 109 818 684 | 198 015 728 | | | |
| 2012 | 97 250 541 | | | | |

What will the future payments amount to?

# Overview

| Important issues | Models treated | Curriculum | Duration (in lectures) |
|---|---|---|---|
| What is driving the result of a non-life insurance company? | insurance economics models | Lecture notes | 0,5 |
| How is claim frequency modelled? | Poisson, Compound Poisson and Poisson regression | Section 8.2-4 EB | 1,5 |
| How can claims reserving be modelled? | Chain ladder, Bernhuetter Ferguson, Cape Cod, | Note by Patrick Dahl | 2 |
| How can claim size be modelled? | Gamma distribution, log-normal distribution | Chapter 9 EB | 2 |
| How are insurance policies priced? | Generalized Linear models, estimation, testing and modelling. CRM models. | Chapter 10 EB | 2 |
| Credibility theory | Buhlmann Straub | Chapter 10 EB | 1 |
| Reinsurance | | Chapter 10 EB | 1 |
| Solvency | | Chapter 10 EB | 1 |
| Repetition | | | 1 |

# The ultimate goal for calculating the pure premium is pricing

Pure premium = Claim frequency x claim severity

$$Claim\ severity = \frac{total\ claim\ amount}{number\ of\ claims}$$

$$Claim\ frequency = \frac{number\ of\ claims}{number\ of\ policy\ years}$$

Parametric and non parametric modelling (section 9.2 EB)

The log-normal and Gamma families (section 9.3 EB)

The Pareto families (section 9.4 EB)

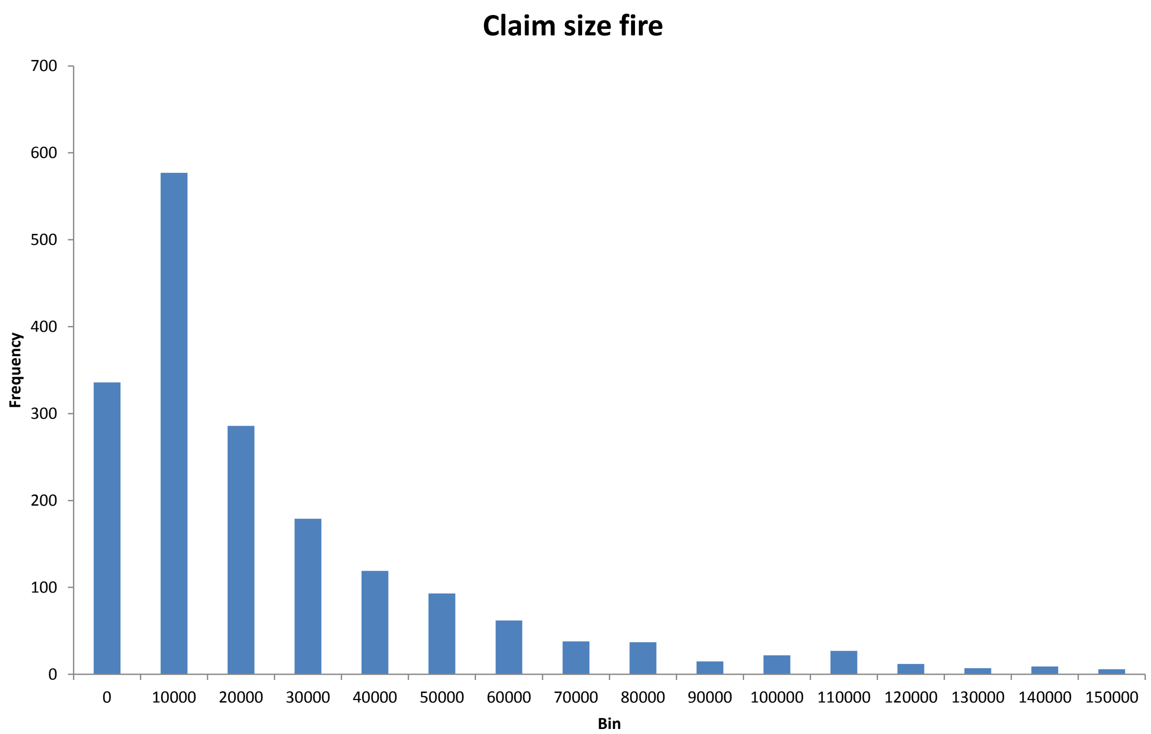
Extreme value methods (section 9.5 EB)

Searching for the model (section 9.6 EB)

# Claim severity modelling is about describing the variation in claim size

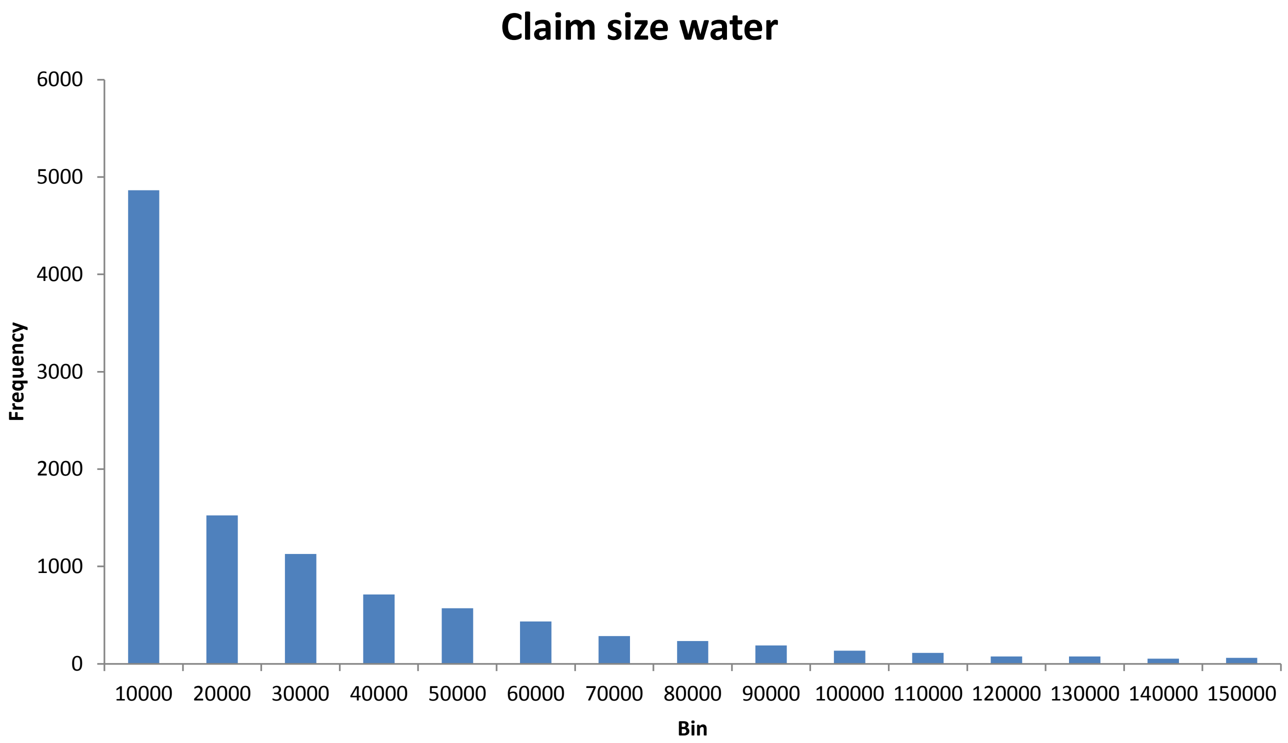
- Non parametric
- Log-normal, Gamma
- The Pareto
- Extreme value
- Searching

- The graph below shows how claim size varies for fire claims for houses
- The graph shows data up to the 88th percentile
- How do we handle «typical claims» ? (claims that occur regurlarly)
- How do we handle large claims? (claims that occur rarely)



# Claim severity modelling is about describing the variation in claim size

- The graph below shows how claim size varies for water claims for houses
- The graph shows data up to the 97th percentile
- The shape of fire claims and water claims seem to be quite different
- What does this suggest about the drivers of fire claims and water claims?
- Any implications for pricing?



# The ultimate goal for calculating the pure premium is pricing

- Claim size modelling can be *parametric* through families of distributions such as the Gamma, log-normal or Pareto with parameters tuned to historical data
- Claim size modelling can also be *non-parametric* where each claim zi of the past is assigned a probability 1/n of re-appearing in the future
- A new claim is then envisaged as a random variable $\hat{z}$ for which

$$\Pr(\hat{Z} = z_i) = \frac{1}{n}, \quad \mathrm{i} = 1, \ldots, \mathrm{n}$$

- This is an entirely proper probability distribution
- It is known as *the empirical distribution* and will be useful in Section 9.5.

Size of claim

Client behavior can affect outcome

- Burglar alarm
- Tidy ship (maintenance etc)
- Garage for the car

Where do we draw the line?

Bad luck

- Electric failure
- Catastrophes
- House fires

Here we sample from the empirical distribution

Here we use special Techniques (section 9.5)

# Example

| | |
|---|---|
| 80 | 45 000 |
| 81 | 45 301 |
| 82 | 48 260 |
| 83 | 50 000 |
| 84 | 52 580 |
| 85 | 56 126 |
| 86 | 60 000 |
| 87 | 64 219 |
| 88 | 69 571 |
| 89 | 74 604 |
| 90 | 80 000 |
| 91 | 85 998 |
| 92 | 95 258 |
| 93 | 100 000 |
| 94 | 112 767 |
| 95 | 134 994 |
| 96 | 159 646 |
| 97 | 200 329 |
| 98 | 286 373 |
| 99 | 500 000 |
| 99,1 | 602 717 |
| 99,2 | 662 378 |
| 99,3 | 810 787 |
| 99,4 | 940 886 |
| 99,5 | 1 386 840 |
| 99,6 | 2 133 580 |
| 99,7 | 2 999 062 |
| 99,8 | 3 612 031 |
| 99,9 | 4 600 301 |
| 100 | 8 876 390 |


Empirical distribution

- The threshold may be set for example at the 99th percentile, i.e., 500 000 NOK for this product
- The threshold is sometimes called the *large claims threshold*

# Scale families of distributions

- All sensible parametric models for claim size are of the form

$$Z = \beta Z_0, \quad \text{where } \beta > 0 \text{ is a parameter}$$

- and Z0 is a standardized random variable corresponding to $\beta = 1$.
- This proportionality os inherited by expectations, standard deviations and percentiles; i.e. if $\xi_0, \sigma_0$ and $q_{0\varepsilon}$ are expectation, standard devation and $\varepsilon$ -percentile for Z0, then the same quantities for Z are

$$\xi = \beta\xi_0, \quad \sigma = \beta\sigma_0 \text{ and } \mathrm{q}_\varepsilon = \beta \mathrm{q}_{0\varepsilon}$$

- The parameter $\beta$ can represent for example the exchange rate.
- The effect of passing from one currency to another does not change the shape of the density function (if the condition above is satisfied)
- In statistics $\beta$ is known as a parameter of scale
- Assume the log-normal model $Z = \exp(\theta + \sigma\varepsilon)$ where $\theta$ and $\sigma$ are parameters and $\varepsilon \sim N(0,1)$. Then $E(Z) = \exp(\theta + (1/2)\sigma^2)$. Assume we rephrase the model as

$$Z = \xi Z_0, \text{ where } Z_0 = \exp(-(1/2)\sigma^2 + \sigma\varepsilon) \text{ and } \xi = \exp(\theta + (1/2)\sigma^2)$$

- Then

$$EZ_0 = E\{\exp(-(1/2)\sigma^2 + \sigma\varepsilon)\} = E\{\exp(-(1/2)\sigma^2 - \theta + \theta + \sigma\varepsilon)\}$$

$$= \exp(-(1/2)\sigma^2 - \theta)E\{\exp(\theta + \sigma\varepsilon)\} = \exp(-(1/2)\sigma^2 - \theta + \theta + (1/2)\sigma^2) = 1$$

# Fitting a scale family

- Models for scale families satisfy

$$\Pr(Z \le z) = \Pr(Z_0 \le z/\beta) \quad \text{or} \quad F(z\,|\,\beta) = F_0(z/\beta)$$

where $F(z\,|\,\beta)$ and $F_0(z/\beta)$ are the distribution functions of Z and $Z_0$.

