Non-life insurance mathematics

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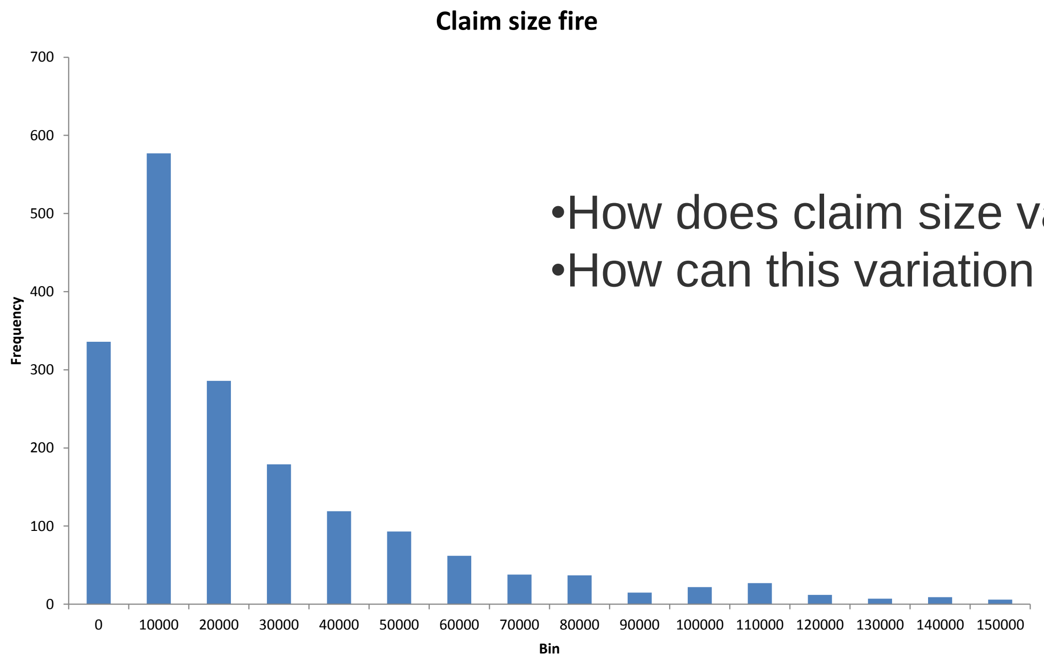
Repetition claim size

Claim severity modelling is about describing the variation in claim size

The concept

- The graph below shows how claim size varies for fire claims for houses
- The graph shows data up to the 88th percentile



•How does claim size vary?
•How can this variation be modelled?

•Truncation is necessary (large claims are rare and disturb the picture)
•0-claims can occur (because of deductibles)
•Two approaches to claim size modelling – non-parametric and parametric

Non-parametric modelling can be useful

- Claim size modelling can be *non-parametric* where each claim z_i of the past is assigned a probability 1/n of re-appearing in the future
- A new claim is then envisaged as a random variable $\hat{Z}$ for which

$$\Pr(\hat{Z} = z_i) = \frac{1}{n}, \quad \mathrm{i} = 1, \ldots, \mathrm{n}$$

- This is an entirely proper probability distribution
- It is known as *the empirical distribution* and will be useful in Section 9.5.

Non-parametric modelling can be useful

- All sensible parametric models for claim size are of the form

$$Z = \beta Z_0, \quad \text{where } \beta > 0 \text{ is a parameter}$$

$\hat{Z}$

- and Z0 is a standardized random variable corresponding to $\beta = 1$.
- The large the scale parameter, the more spread out the distribution

$$Z = \beta Z_0, Z_0 \sim N(1,1)$$

$$\beta = 1 \Rightarrow Z \sim N(1,1)$$

$$\beta = 2 \Rightarrow Z \sim N(2,2)$$

$$\beta = 3 \Rightarrow Z \sim N(3,3)$$

Fitting a scale family

- Models for scale families satisfy

$$\Pr(Z \le z) = \Pr(Z_0 \le z/\beta) \quad \text{or} \quad F(z|\beta) = F_0(z/\beta)$$

where $F(z|\beta)$ and $F_0(z/\beta)$ are the distribution functions of Z and Z_0.

- Differentiating with respect to z yields the family of density functions

$$f(z|\beta) = \frac{1}{\beta} f_0(\frac{z}{\beta}), \; z > 0 \quad \text{where} \quad f_0(z|\beta) = \frac{dF_0(z)}{dz}$$

- The standard way of fitting such models is through likelihood estimation. If $z_1,\ldots,z_n$ are the historical claims, the criterion becomes

$$L(\beta, f_0) = -n\log(\beta) + \sum_{i=1}^{n} \log\{f_0(z_i/\beta)\},$$

which is to be maximized with respect to β and other parameters.

- A useful extension covers situations with *censoring*.

Fitting a scale family

- Full value insurance:
  - The insurance company is liable that the object at all times is insured at its true value
- First loss insurance
  - The object is insured up to a pre-specified sum.
  - The insurance company will cover the claim if the claim size does not exceed the pre-specified sum

- The chance of a claim Z exceeding b is $1 - F_0(b/\beta)$, and for nb such events with lower bounds b1,…,bnb the analogous joint probability becomes

$$\{1 - F_0(b_1/\beta)\} x \ldots x \{1 - F_0(b_{n_b}/\beta)\}.$$

Take the logarithm of this product and add it to the log likelihood of the fully observed claims $z_1, \ldots, z_n$. The criterion then becomes

$$L(\beta, f_0) = -n \log(\beta) + \sum_{i=1}^{n} \log\{ f_0(z_i/\beta)\} + \sum_{i=1}^{n_b} \log\{1 - F_0(z_i/\beta)\},$$

complete information (for objects fully insured)

censoring to the right (for first loss insured)

Shifted distributions

- The distribution of a claim may start at some treshold b instead of the origin.
- Obvious examples are deductibles and re-insurance contracts.
- Models can be constructed by adding b to variables starting at the origin; i.e. where Z_0 is a standardized variable as before. Now

$$\Pr(Z \leq z) = \Pr(b + \beta Z_0 \leq z) = \Pr(Z_0 \leq \frac{z-b}{\beta})$$

- Example:
  - Re-insurance company will pay if claim exceeds 1 000 000 NOK

$$Z = 1000000 + \beta Z_0$$

The payout of the insurance company

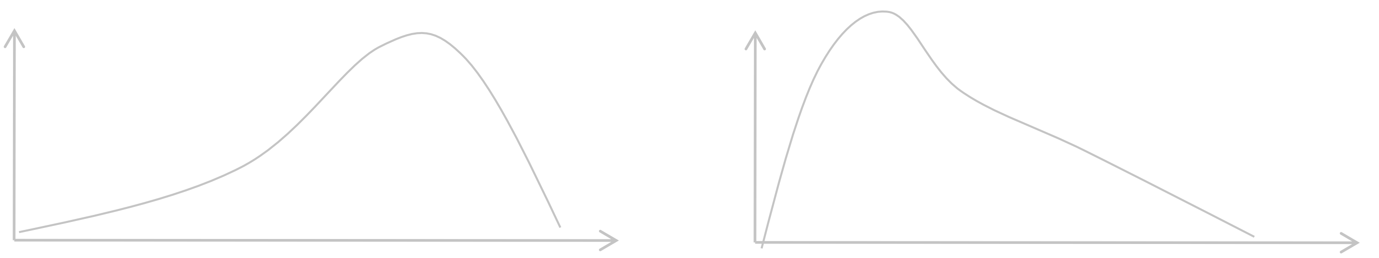
Total claim amount

Currency rate for example NOK per EURO, for example 8 NOK per EURO

Skewness as simple description of shape

- A major issue with claim size modelling is asymmetry and the right tail of the distribution. A simple summary is the coefficient of skewness

$$\varsigma = skew(Z) = \frac{\nu^3}{\sigma^3} \quad \text{where } \nu^3 = E(Z-\xi)^3$$



Negative skewness   Positive skewness

Negative skewness: the left tail is longer; the mass of the distribution
Is concentrated on the right of the figure. It has relatively few low values

Positive skewness: the right tail is longer; the mass of the distribution
Is concentrated on the left of the figure. It has relatively few high values

Non-parametric estimation

- The random variable $\hat{Z}$ that attaches probabilities 1/n to all claims z_i of the past is a possible model for *future* claims.
- Expectation, standard deviation, skewness and percentiles are all closely related to the ordinary sample versions. For example

$$E(\hat{Z}) = \sum_{i=1}^{n} \Pr(\hat{Z} = z_i) z_i = \sum_{i=1}^{n} \frac{1}{n} z_i = \bar{z}.$$

- Furthermore,

$$\operatorname{var}(\hat{Z}) = E(\hat{Z} - E(\hat{Z}))^2 = \sum_{i=1}^{n} \Pr(\hat{Z} = z_i)(z_i - \bar{z})^2 = \sum_{i=1}^{n} \frac{1}{n}(z_i - \bar{z})^2$$

$$\Rightarrow sd(\hat{Z}) = \sqrt{\frac{n-1}{n}} s, \quad s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (z_i - \bar{z})^2}$$

- Third order moment and skewness becomes

$$\hat{v}_3(\hat{Z}) = \frac{1}{n} \sum_{i=1}^{n} (z_i - \bar{z})^3 \quad \text{and} \quad \operatorname{skew}(\hat{Z}) = \frac{\hat{v}_3(\hat{Z})}{\{sd(\hat{Z})\}^3}$$

The log-normal family

- A convenient definition of the log-normal model in the present context is as $Z = \xi Z_0$ where $Z_0 = e^{-\sigma^2/2+\sigma\varepsilon}$ for $\varepsilon \sim N(0,1)$
- Mean, standard deviation and skewness are

$$E(Z) = \xi, \quad \mathrm{sd}(Z) = \xi\sqrt{e^{\sigma^2-1}}, \quad skew(Z) = (e^{\sigma^2}+2)\sqrt{e^{\sigma^2-1}}$$

see section 2.4.

- Parameter estimation is usually carried out by noting that logarithms are Gaussian. Thus

$$Y = \log(Z) = \log(\xi) - 1/2\sigma^2 + \sigma\varepsilon$$

and when the original log-normal observations $z_1,\ldots,z_n$ are transformed to Gaussian ones through $y_1=\log(z_1),\ldots,y_n=\log(z_n)$ with sample mean and variance $\bar{y}$ and s_y, the estimates of ξ and σ become

$$\log(\hat{\xi}) - 1/2\hat{\sigma}^2 = \bar{y}, \quad \hat{\sigma} = s_y \quad \text{or} \quad \hat{\xi} = e^{s_y^2/2+\bar{y}}, \hat{\sigma} = s_y.$$

The Gamma family

- The Gamma family is an important family for which the density function is

$$f(x) = \frac{(\alpha/\xi)^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\alpha x/\xi}, \quad x > 0, \text{where } \Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

- It was defined in Section 2.5 as $Z = \xi G$ where $G \sim \text{Gamma}(\alpha)$ is the standard Gamma with mean one and shape alpha. The density of the standard Gamma simplifies to

$$f(x) = \frac{\alpha^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\alpha x}, \quad x > 0, \text{where } \Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

Mean, standard deviation and skewness are

$$E(Z) = \xi, \quad \text{sd}(Z) = \xi/\sqrt{\alpha}, \quad \text{skew}(Z) = 2/\sqrt{\alpha}$$

and there is a convolution property. Suppose $G_1, \ldots, G_n$ are independent with $G_i \sim Gamma(\alpha_i)$. Then

$$\overline{G} \sim Gamma(\alpha_1 + \ldots + \alpha_n) \quad \text{if} \quad \overline{G} = \frac{\alpha_1 G_1 + \ldots + \alpha_n G_n}{\alpha_1 + \ldots + \alpha_n}$$

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The Gamma family

$$\log(f_0(z)) = \alpha\log(\alpha) - \log\Gamma(\alpha) + (\alpha-1)\log(z) - \alpha z$$

$$Z = \xi G$$

$$L(\xi,\alpha) = -n\log(\xi) + \sum_{i=1}^{n}\log(f_0(z/\xi))$$

$$= -n\log(\xi) + \sum_{i=1}^{n}\alpha\log(\alpha) - \log\Gamma(\alpha) + (\alpha-1)\log(z_i/\xi) - \alpha z/\xi$$

$$= -n\log(\xi) + n\alpha\log(\alpha) - n\log\Gamma(\alpha) + (\alpha-1)\sum_{i=1}^{n}\log(z_i/\xi) - \alpha/\xi\sum_{i=1}^{n}z_i$$

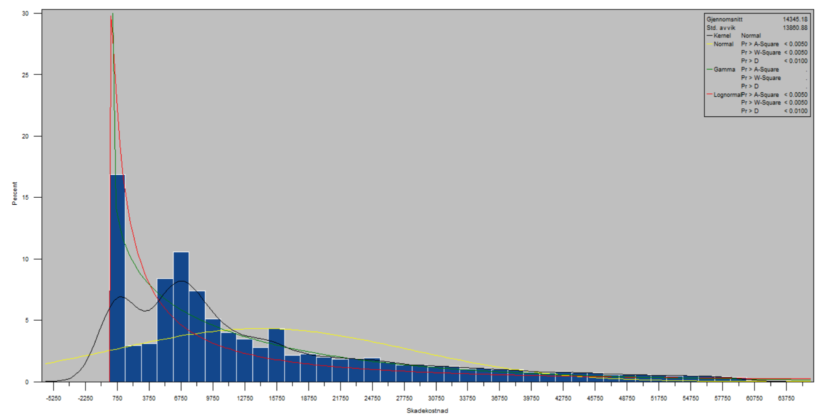
$$= -n\log(\xi) + n\alpha\log(\alpha) - n\log\Gamma(\alpha) + (\alpha-1)\sum_{i=1}^{n}\log(z_i)$$

$$+ (\alpha-1)(-n\log(\xi)) - \alpha/\xi\sum_{i=1}^{n}z_i$$

$$= n\alpha\log(\alpha/\xi) - n\log\Gamma(\alpha) + (\alpha-1)\sum_{i=1}^{n}\log(z_i) - \alpha/\xi\sum_{i=1}^{n}z_i$$