- Differentiating with respect to z yields the family of density functions

$$f(z\,|\,\beta) = \frac{1}{\beta} f_0\left(\frac{z}{\beta}\right), \; z > 0 \quad \text{where} \quad f_0(z\,|\,\beta) = \frac{dF_0(z)}{dz}$$

- The standard way of fitting such models is through likelihood estimation. If $z_1,\ldots,z_n$ are the historical claims, the criterion becomes

$$L(\beta, f_0) = -n\log(\beta) + \sum_{i=1}^{n} \log\{f_0(z_i/\beta)\},$$

which is to be maximized with respect to $\beta$ and other parameters.

- A useful extension covers situations with censoring.
- Perhaps the situation where the actual loss is only given as some lower bound b is most frequent.
- Example:
    - travel insurance. Expenses by loss of tickets (travel documents) and passport are covered up to 10 000 NOK if the loss is not covered by any of the other clauses.

# Fitting a scale family

- The chance of a claim Z exceeding b is $1-F_0(b/\beta)$, and for nb such events with lower bounds b1,…,bnb the analogous joint probability becomes

$$\{1-F_0(b_1/\beta)\}x...x\{1-F_0(b_{n_b}/\beta)\}.$$

Take the logarithm of this product and add it to the log likelihood of the fully observed claims $z_1,\dots,z_n$. The criterion then becomes

$$L(\beta, f_0) = -n\log(\beta) + \sum_{i=1}^{n}\log\{f_0(z_i/\beta)\} + \sum_{i=1}^{n_b}\log\{f_0(z_i/\beta)\},$$

*complete information* *censoring to the right*

# Shifted distributions

- The distribution of a claim may start at some treshold b instead of the origin.
- Obvious examples are deductibles and re-insurance contracts.
- Models can be constructed by adding b to variables starting at the origin; i.e. $Z = b + \beta Z_0$ where $Z_0$ is a standardized variable as before. Now

$$\Pr(Z \leq z) = \Pr(b + \beta Z_0 \leq z) = \Pr(Z_0 \leq \frac{z-b}{\beta})$$

and differentiation with respect to z yields

$$f(z \mid \beta) = \frac{1}{\beta} f_0(\frac{z-b}{\beta}), \quad z > b$$

which is the density function of random variables with b as a lower limit.

# Skewness as simple description of shape

- A major issue with claim size modelling is asymmetry and the right tail of the distribution. A simple summary is the coefficient of skewness

$$\varsigma = skew(Z) = \frac{\nu^3}{\sigma^3} \quad \text{where } \nu^3 = E(Z-\xi)^3$$

- The numerator is the third order moment. Skewness should not depend on currency and doesn't since

$$skew(Z) = \frac{E(Z-\xi)^3}{\sigma^3} = \frac{E(\beta Z_0 - \beta\xi_0)^3}{(\beta\sigma_0)^3} = \frac{E(Z_0-\xi_0)^3}{\sigma_0^3} = skew(Z_0)$$

- Skewness is often used as a simplified measure of shape
- The standard estimate of the skewness coefficient $\varsigma$ from observations $z_1,\ldots,z_n$ is

$$\hat{\varsigma} = \frac{\hat{\nu}^3}{s^3} \quad \text{where } \hat{\nu}^3 = \frac{1}{n-3+2/n}\sum_{i=1}^{n}(z_i-\bar{z})^3$$

# Non-parametric estimation

- The random variable $\hat{Z}$ that attaches probabilities 1/n to all claims $z_i$ of the past is a possible model for *future* claims.
- Expectation, standard deviation, skewness and percentiles are all closely related to the ordinary sample versions. For example

$$E(\hat{Z}) = \sum_{i=1}^{n} \Pr(\hat{Z} = z_i) z_i = \sum_{i=1}^{n} \frac{1}{n} z_i = \bar{z}.$$

- Furthermore,

$$\operatorname{var}(\hat{Z}) = E(\hat{Z} - E(\hat{Z}))^2 = \sum_{i=1}^{n} \Pr(\hat{Z} = z_i)(z_i - \bar{z})^2 = \sum_{i=1}^{n} \frac{1}{n}(z_i - \bar{z})^2$$

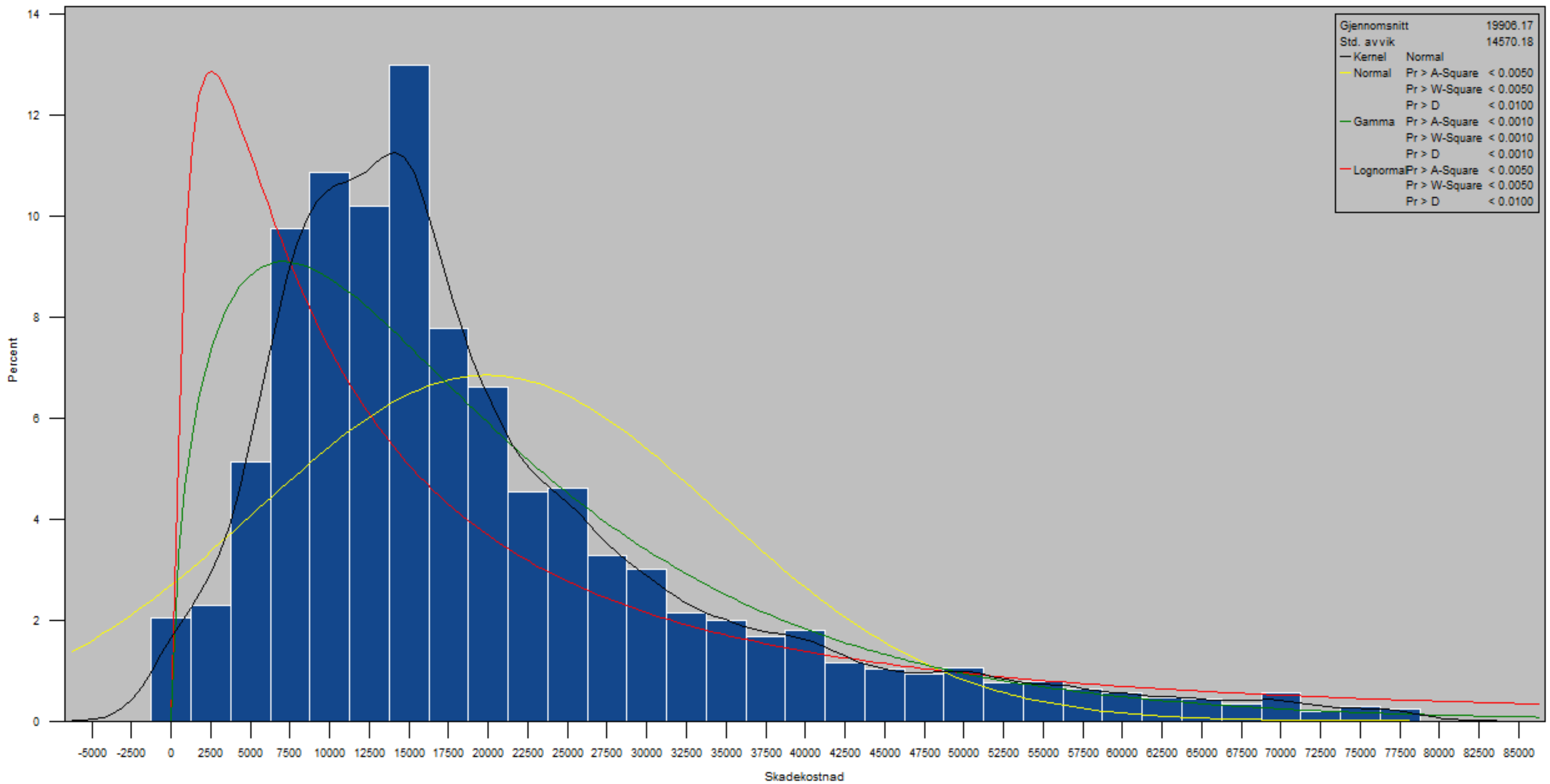
$$\Rightarrow sd(\hat{Z}) = \sqrt{\frac{n-1}{n}} s, \quad \mathrm{s} = \sqrt{\frac{1}{\mathrm{n}-1} \sum_{i=1}^{n} (z_i - \bar{z})^2}$$

- Third order moment and skewness becomes

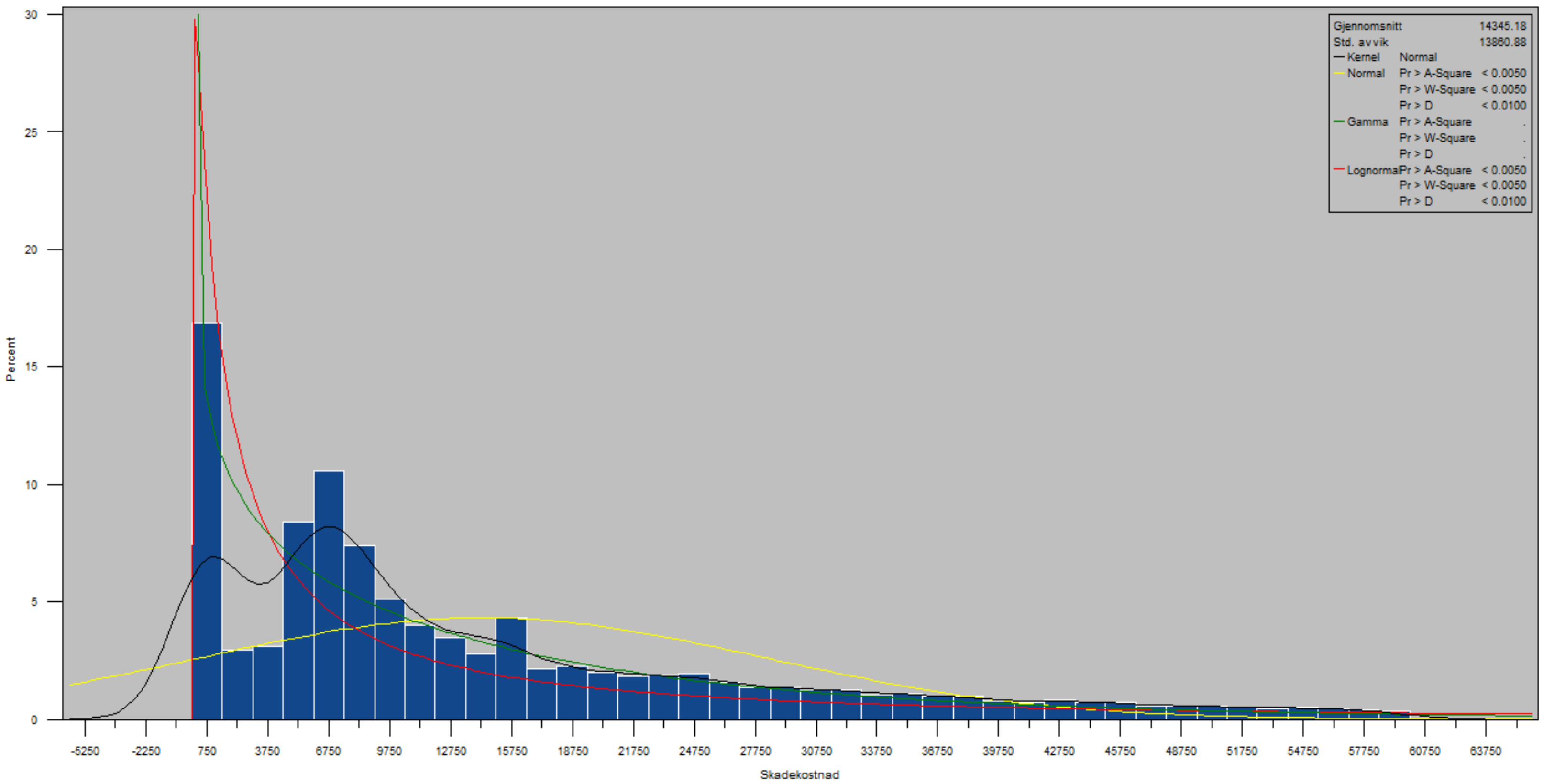
$$\hat{v}_3(\hat{Z}) = \frac{1}{n} \sum_{i=1}^{n} (z_i - \bar{z})^3 \quad \text{and} \quad \operatorname{skew}(\hat{Z}) = \frac{\hat{v}_3(\hat{Z})}{\{sd(\hat{Z})\}^3}$$