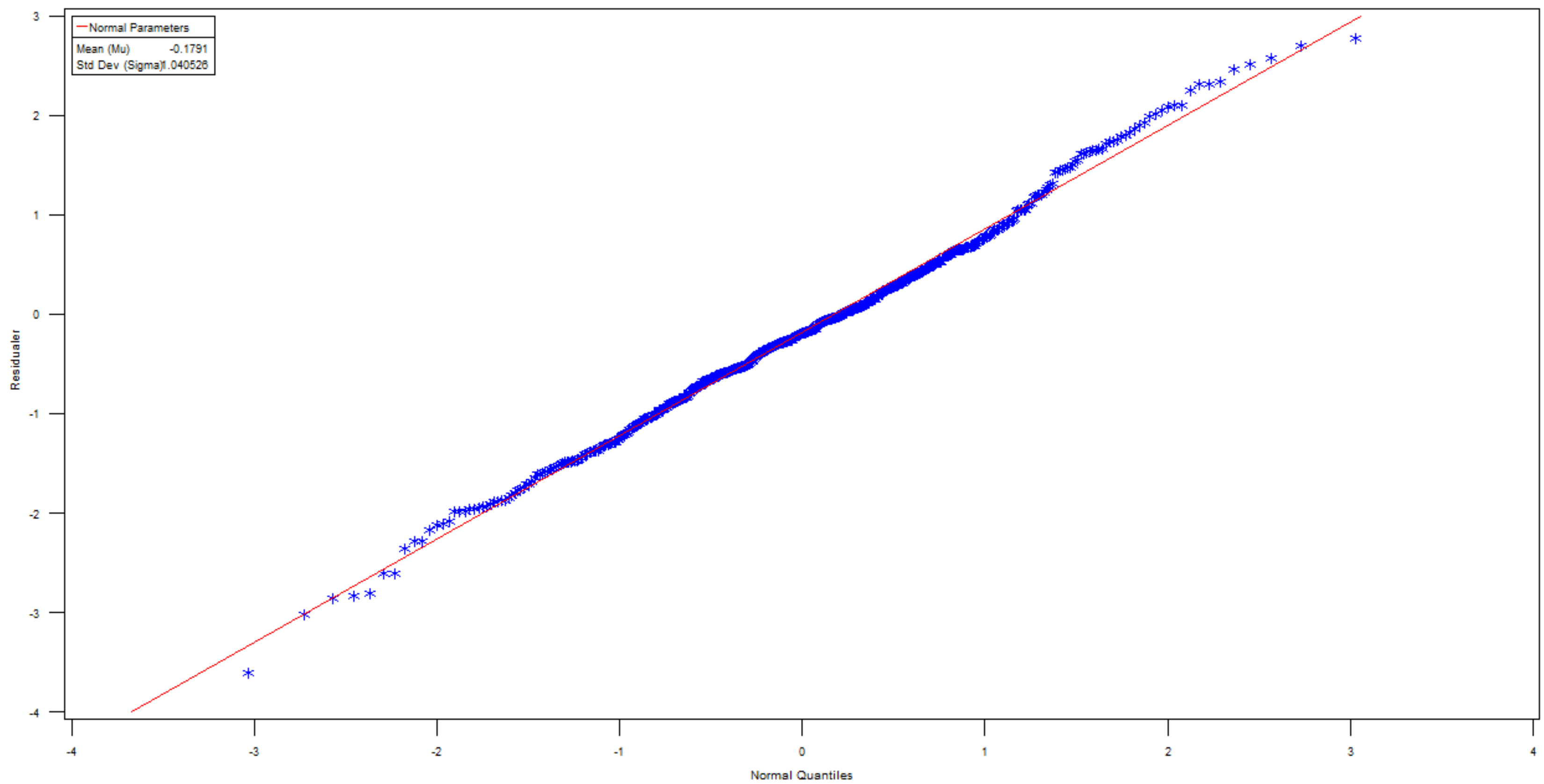
Example: car insurance

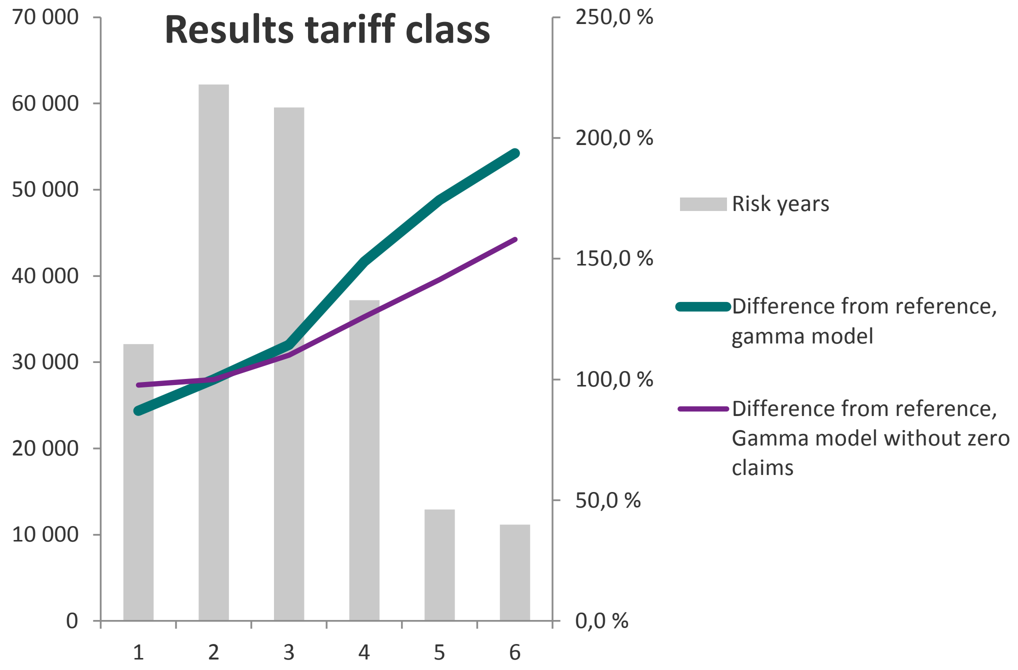
- Hull coverage (i.e., damages on own vehicle in a collision or other sudden and unforeseen damage)
- Time period for parameter estimation: 2 years
- Covariates:
  - Car age
  - Region of car owner
  - Tariff class
  - Bonus of insured vehicle