- Skewness tends to be small
- No simulated claim can be largeer than what has been observed in the past
- These drawbacks imply underestimation of risk

# TPL


Gjennomsnitt 19906.17
Std. avvik 14570.18
Kernel Normal
Normal Pr > A-Square < 0.0050
Pr > W-Square < 0.0050
Pr > D < 0.0100
Gamma Pr > A-Square < 0.0010
Pr > W-Square < 0.0010
Pr > D < 0.0010
Lognormal Pr > A-Square < 0.0050
Pr > W-Square < 0.0050
Pr > D < 0.0100
Percent
0 2 4 6 8 10 12 14
-5000 -2500 0 2500 5000 7500 10000 12500 15000 17500 20000 22500 25000 27500 30000 32500 35000 37500 40000 42500 45000 47500 50000 52500 55000 57500 60000 62500 65000 67500 70000 72500 75000 77500 80000 82500 85000
Skadekostnad

# Hull


Gjennomsnitt 14345.18
Std. avvik 13860.88
Kernel Normal
Normal Pr > A-Square < 0.0050
Pr > W-Square < 0.0050
Pr > D < 0.0100
Gamma Pr > A-Square .
Pr > W-Square .
Pr > D .
Lognormal Pr > A-Square < 0.0050
Pr > W-Square < 0.0050
Pr > D < 0.0100
Percent
0
5
10
15
20
25
30
-5250
-2250
750
3750
6750
9750
12750
15750
18750
21750
24750
27750
30750
33750
36750
39750
42750
45750
48750
51750
54750
57750
60750
63750
Skadekostnad

# The log-normal family

- A convenient definition of the log-normal model in the present context is as $Z = \xi Z_0$ where $Z_0 = e^{-\sigma^2/2+\sigma\varepsilon}$ for $\varepsilon \sim N(0,1)$
- Mean, standard deviation and skewness are

$$E(Z) = \xi, \quad \mathrm{sd}(Z) = \xi\sqrt{e^{\sigma^2-1}}, \quad skew(Z) = (e^{\sigma^2}+2)\sqrt{e^{\sigma^2-1}}$$

see section 2.4.
- Parameter estimation is usually carried out by noting that logarithms are Gaussian. Thus

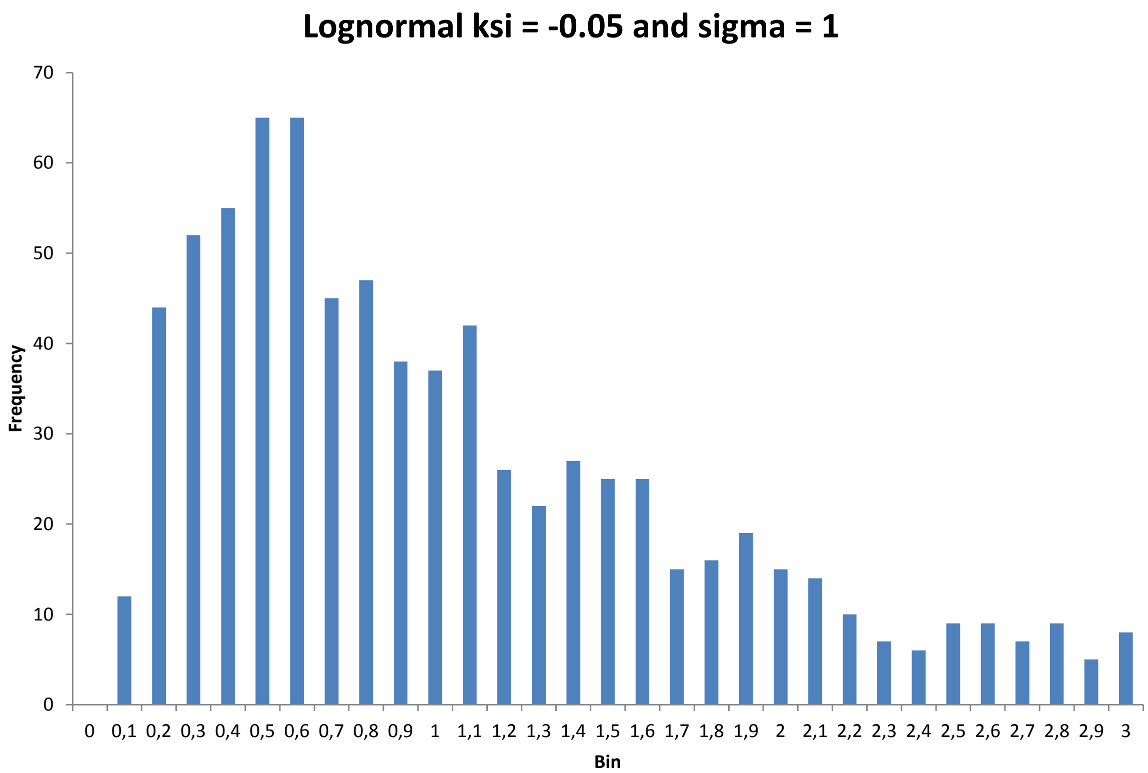
$$Y = \log(Z) = \log(\xi) - 1/2\sigma^2 + \sigma\varepsilon$$

and when the original log-normal observations $z_1,\ldots,z_n$ are transformed to Gaussian ones through $y_1=\log(z_1),\ldots,y_n=\log(z_n)$ with sample mean and variance $\bar{y}$ and $s_y$, the estimates of $\xi$ and $\sigma$ become

$$\log(\hat{\xi}) - 1/2\hat{\sigma}^2 = \bar{y}, \quad \hat{\sigma} = s_y \quad \text{or} \quad \hat{\xi} = e^{s_y^2/2+\bar{y}}, \hat{\sigma} = s_y.$$

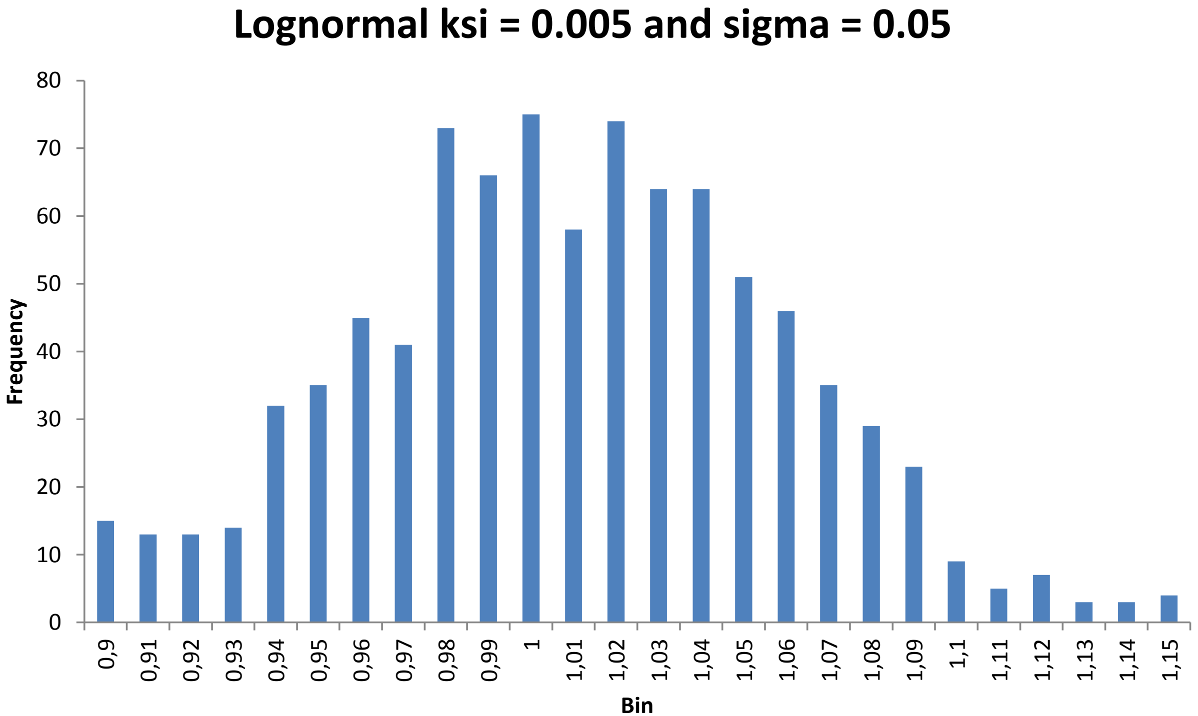
# Log-normal sampling (Algoritm 2.5)

1. Input: $\xi, \sigma$
2. Draw $U^* \sim uniform$ and $\varepsilon^* \leftarrow \Phi^{-1}(U^*)$
3. Return $Z \leftarrow e^{\xi + \sigma\varepsilon^*}$



# The lognormal family

- Different choice of ksi and sigma
- The shape depends heavily on sigma and is highly skewed when sigma is not too close to zero



# The Gamma family

- The Gamma family is an important family for which the density function is

$$f(x) = \frac{(\alpha/\xi)^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\alpha x/\xi}, \quad x > 0, \text{ where } \Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

- It was defined in Section 2.5 as $Z = \xi G$ where $G \sim \text{Gamma}(\alpha)$ is the standard Gamma with mean one and shape alpha. The density of the standard Gamma simplifies to

$$f(x) = \frac{\alpha^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\alpha x}, \quad x > 0, \text{ where } \Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

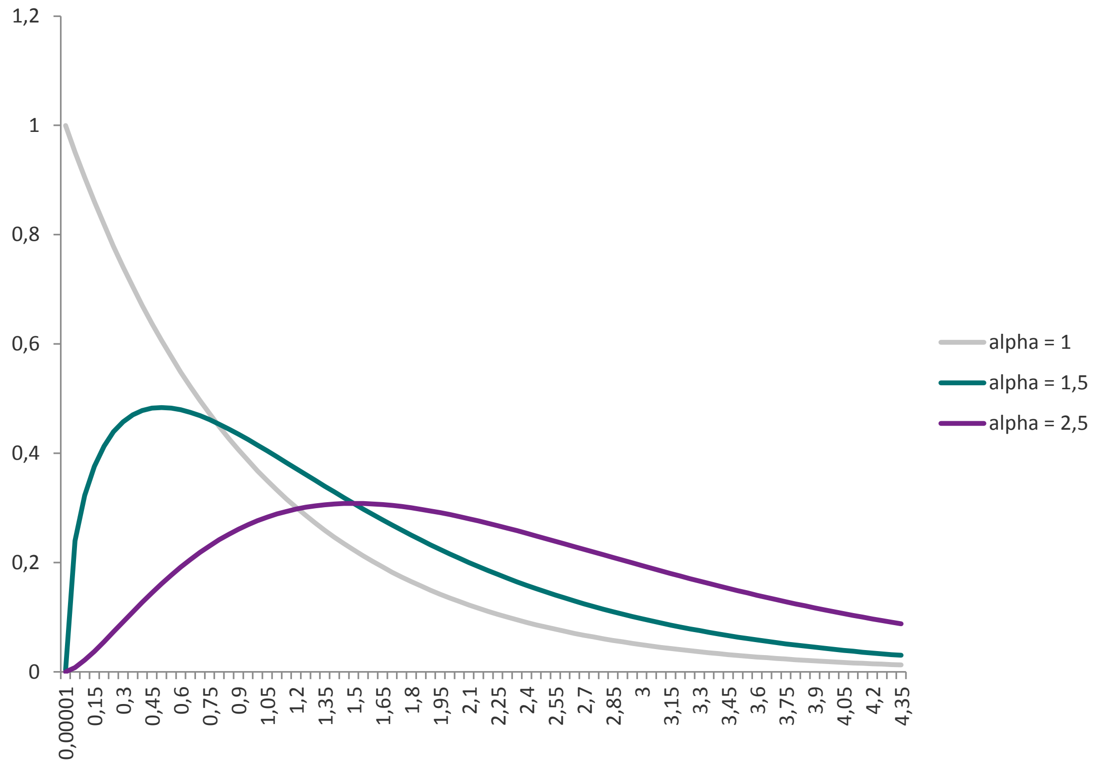
Mean, standard deviation and skewness are

$$E(Z) = \xi, \quad \text{sd}(Z) = \xi/\sqrt{\alpha}, \quad \text{skew}(Z) = 2/\sqrt{\alpha}$$

and there is a convolution property. Suppose $G_1, \ldots, G_n$ are independent with $G_i \sim Gamma(\alpha_i)$. Then

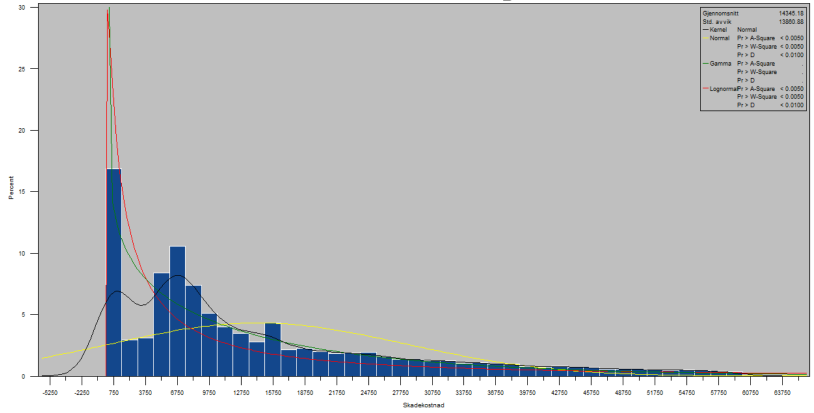
$$\overline{G} \sim Gamma(\alpha_1 + \ldots + \alpha_n) \quad \text{if} \quad \overline{G} = \frac{\alpha_1 G_1 + \ldots + \alpha_n G_n}{\alpha_1 + \ldots + \alpha_n}$$

# Example of Gamma distribution



# Example: car insurance

- Hull coverage (i.e., damages on own vehicle in a collision or other sudden and unforeseen damage)
- Time period for parameter estimation: 2 years
- Covariates:
  - Driving length
  - Car age
  - Region of car owner
  - Tariff class
  - Bonus of insured vehicle
- 2 models are tested and compared – Gamma and lognormal



# Comparisons of Gamma and lognormal

- The models are compared with respect to fit, results, validation of model, type 3 analysis and QQ plots
- Fit: ordinary fit measures are compared
- Results: parameter estimates of the models are compared
- Validation of model: the data material is split in two, independent groups. The model is calibrated (i.e., estimated) on one half and validated on the other half
- Type 3 analysis of effects: Does the fit of the model improve significantly by including the specific variable?

# Comparison of Gamma and lognormal - fit

Gamma fit

| Criterion | Deg. fr. | Verdi | Value/DF |
|---|---|---|---|
| Deviance | 546 | 12 926,1628 | 23,6743 |
| Scaled Deviance | 546 | 669,2070 | 1,2257 |
| Pearson Chi-Square | 546 | 7 390,8283 | 13,5363 |
| Scaled Pearson X2 | 546 | 382,6344 | 0,7008 |
| Log Likelihood | _ | - 5 278,7043 | _ |
| Full Log Likelihood | _ | - 5 278,7043 | _ |
| AIC (smaller is better) | _ | 10 595,4086 | _ |
| AICC (smaller is better) | _ | 10 596,8057 | _ |
| BIC (smaller is better) | _ | 10 677,7747 | _ |

Lognormal fit

| Criterion | Deg. fr. | Verdi | Value/DF |
|---|---|---|---|
| Deviance | 2 814 | 119 523,2128 | 42,4745 |
| Scaled Deviance | 2 814 | 2 838,0000 | 1,0085 |
| Pearson Chi-Square | 2 814 | 119 523,2128 | 42,4745 |
| Scaled Pearson X2 | 2 814 | 2 838,0000 | 1,0085 |
| Log Likelihood | _ | - 7 145,8679 | _ |
| Full Log Likelihood | _ | - 7 145,8679 | _ |
| AIC (smaller is better) | _ | 14 341,7357 | _ |
| AICC (smaller is better) | _ | 14 342,1980 | _ |
| BIC (smaller is better) | _ | 14 490,5071 | _ |

# Comparison of Gamma and lognormal – type 3

Gamma fit

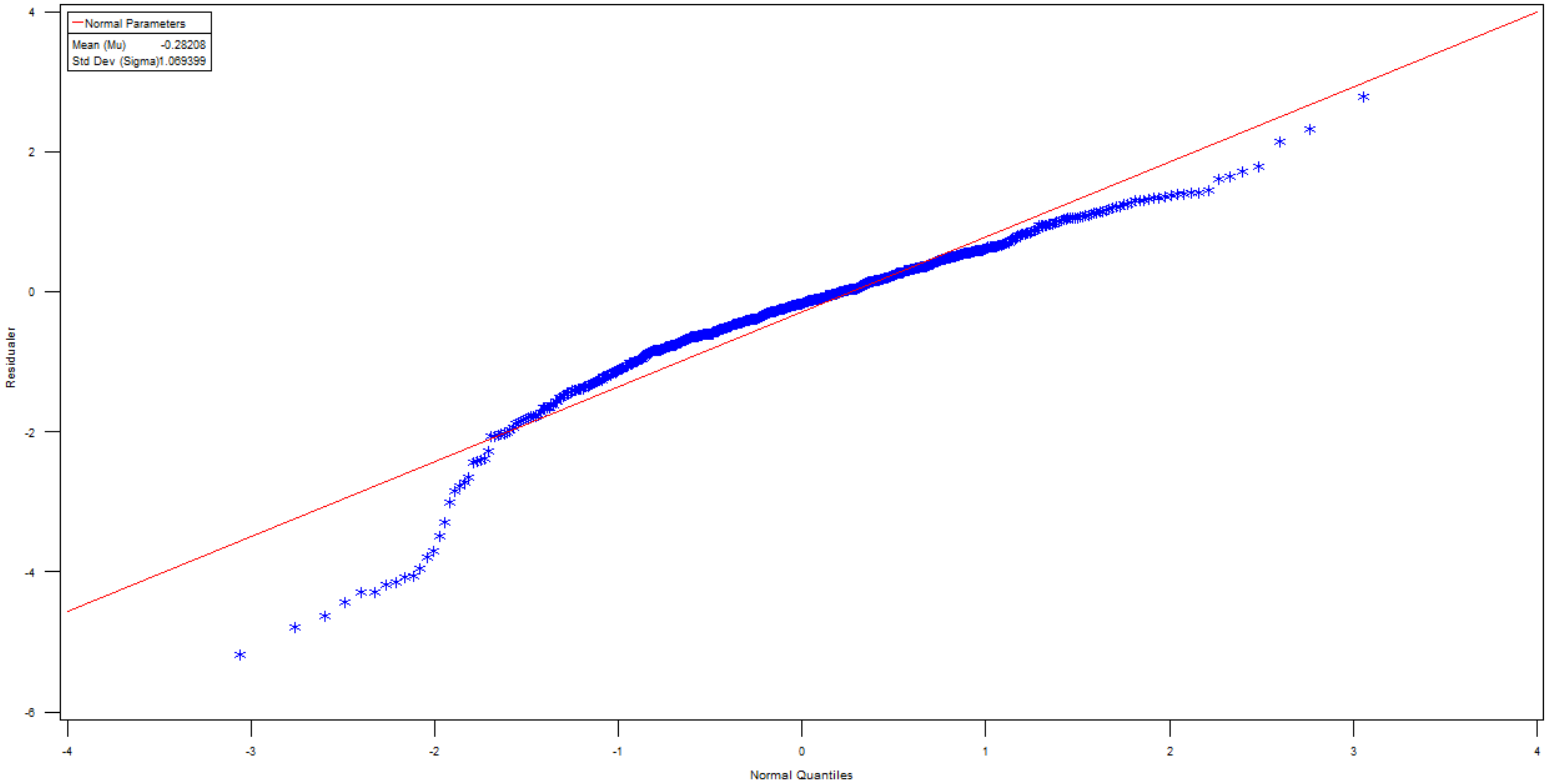
| Source | Deg. fr. | Chi-square | Pr>Chi-sq | Method |
|---|---|---|---|---|
| Tariff class | 5 | 70,75 | <.0001 | LR |
| Bonus | 2 | 19,32 | <.0001 | LR |
| Region | 7 | 20,15 | 0,0053 | LR |
| Car age | 3 | 342,49 | <.0001 | LR |

Lognormal fit

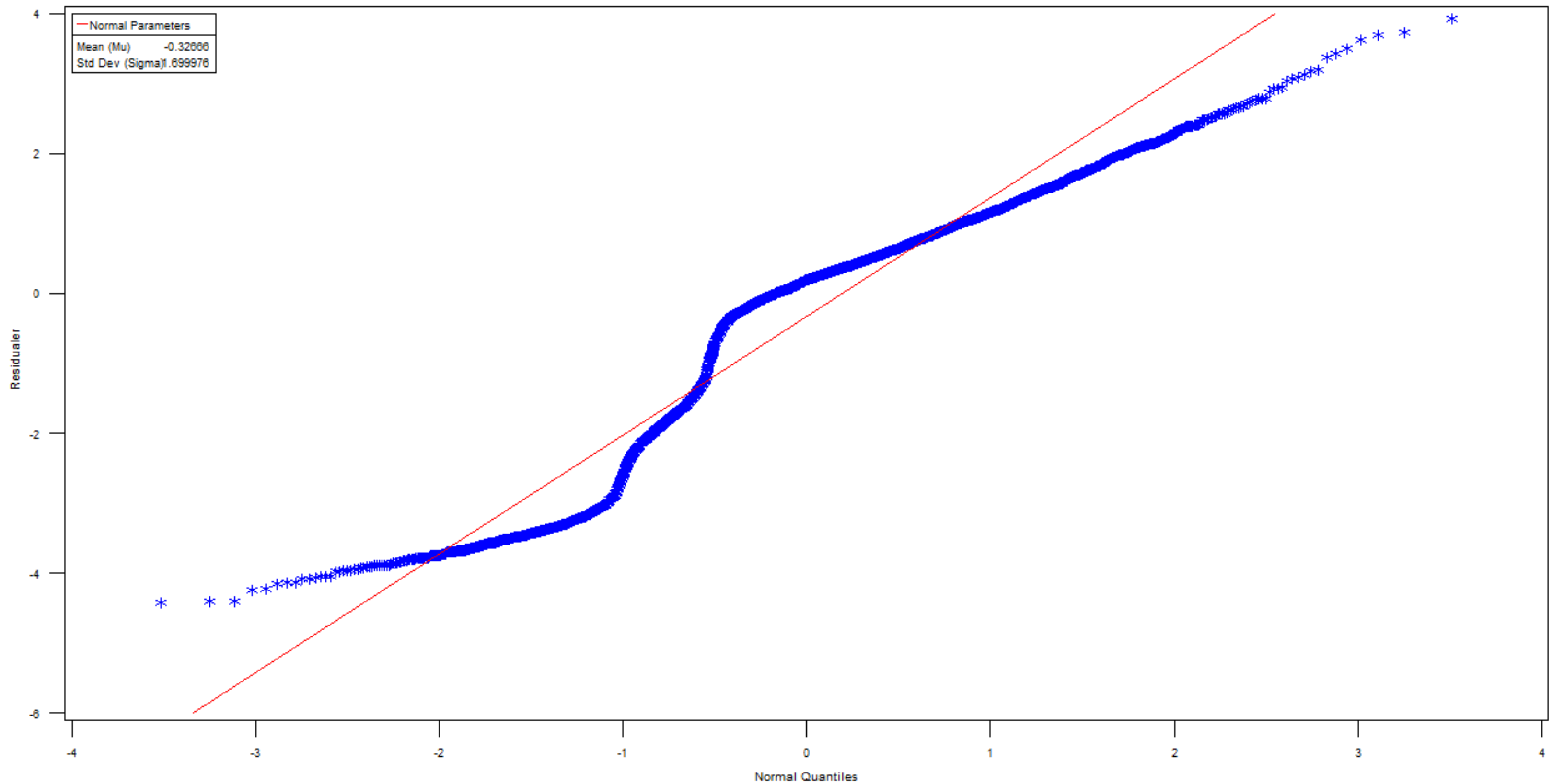
| Source | Deg. fr. | Chi-square | Pr>Chi-sq | Method |
|---|---|---|---|---|
| Tariff class | 5 | 51,75 | <.0001 | LR |
| Bonus | 2 | 177,74 | <.0001 | LR |
| Region | 7 | 48,14 | <.0001 | LR |
| Driving length | 6 | 70,18 | <.0001 | LR |
| Car age | 3 | 939,46 | <.0001 | LR |