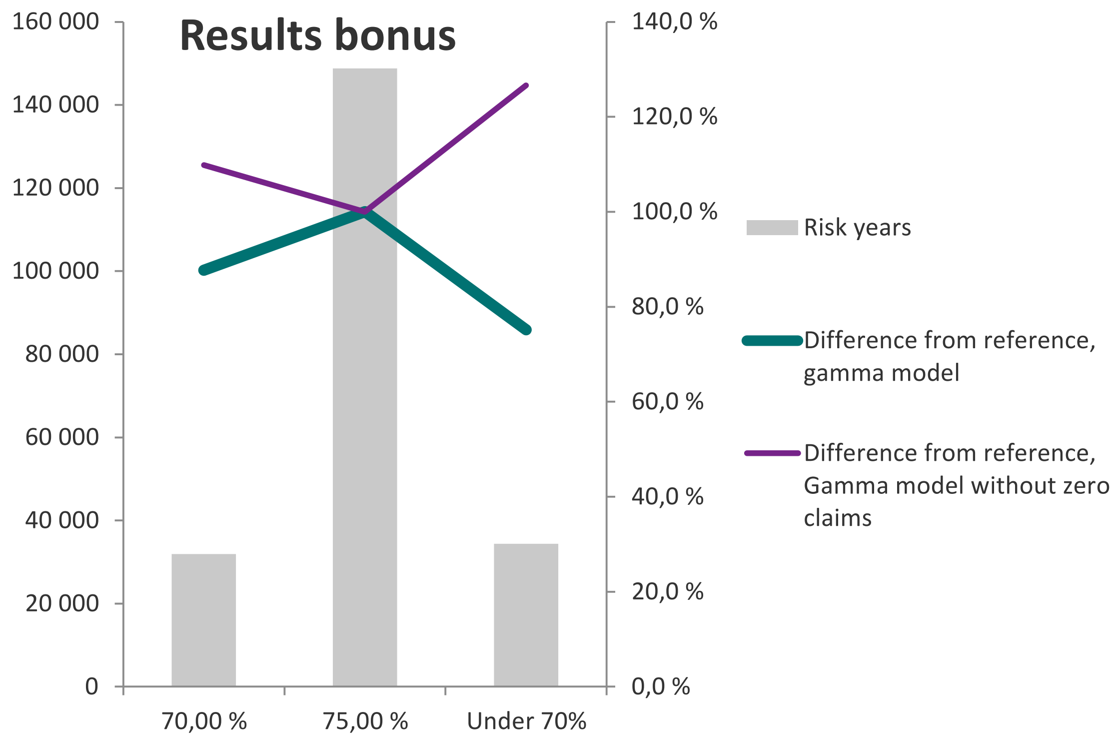


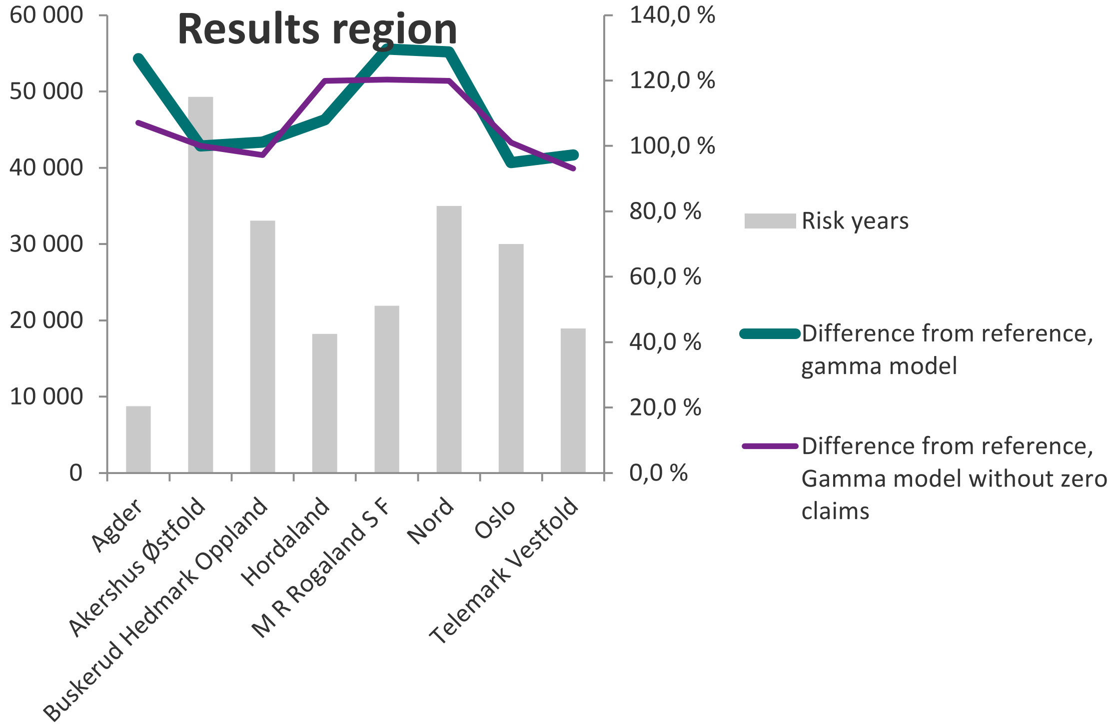
- Gamma without zero claims the best model

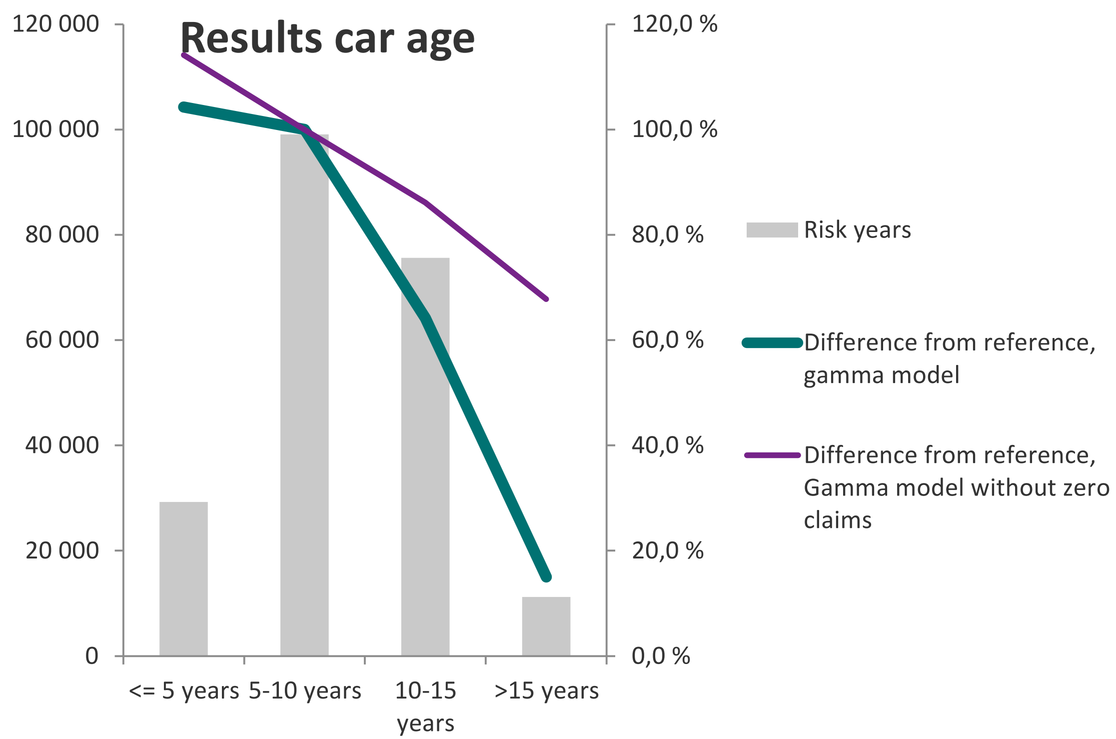
QQ plot Gamma model without zero claims




Results tariff class
70 000
60 000
50 000
40 000
30 000
20 000
10 000
0
250,0 %
200,0 %
150,0 %
100,0 %
50,0 %
0,0 %
1
2
3
4
5
6
Risk years
Difference from reference, gamma model
Difference from reference, Gamma model without zero claims


Results bonus
160 000
140 000
120 000
100 000
80 000
60 000
40 000
20 000
0
140,0 %
120,0 %
100,0 %
80,0 %
60,0 %
40,0 %
20,0 %
0,0 %
70,00 %
75,00 %
Under 70%
Risk years
Difference from reference, gamma model
Difference from reference, Gamma model without zero claims