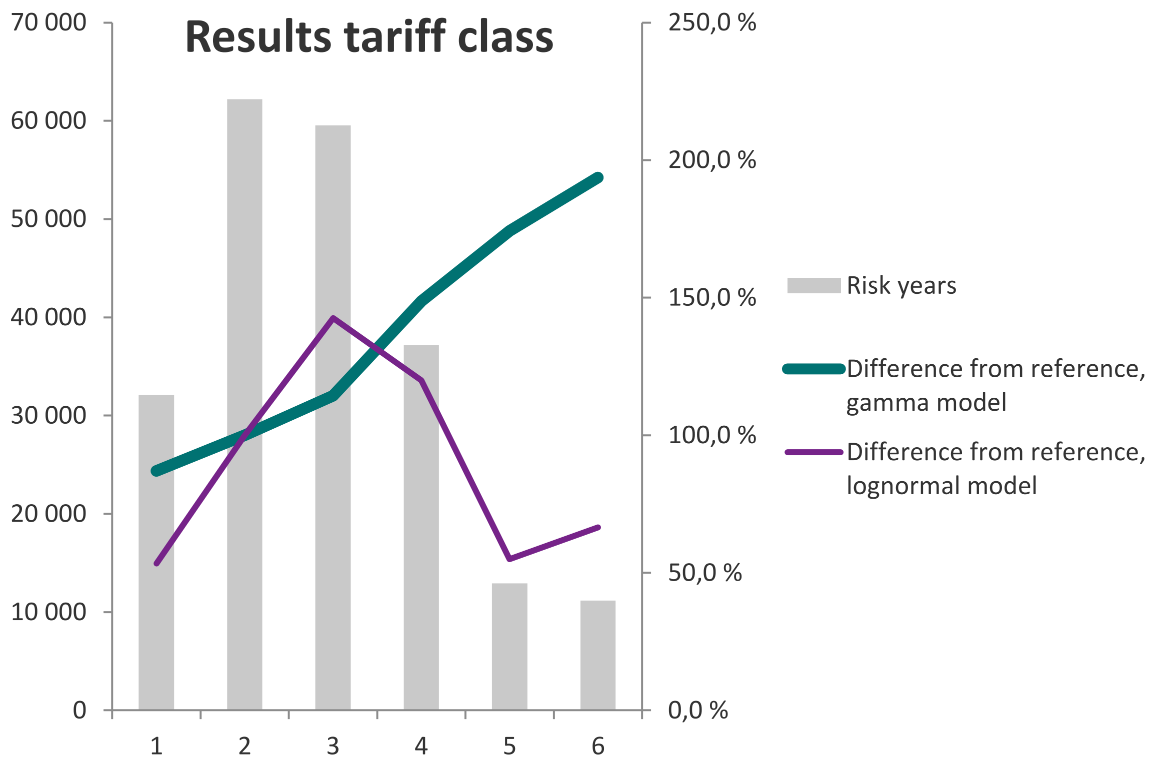
# QQ plot Gamma model

- Non parametric
- Log-normal, Gamma
- The Pareto
- Extreme value
- Searching

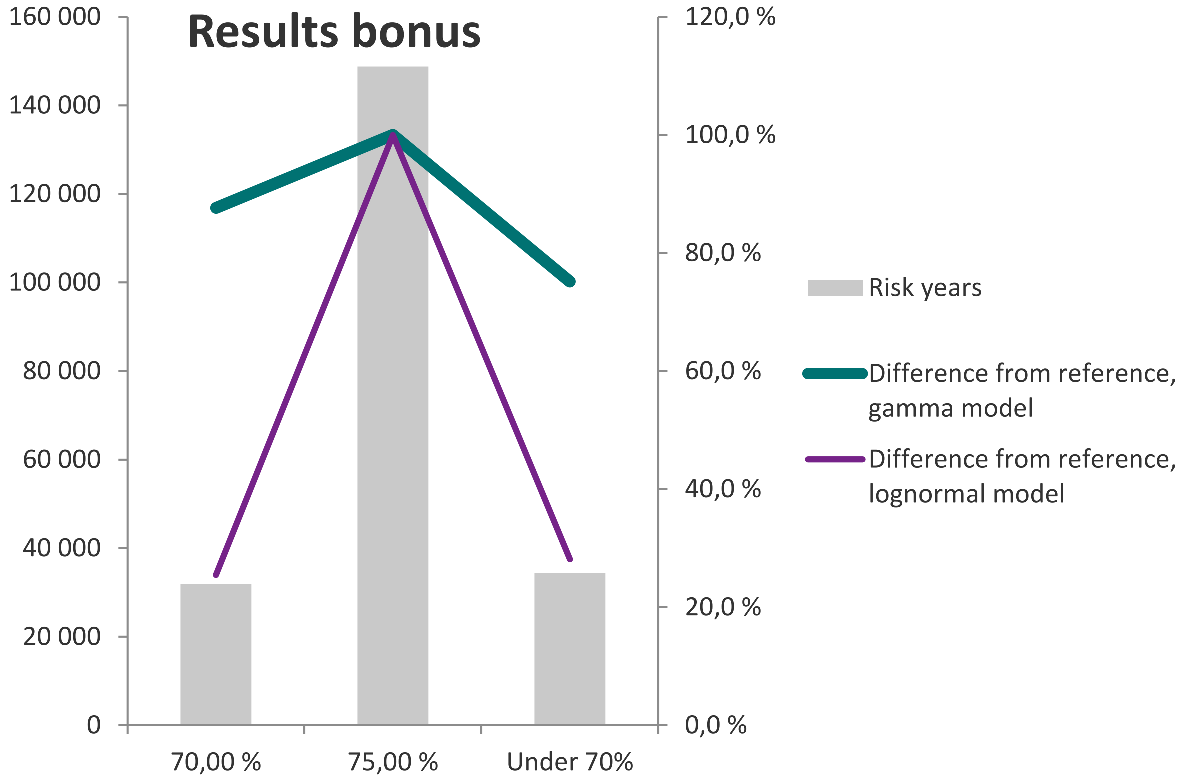


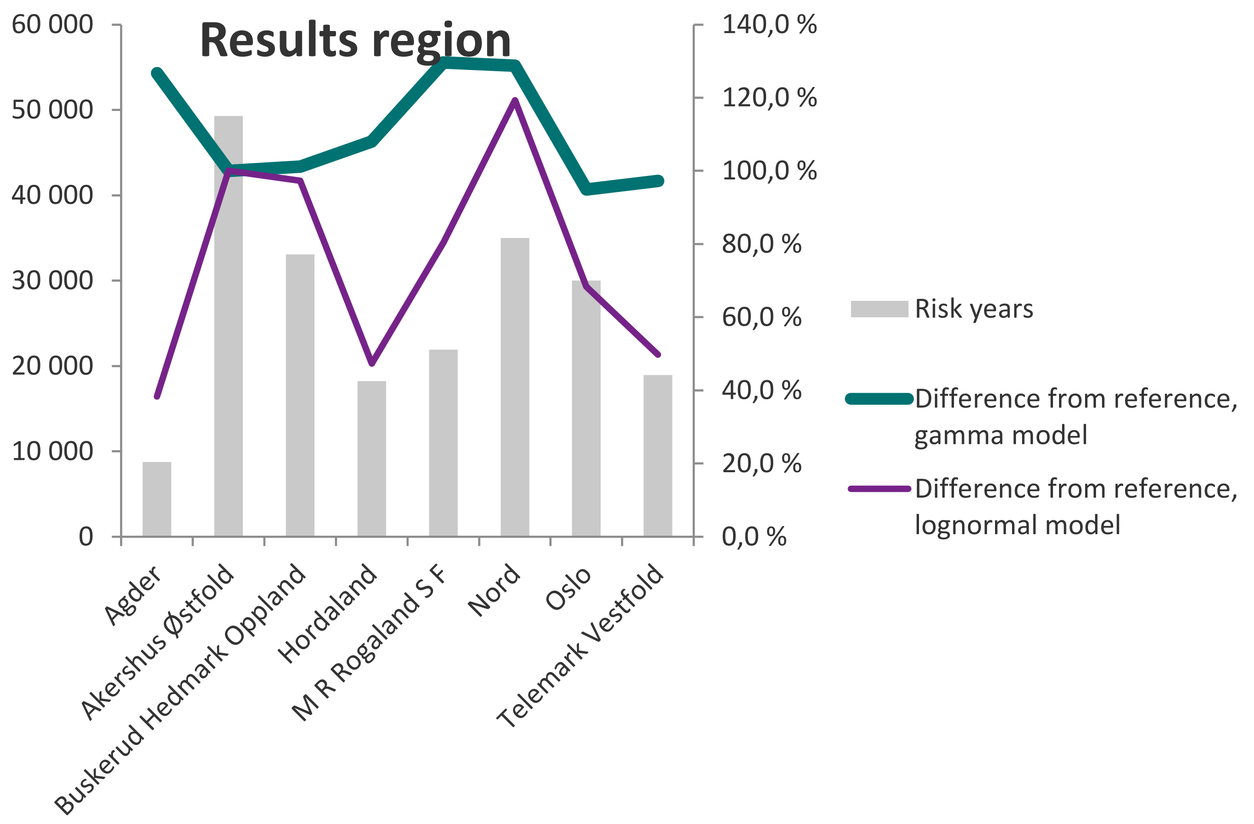
# QQ plot log normal model

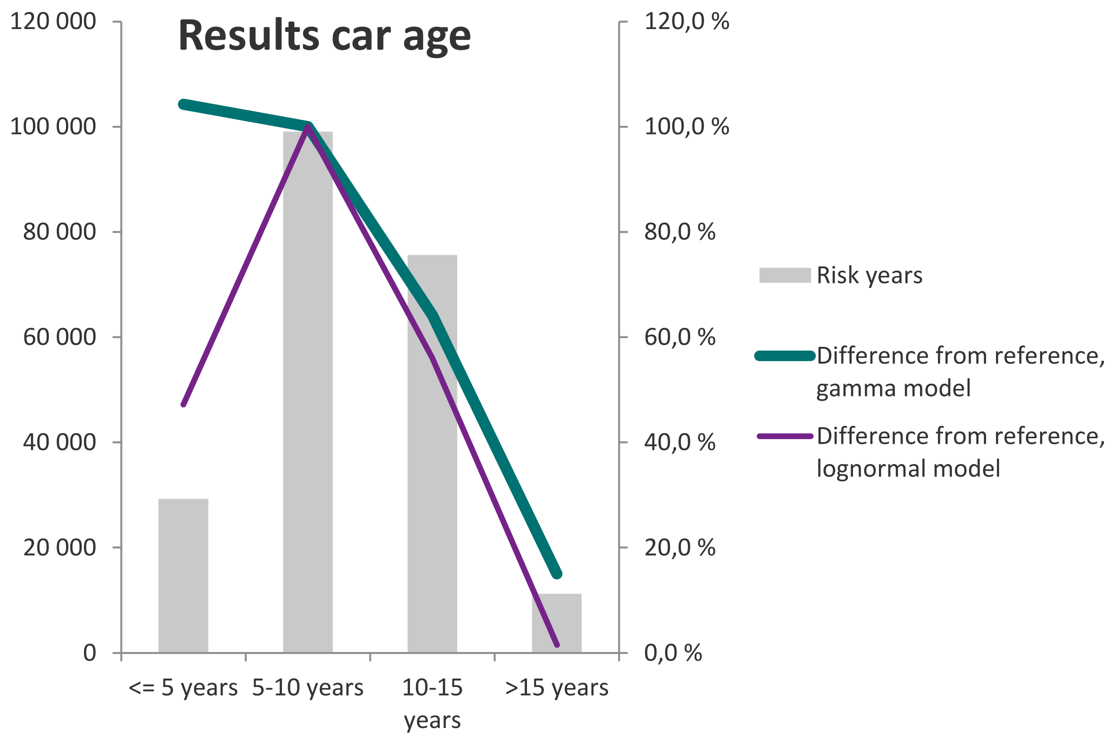



Results tariff class
70 000
60 000
50 000
40 000
30 000
20 000
10 000
0
250,0 %
200,0 %
150,0 %
100,0 %
50,0 %
0,0 %
1
2
3
4
5
6
Risk years
Difference from reference, gamma model
Difference from reference, lognormal model

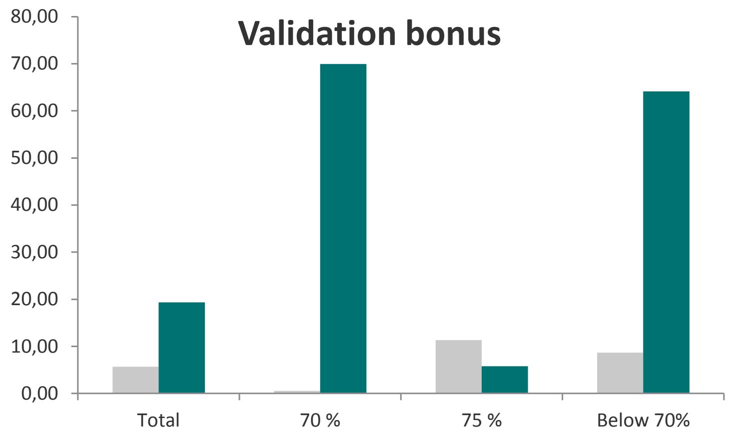
Non parametric
Log-normal, Gamma
The Pareto
Extreme value
Searching

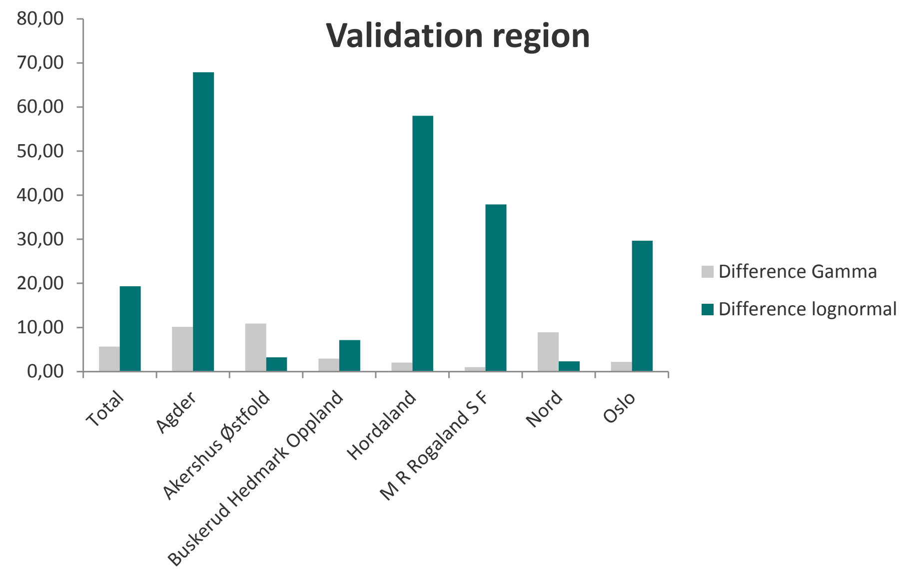


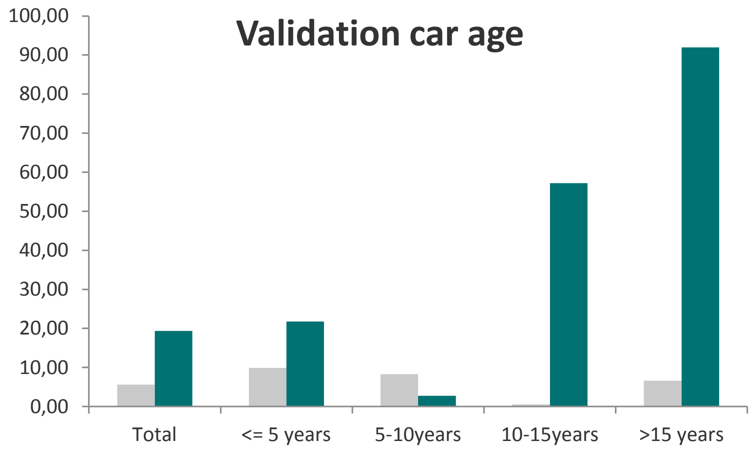

Results region
60 000
50 000
40 000
30 000
20 000
10 000
0
140,0 %
120,0 %
100,0 %
80,0 %
60,0 %
40,0 %
20,0 %
0,0 %
Agder
Akershus Østfold
Buskerud Hedmark Oppland
Hordaland
M R Rogaland S F
Nord
Oslo
Telemark Vestfold
Risk years
Difference from reference, gamma model
Difference from reference, lognormal model

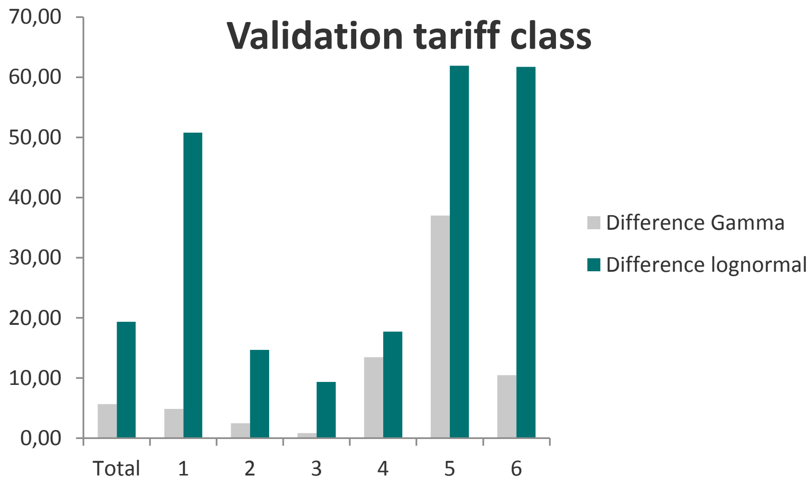

Results car age
120 000
100 000
80 000
60 000
40 000
20 000
0
120,0 %
100,0 %
80,0 %
60,0 %
40,0 %
20,0 %
0,0 %
<= 5 years
5-10 years
10-15 years
>15 years
Risk years
Difference from reference, gamma model
Difference from reference, lognormal model

Non parametric
Log-normal, Gamma
The Pareto
Extreme value
Searching




Validation car age
100,00
90,00
80,00
70,00
60,00
50,00
40,00
30,00
20,00
10,00
0,00
Total
<= 5 years
5-10years
10-15years
>15 years


Validation tariff class
70,00
60,00
50,00
40,00
30,00
20,00
10,00
0,00
Total
1
2
3
4
5
6
Difference Gamma
Difference lognormal

# Conclusions so far

- None of the models seem to be perfect
- Lognormal behaves worst and can be discarded
- Can we do better?
- We try Gamma once more, now exluding the 0 claims (about 17% of the claims)
  - Claims where the policy holder has no guilt (other party is to blame)

# Comparison of Gamma and lognormal - fit

Gamma fit

| Criterion | Deg. fr. | Verdi | Value/DF |
|---|---|---|---|
| Deviance | 546 | 12 926,1628 | 23,6743 |
| Scaled Deviance | 546 | 669,2070 | 1,2257 |
| Pearson Chi-Square | 546 | 7 390,8283 | 13,5363 |
| Scaled Pearson X2 | 546 | 382,6344 | 0,7008 |
| Log Likelihood | _ | - 5 278,7043 | _ |
| Full Log Likelihood | _ | - 5 278,7043 | _ |
| AIC (smaller is better) | _ | 10 595,4086 | _ |
| AICC (smaller is better) | _ | 10 596,8057 | _ |
| BIC (smaller is better) | _ | 10 677,7747 | _ |

Gamma without zero claims fit

| Criterion | Deg. fr. | Verdi | Value/DF |
|---|---|---|---|
| Deviance | 494 | 968,9122 | 1,9614 |
| Scaled Deviance | 494 | 546,4377 | 1,1061 |
| Pearson Chi-Square | 494 | 949,1305 | 1,9213 |
| Scaled Pearson X2 | 494 | 535,2814 | 1,0836 |
| Log Likelihood | _ | - 5 399,8298 | _ |
| Full Log Likelihood | _ | - 5 399,8298 | _ |
| AIC (smaller is better) | _ | 10 837,6596 | _ |
| AICC (smaller is better) | _ | 10 839,2043 | _ |
| BIC (smaller is better) | _ | 10 918,1877 | _ |

# Comparison of Gamma and lognormal – type 3

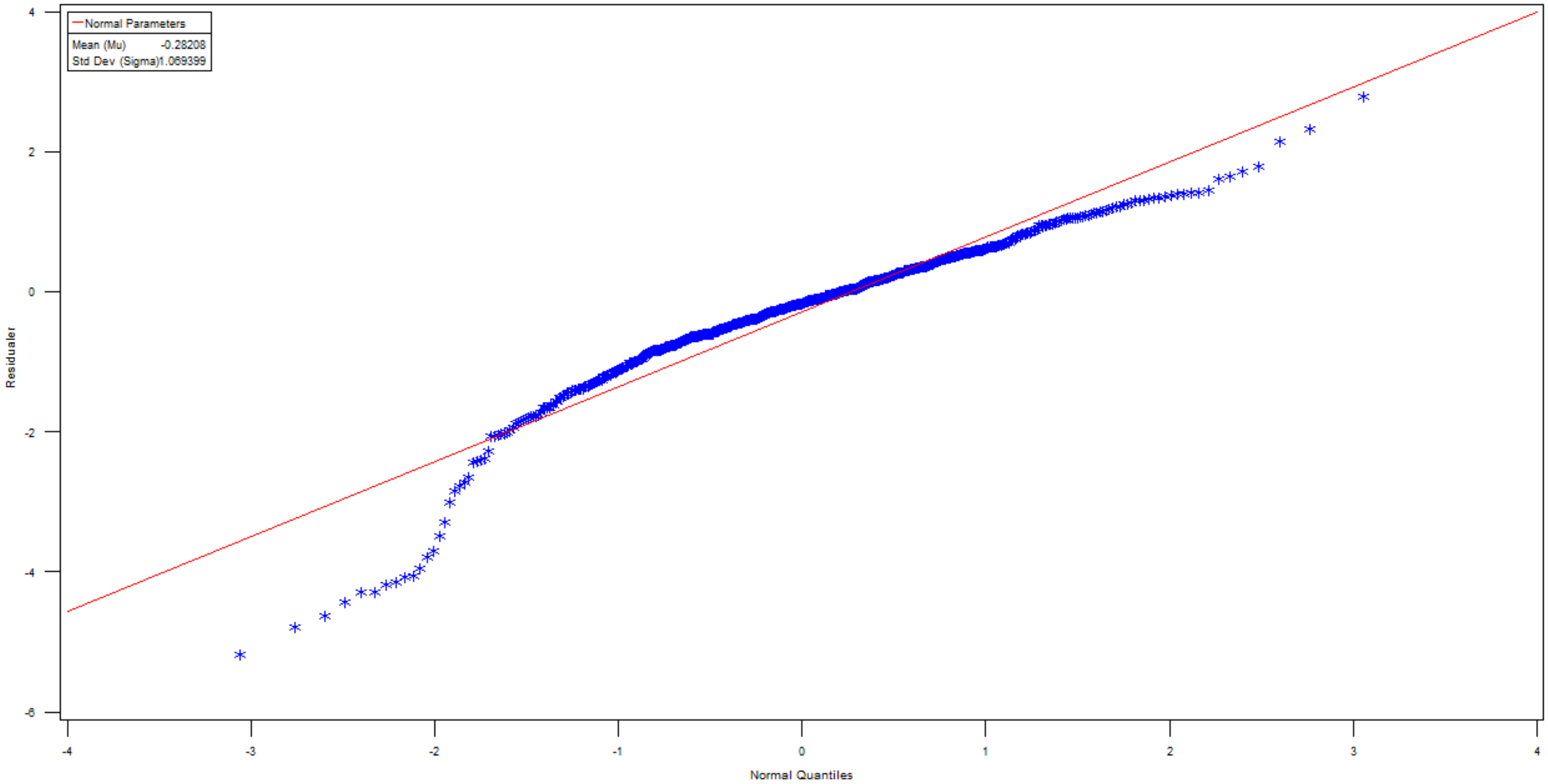
Gamma fit

| Source | Deg. fr. | Chi-square | Pr>Chi-sq | Method |
|---|---|---|---|---|
| Tariff class | 5 | 70,75 | <.0001 | LR |
| Bonus | 2 | 19,32 | <.0001 | LR |
| Region | 7 | 20,15 | 0,0053 | LR |
| Car age | 3 | 342,49 | <.0001 | LR |