Results region
60 000
50 000
40 000
30 000
20 000
10 000
0
140,0 %
120,0 %
100,0 %
80,0 %
60,0 %
40,0 %
20,0 %
0,0 %
Agder
Akershus Østfold
Buskerud Hedmark Oppland
Hordaland
M R Rogaland S F
Nord
Oslo
Telemark Vestfold
Risk years
Difference from reference, gamma model
Difference from reference, Gamma model without zero claims


Results car age
120 000
100 000
80 000
60 000
40 000
20 000
0
120,0 %
100,0 %
80,0 %
60,0 %
40,0 %
20,0 %
0,0 %
<= 5 years
5-10 years
10-15 years
>15 years
Risk years
Difference from reference, gamma model
Difference from reference, Gamma model without zero claims

Overview

Important issues	Models treated	Curriculum	Duration (in lectures)
What is driving the result of a non-life insurance company?	insurance economics models	Lecture notes	0,5
How is claim frequency modelled?	Poisson, Compound Poisson and Poisson regression	Section 8.2-4 EB	1,5
How can claims reserving be modelled?	Chain ladder, Bernhuetter Ferguson, Cape Cod,	Note by Patrick Dahl	2
How can claim size be modelled?	Gamma distribution, log-normal distribution	Chapter 9 EB	2
How are insurance policies priced?	Generalized Linear models, estimation, testing and modelling. CRM models.	Chapter 10 EB	2
Credibility theory	Buhlmann Straub	Chapter 10 EB	1
Reinsurance		Chapter 10 EB	1
Solvency		Chapter 10 EB	1
Repetition			1

The ultimate goal for calculating the pure premium is pricing

Pure premium = Claim frequency x claim severity

$$Claim\ severity = \frac{total\ claim\ amount}{number\ of\ claims}$$

$$Claim\ frequency = \frac{number\ of\ claims}{number\ of\ policy\ years}$$

Parametric and non parametric modelling (section 9.2 EB)

The log-normal and Gamma families (section 9.3 EB)

The Pareto families (section 9.4 EB)

Extreme value methods (section 9.5 EB)

Searching for the model (section 9.6 EB)

The Pareto distribution

The Pareto distributions, introduced in Section 2.5, are among the most heavy-tailed of all models in practical use and potentially a conservative choice when evaluating risk. Density and distribution functions are

$$f(z)=\frac{\alpha/\beta}{(1+z/\beta)^{1+\alpha}} \quad \text{and} \quad F(z)=1-\frac{1}{(1+z/\beta)^{\alpha}}, \quad z>0.$$

Simulation can be done using Algorithm 2.13:

1. Input alpha and beta
2. Generate U~Uniform
3. Return X = beta(U^^(-(1/alpha))-1)

Pareto models are so heavy-tailed that even the mean may fail to exist (that's why another parameter beta must be used to represent scale). Formulae for expectation, standard deviation and skewness are

$$\xi=E(Z)=\frac{\beta}{\alpha-1}, \quad \text{sd}(Z)=\xi\left(\frac{\alpha}{\alpha-2}\right)^{1/2}, \quad \text{skew}(Z)=2\left(\frac{\alpha-2}{\alpha}\right)^{1/2}\frac{\alpha+1}{\alpha-3}$$

valid for alpha>1, alpha>2 and alpha >3 respectively.

The Pareto distribution

The median is given by

$$med(Z) = \beta(2^{1/\alpha} - 1)$$

- The exponential distribution appears in the limit when the ratio $\xi = \beta/(\alpha - 1)$ is kept fixed and $\alpha \to \infty$.
- There is in this sense overlap between the Pareto and the Gamma families.
- The exponential distribution is a heavy-tailed Gamma and the most light-tailed Pareto and it is common to regard it as a member of both families

Likelihood estimation

- The Pareto model was used as illustration in Section 7.3, and likelihood estimation was developed there
- Censored information is now added. Suppose observations are in two groups, either the ordinary, fully observed claims $z_1,..,z_n$ or those only to known to have exceeded certain thresholds $b_1,..,b_n$ but not by how much.
- The log likelihood function for the first group is as in Section 7.3

$$n\log(\alpha/\beta) - (1+\alpha)\sum_{i=1}^{n}\log(1+\frac{z_i}{\beta}),$$

The Pareto distribution

whereas the censored part adds contribution from knowing that $Z_i > b_i$.
The probability is

$$\Pr(Z_i > b_i) = \frac{1}{(1 + b_i/\beta)^{\alpha}} \quad \text{or} \quad \log\{\Pr(Z_i > b_i)\} = -\alpha \sum_{i=1}^{n_b} \log(1 + b_i/\beta)$$

and the full likelihood becomes

$$L(\alpha, \beta) = n \log(\alpha/\beta) - (1+\alpha)\sum_{i=1}^{n} \log(1 + z_i/\beta) - \alpha \sum_{i=1}^{n_b} \log(1 + b_i/\beta)$$

Complete information — Censoring to the right

This is to be maximised with respect to α and β, a numerical problem very much the same as in Section 7.3.

Over-threshold under Pareto

One of the most important properties of the Pareto family is the behaviour at the extreme right tail. The issue is defined by the **over-threshold** model which is the distribution of $Z_b = Z - b$ given $Z > b$. Its density function is

$$\Pr(Z - b \le z \mid Z > b) = \frac{\Pr(Z - b \le z, Z > b)}{1 - F(b)} = \frac{F(z+b)}{1 - F(b)}$$

$$\Rightarrow f_b(z) = \frac{f(z+b)}{1 - F_Z(b)}, \quad z > 0.$$

The over-threshold density becomes Pareto:

$$f_b(z) = \frac{(1 + b/\beta)^{\alpha} \alpha / \beta}{(1 + (z+b)/\beta)^{1+\alpha}} = \frac{\alpha / (\beta + b)}{(1 + (z+b)/(\beta + b))^{1+\alpha}}$$

Pareto density function

- The shape alpha is the same as before, but the parameter of scale has now changed to $\beta_b = \beta + b$
- Over-threshold distributions preserve the Pareto model and its shape.
- The mean is given by (alpha must exceed 1)

$$E(Z_b \mid Z > b) = \frac{\beta_b}{\alpha - 1} = \frac{\beta + b}{\alpha - 1} = \xi + \frac{b}{\alpha - 1}$$

The extended Pareto family

Add the numerator $(z/\beta)^{\theta}$ to the Pareto density function, and it reads

$$f(z) = \frac{\Gamma(\alpha+\theta)}{\Gamma(\alpha)\Gamma(\theta)} \frac{1}{\beta} \frac{(z/\beta)^{\theta-1}}{(1+z/\beta)^{\alpha+\theta}} \quad \text{where} \quad \alpha,\beta,\theta > 0$$

which defines the **extended** Pareto model.

- Shape is now defined by two parameters α and θ, and this creates useful flexibility.
- The density function is either decreasing over the positive real line (if theta <= 1) or has a single maximum (if theta >1). Mean and standard deviation are

$$\xi = E(Z) = \frac{\theta\beta}{\alpha-1} \quad \text{and} \quad \mathrm{sd}(Z) = \xi\left(\frac{\alpha+\beta-1}{\theta(\alpha-2)}\right)^{1/2}$$

Pareto density function

which are valid when alpha > 1 and alpha>2 respectively whereas skewness is

$$skew(Z) = 2\left(\frac{\alpha-2}{\theta(\alpha+\theta-1)}\right)^{1/2} \frac{\alpha+2\theta-1}{\alpha-3},$$

provided alpha > 3. These results verified in Section 9.7 reduce to those for the ordinary Pareto distribution when theta=1.