Gamma without zero claims fit

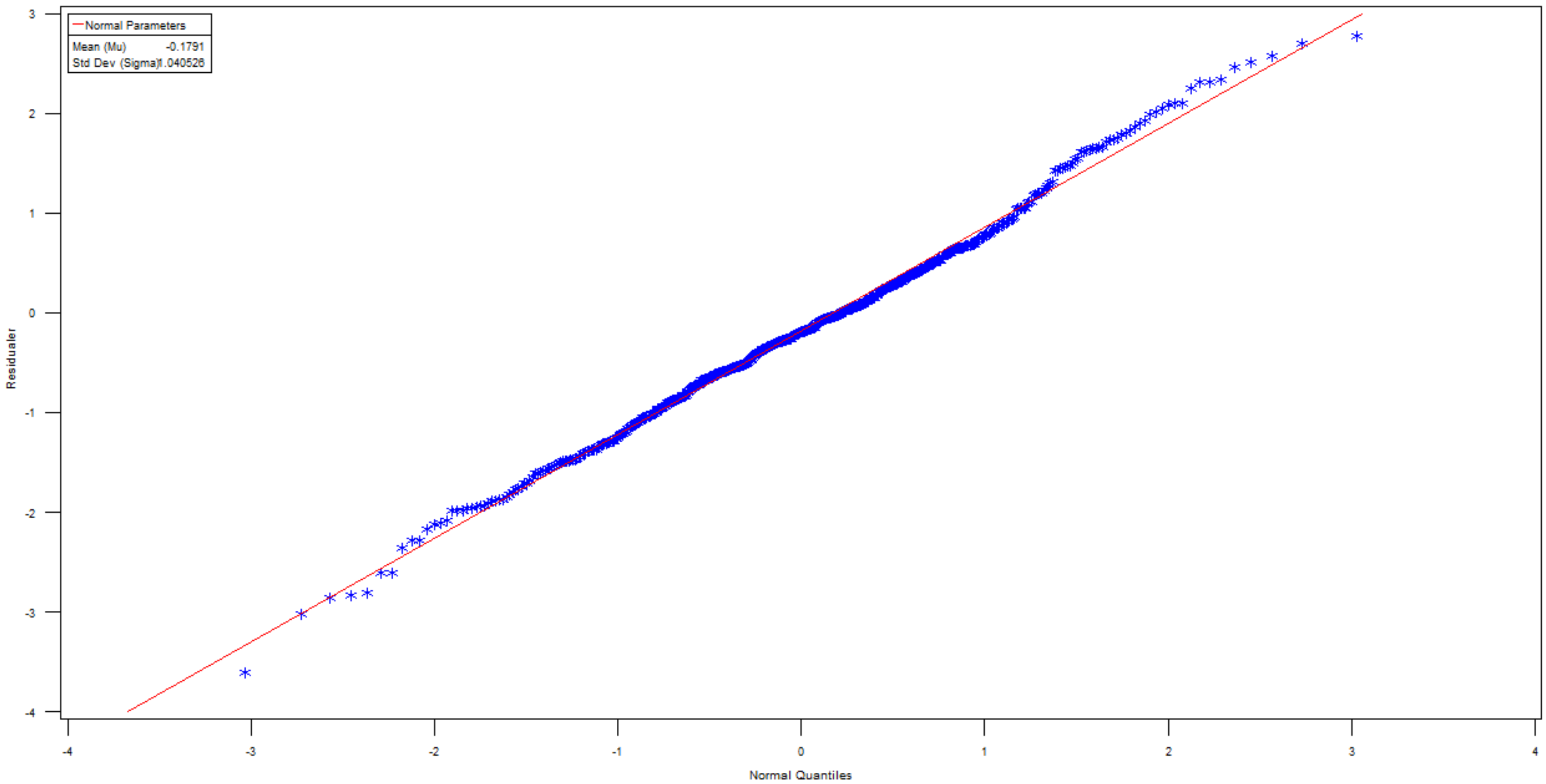
| Source | Deg. fr. | Chi-square | Pr>Chi-sq | Method |
|---|---|---|---|---|
| BandCode1 | 5 | 101,22 | <.0001 | LR |
| CurrNCD_Cd | 2 | 43,04 | <.0001 | LR |
| KundeFylkeNavn | 7 | 48,08 | <.0001 | LR |
| Side1Verdi6 | 3 | 70,76 | <.0001 | LR |

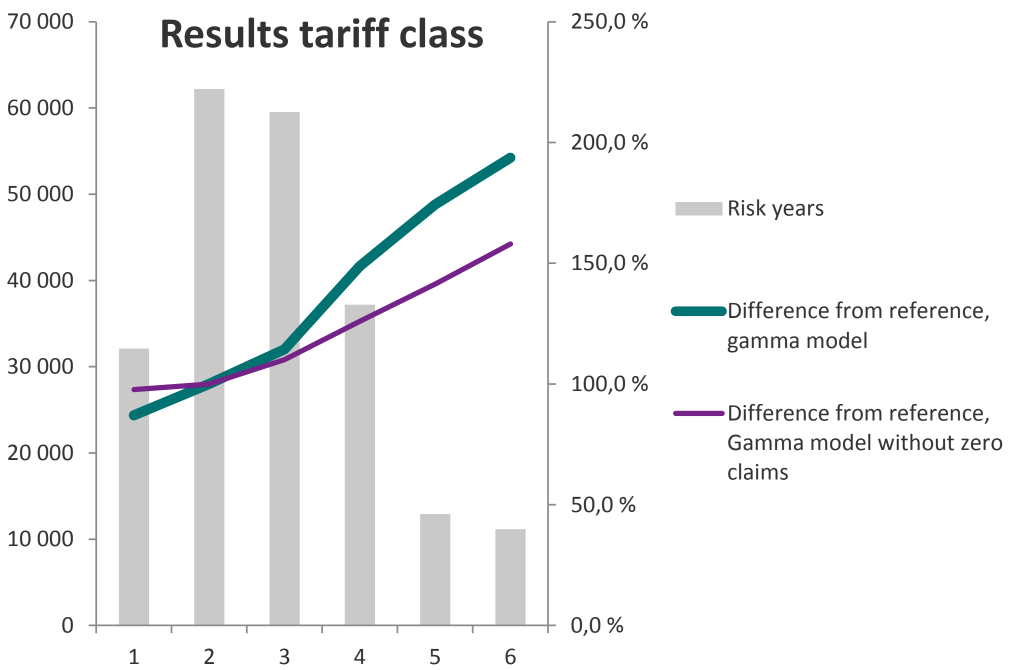
# QQ plot Gamma



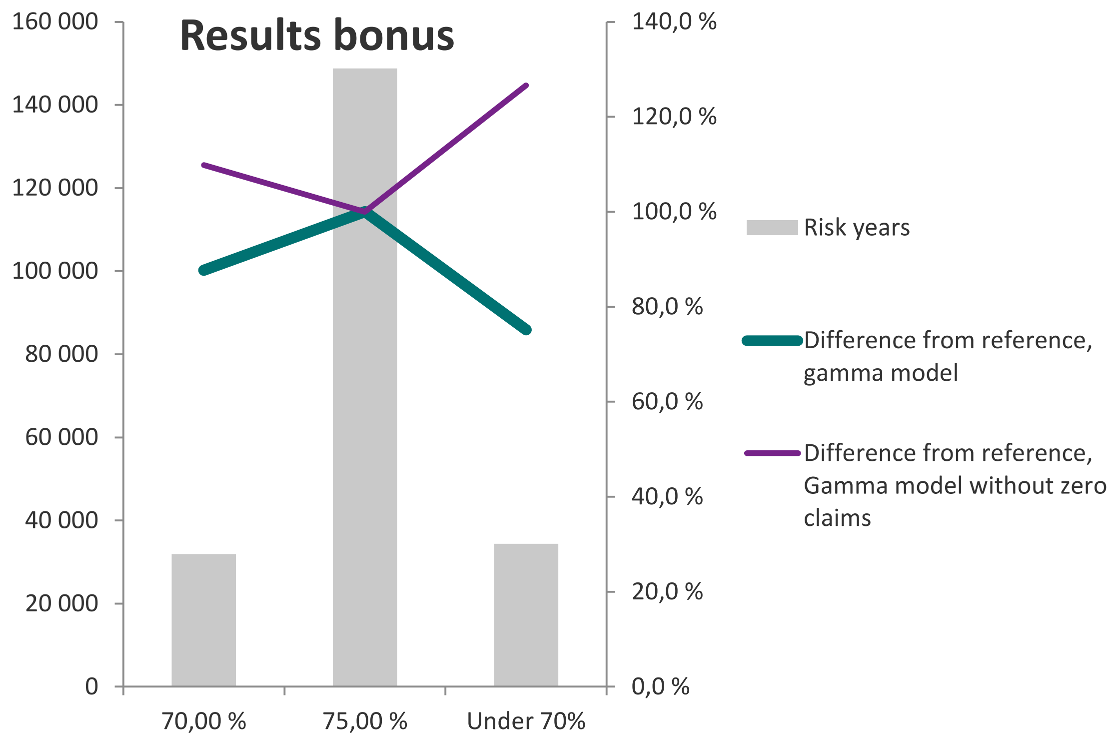
# QQ plot Gamma model without zero claims