Sampling the extended Pareto family

An extended Pareto variable with parameters (α, θ, β) can be written

$$Z = \frac{\theta\beta}{\alpha}\frac{G_1}{G_2} \quad \text{where} \quad G_1 \sim Gamma(\theta),\ G_2 \sim Gamma(\alpha)$$

Here G_1 and G_2 are two independent Gamma variables with mean one.

- The representation which is provided in Section 9.7 implies that 1/Z is extended Pareto distributed as well and leads to the following algorithm:

Algorithm 9.1 The extended Pareto sampler

1. Input (α, θ, β) and $\eta = \theta\beta / \alpha$
2. Draw G_1 ~ Gamma(theta)
3. Draw G_2 ~ Gamma(alpha)
4. Return Z <- etta G_1/G_2

Extreme value methods

Large claims play a special role because of their importance financially

- The share of large claims is the most important driver for profitability volatility
- «The larger claim the greater is the degree of randomness»

But experience is often limited

- How should such situations be tackled?

Theory

- Pareto distributions are preserved over thresholds
- If Z is continuous and unbounded and b is some threshold, then Z-b given Z>b will be Pareto as b grows to infinity!!

…..Ok….

- How do we use this?
- How large does b has to be?

Extreme value methods

Our target is Z_b=Z-b given Z>b. Consider its tail distribution function

$$\bar{F}_b(z) = \Pr(Z_b > z) = \frac{1-F(b+z)}{1-F(b)} \quad \text{where } \mathrm{F(z) = Pr(Z \leq z)}$$

and let $Y_b = Z_b / \beta_b$ where β_b is a scale parameter depending on b. We are assuming that Z>b, and Y_b is then positive with tail distribution $\Pr(Y_b > b) = \bar{F}_b(\beta_b y)$. The general result says that there exists a parameter alpha (not depending on b and possibly infinite) and some sequence beta_b such that

$$\bar{F}_b(\beta_b y) \to \bar{P}(y;\alpha) \text{ as } b \to \infty \quad \text{where } \bar{P}(y;\alpha) = \begin{cases} (1+y)^{-\alpha}, & 0 < \alpha < \infty \\ e^{-y}, & \alpha = \infty \end{cases}$$

The limit $\bar{P}(y;\alpha)$ is the tail distribution of the Pareto model which shows that Z_b becomes $Pareto(\alpha, \beta_b)$ as $b \to \infty$. Both the shape alpha and the scale parameter beta_b depend on the original model but only the latter varies with b.

Extreme value methods

- The decay rate α can be determined from historical data
- One possibility is to select observations exceeding some threshold, impose the Pareto distribution and use likelihood estimation as explained in Section 9.4. We will revert to this
- An alternative often referred to in the literature of extreme value is the *Hill estimate*
- Start by sorting the data in ascending order $z_{(1)} \leq \ldots \leq z_{(n)}$ and take

$$\hat{\alpha}^{-1} = \frac{1}{n - n_1} \sum_{i=n_1+1}^{n} \log\left(\frac{z_{(i)}}{z_{(n_1)}}\right) \quad \text{where } n_1 = (1-p)n$$

- Here p is some small, user-selected number.
- The method is non-parametric (no model is assumed)
- We may want to use $\hat{\alpha}$ as an estimate of α in a Pareto distribution imposed over the threshold $b = z_{(n_1)}$ and would then need an estimate of the scale parameter β_b.
- A practical choice is then

$$\hat{\beta}_b = \frac{z_{(n_2)} - z_{(n_1)}}{2^{1/\hat{\alpha}} - 1} \quad \text{where } n_2 = 1 + \frac{n_1 + n}{2}.$$

The entire distribution through mixtures

- Assume some large claim threshold b is selected
- Then there are many values in the small and medium range below and up to b and few above b
- How to select b?
- One way: choose some small probability p and let n_1 = integer(n(1-p)) and let $b=z_{(n_1)}$
- Another way: study the percentiles
- Modelling may be divided into separate parts defined by the threshold b

$$Z = (1 - I_b) Z_{\leq b} + I_b Z_{>b} \quad \text{where} \quad I_b = \begin{cases} 0 & \text{if} \quad Z \leq b \\ 1 & \text{if} \quad Z > b \end{cases}$$

Central Region (plenty of data) — Extreme right tail (data is scarce)

- Modelling in the central region: non-parametric (empirical distribution) or some selected distribution (i.e., log-normal gamma etc)
- Modelling in the extreme right tail:
  - The result due to Pickands suggests a Pareto distribution, provided b is large enough
  - But is b large enough??
  - Other distributions may perform better, more about this in Section 9.6.

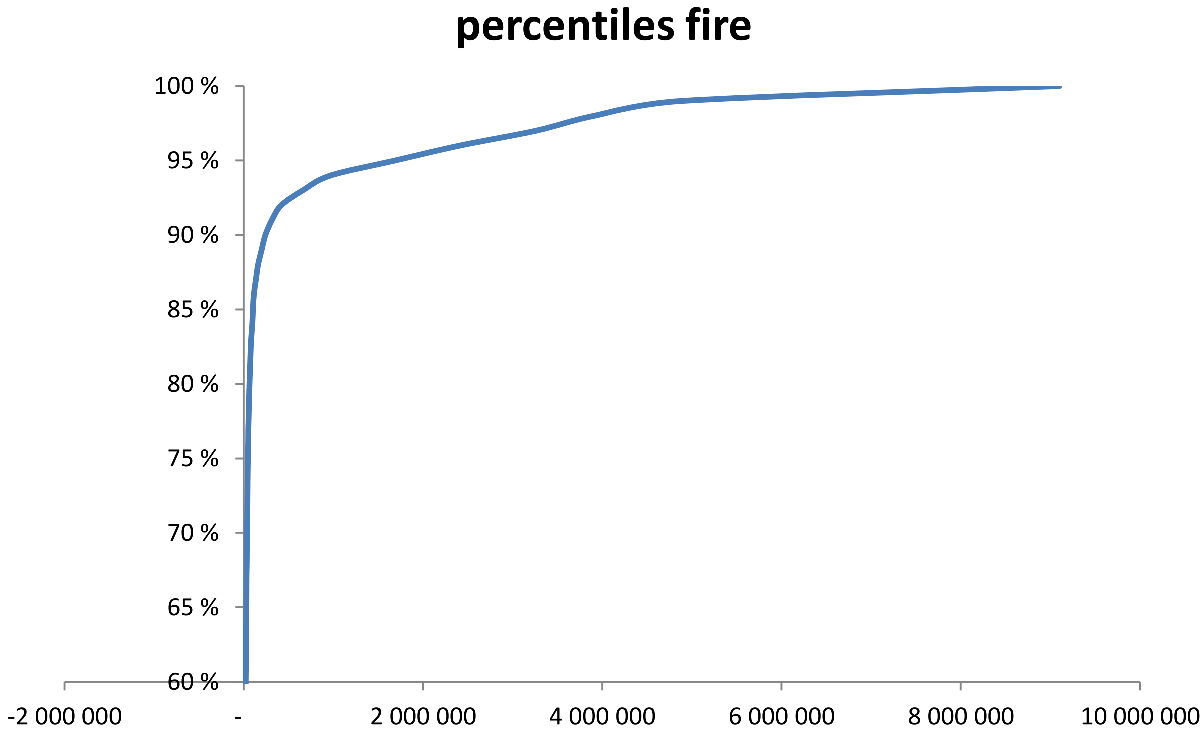
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



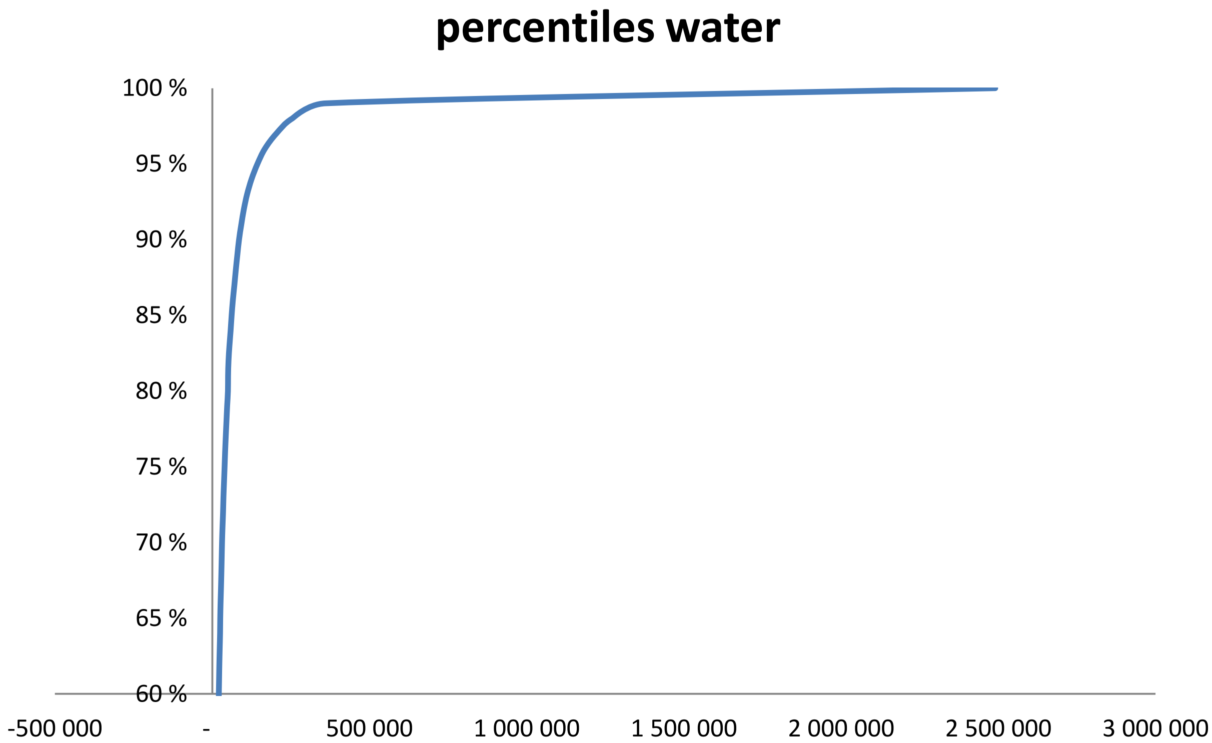
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



Non parametric

Log-normal, Gamma

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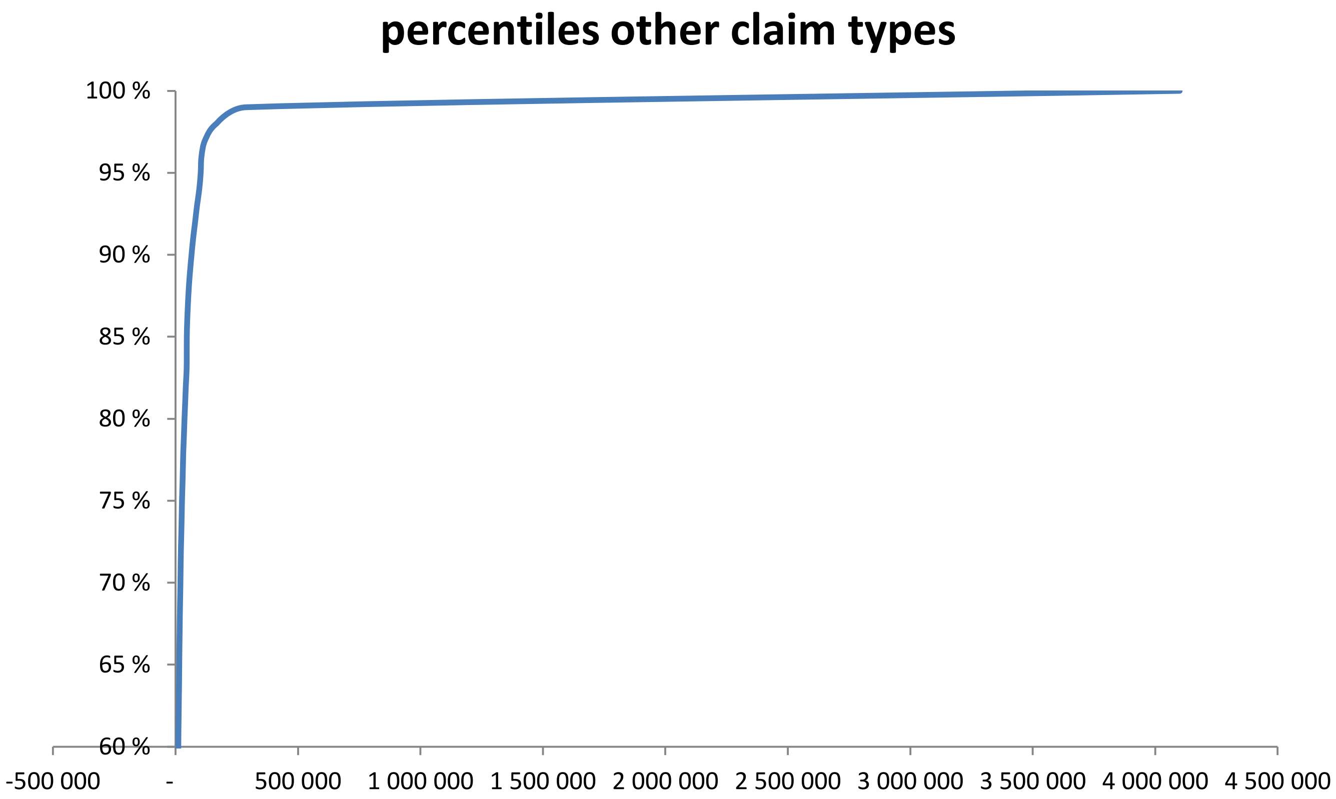