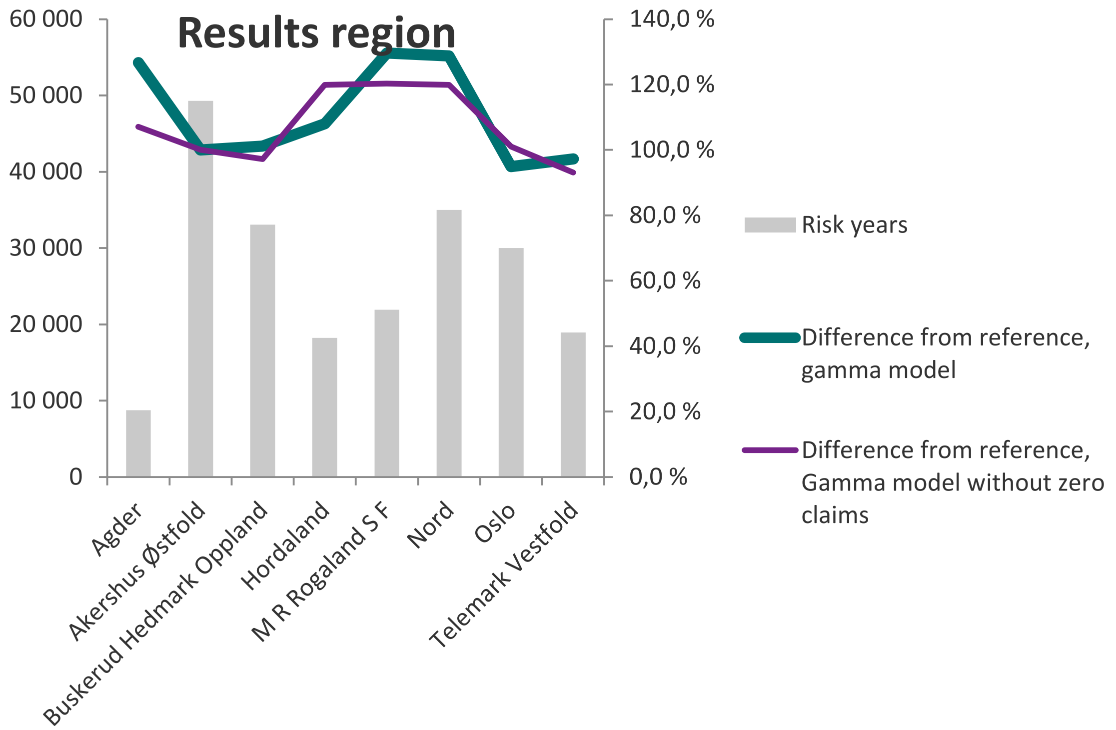


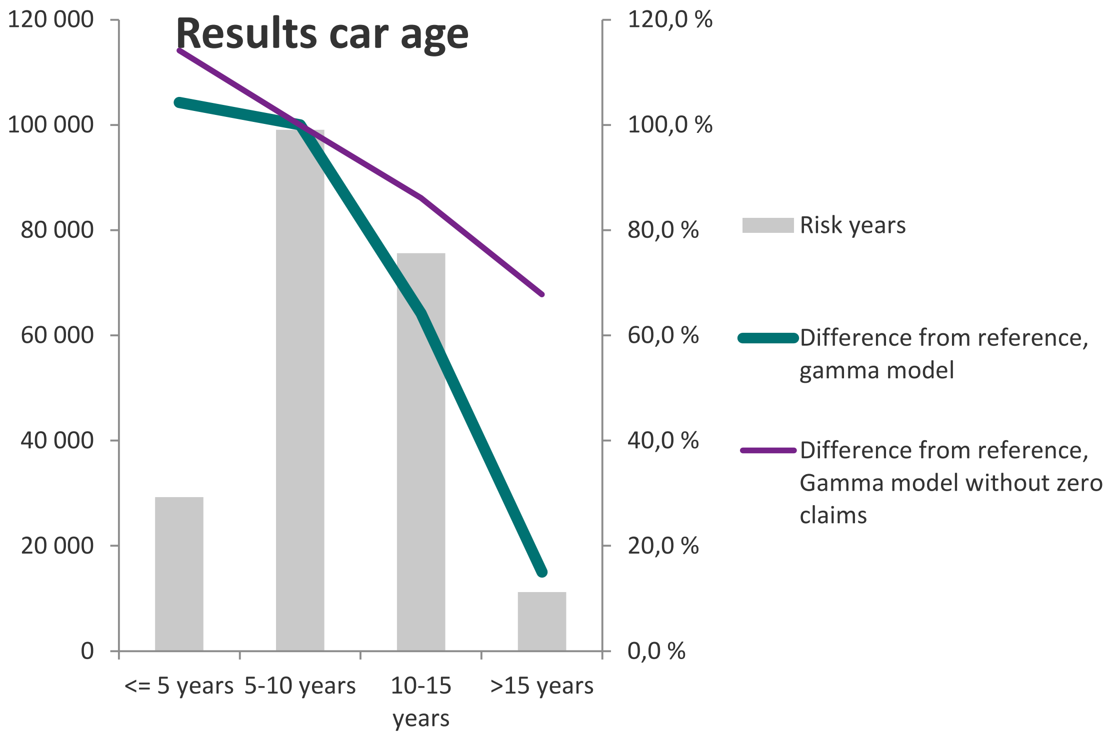


Results tariff class
70 000
60 000
50 000
40 000
30 000
20 000
10 000
0
250,0 %
200,0 %
150,0 %
100,0 %
50,0 %
0,0 %
1
2
3
4
5
6
Risk years
Difference from reference, gamma model
Difference from reference, Gamma model without zero claims

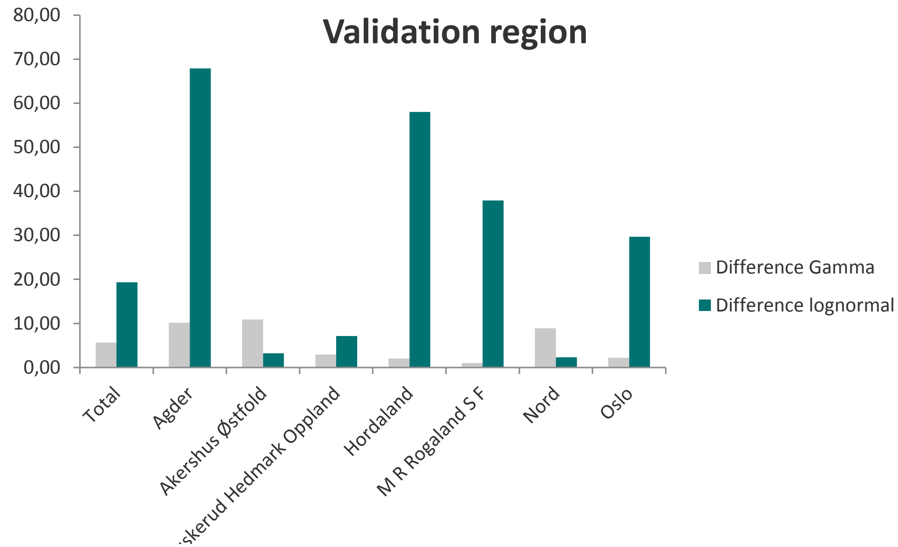

Results bonus
160 000
140 000
120 000
100 000
80 000
60 000
40 000
20 000
0
140,0 %
120,0 %
100,0 %
80,0 %
60,0 %
40,0 %
20,0 %
0,0 %
70,00 %
75,00 %
Under 70%
Risk years
Difference from reference, gamma model
Difference from reference, Gamma model without zero claims

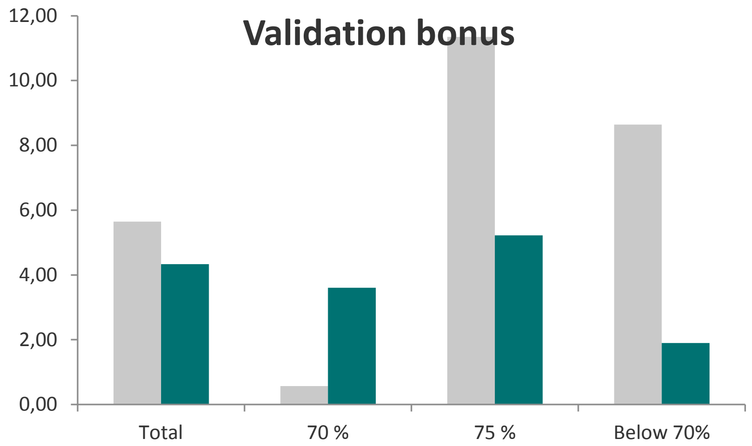
Non parametric
Log-normal, Gamma
The Pareto
Extreme value
Searching

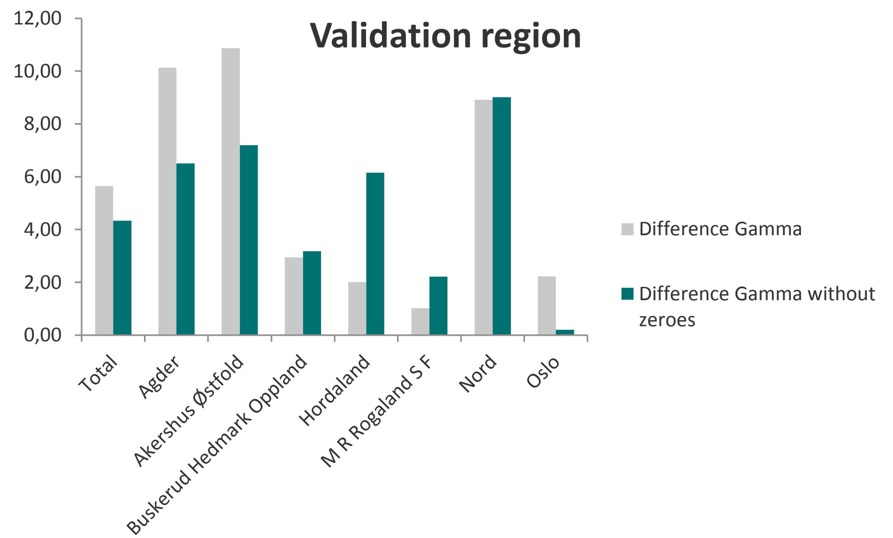


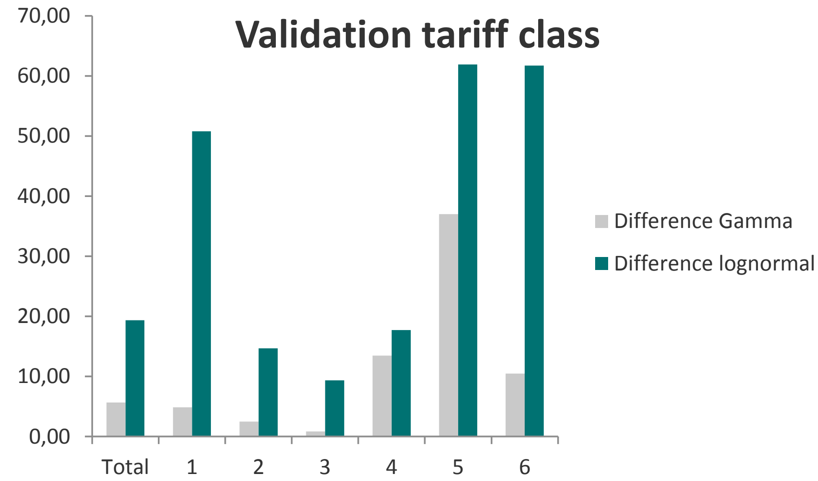

Results car age
120 000
100 000
80 000
60 000
40 000
20 000
0
120,0 %
100,0 %
80,0 %
60,0 %
40,0 %
20,0 %
0,0 %
<= 5 years
5-10 years
10-15 years
>15 years
Risk years
Difference from reference, gamma model
Difference from reference, Gamma model without zero claims

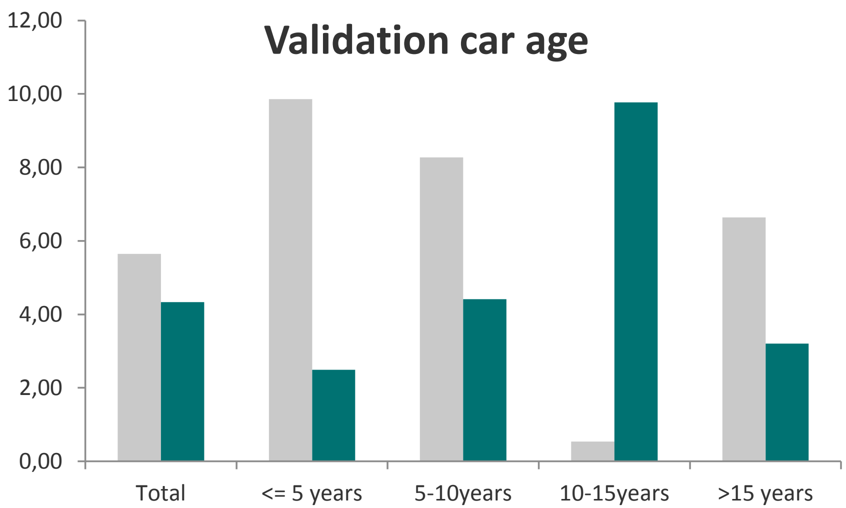
Non parametric
Log-normal, Gamma
The Pareto
Extreme value
Searching

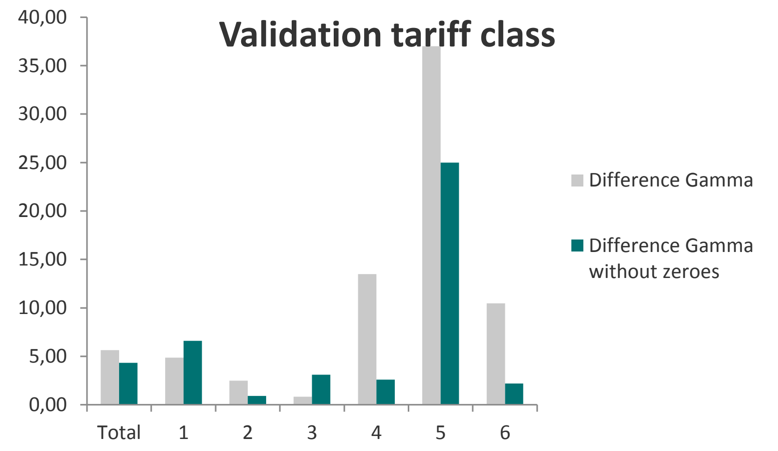



Validation tariff class
70,00
60,00
50,00
40,00
30,00
20,00
10,00
0,00
Total
1
2
3
4
5
6
Difference Gamma
Difference lognormal


Validation car age
12,00
10,00
8,00
6,00
4,00
2,00
0,00
Total
<= 5 years
5-10years
10-15years
>15 years


Validation tariff class
40,00
35,00
30,00
25,00
20,00
15,00
10,00
5,00
0,00
Total
1
2
3
4
5
6
Difference Gamma
Difference Gamma without zeroes