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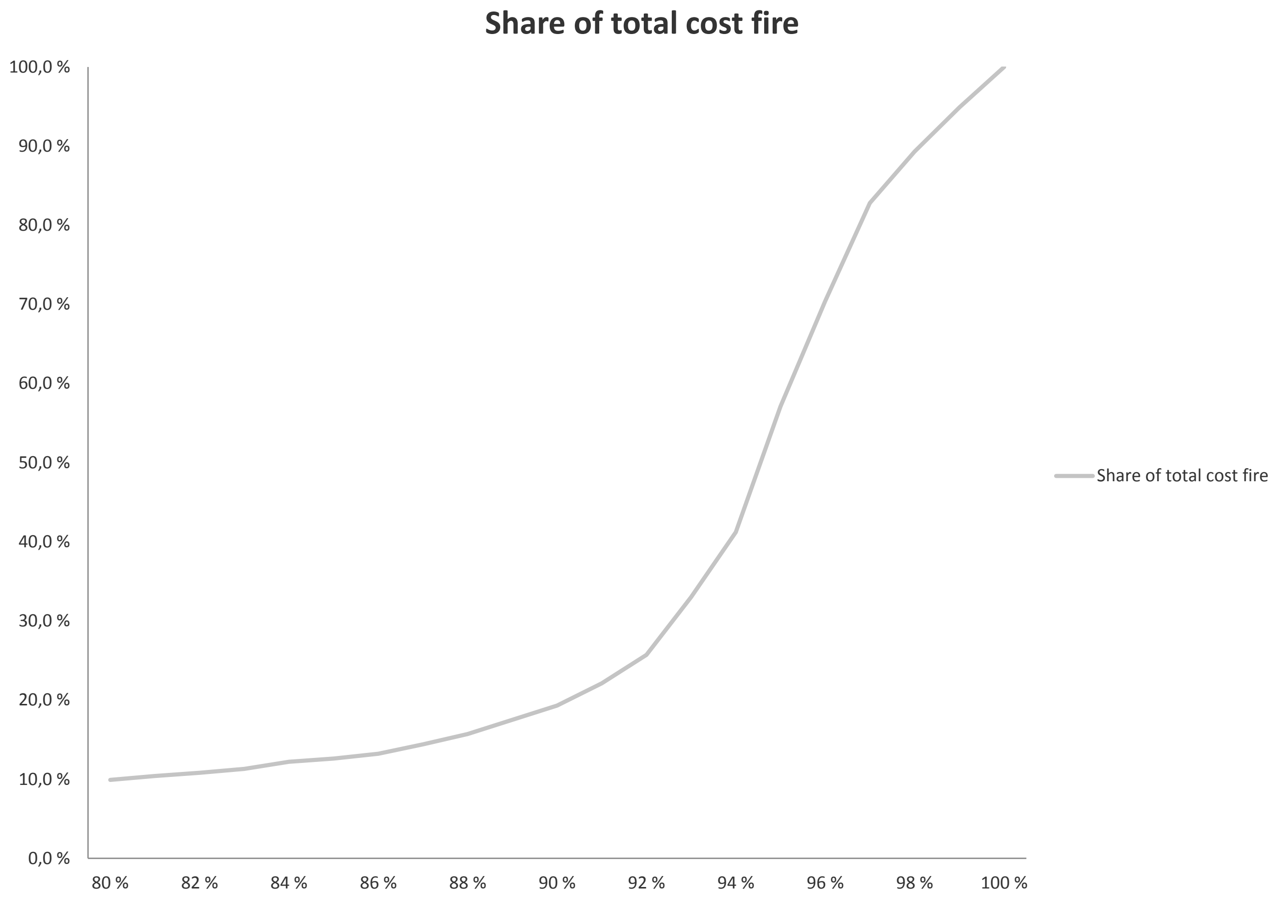


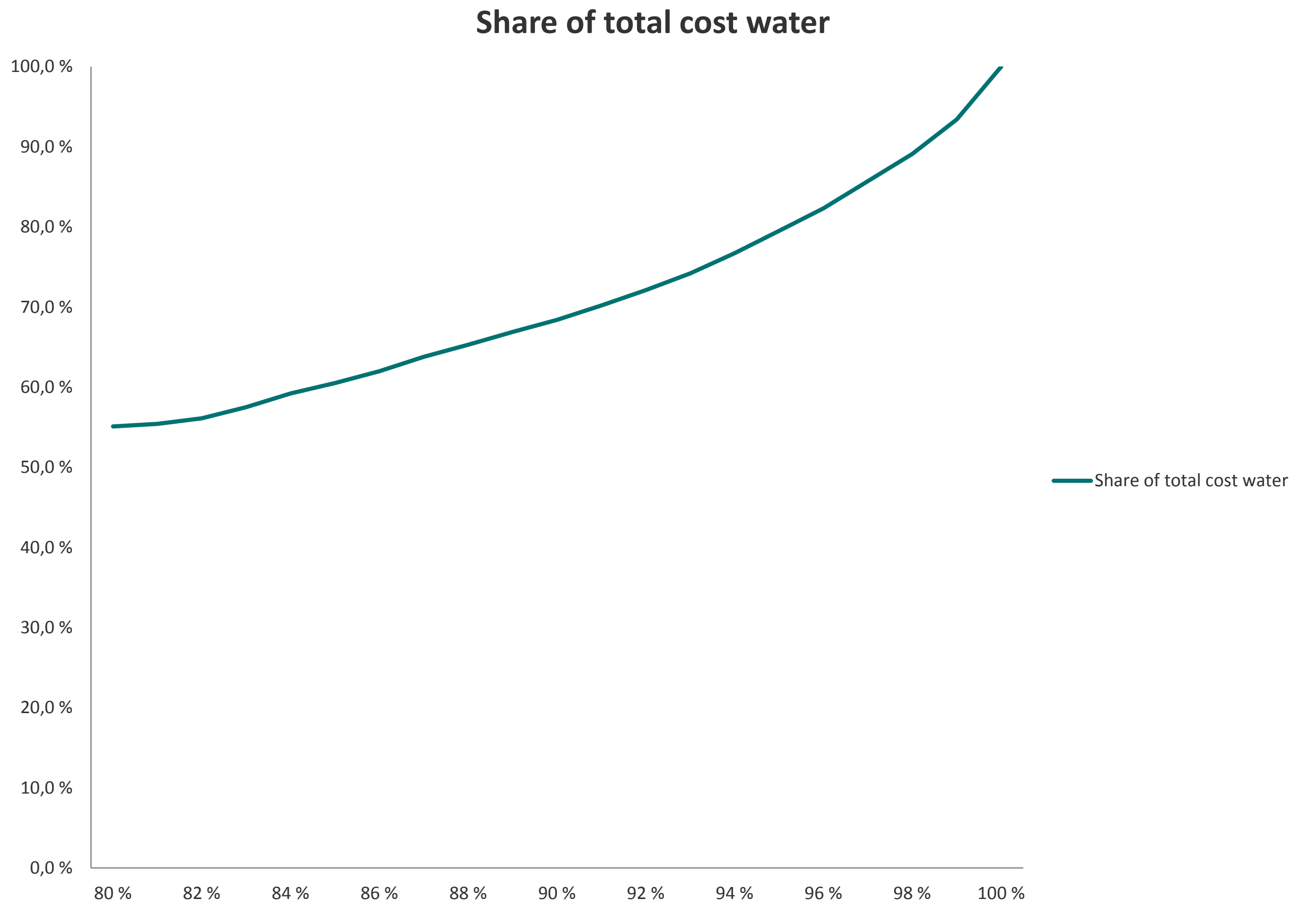
Non parametric

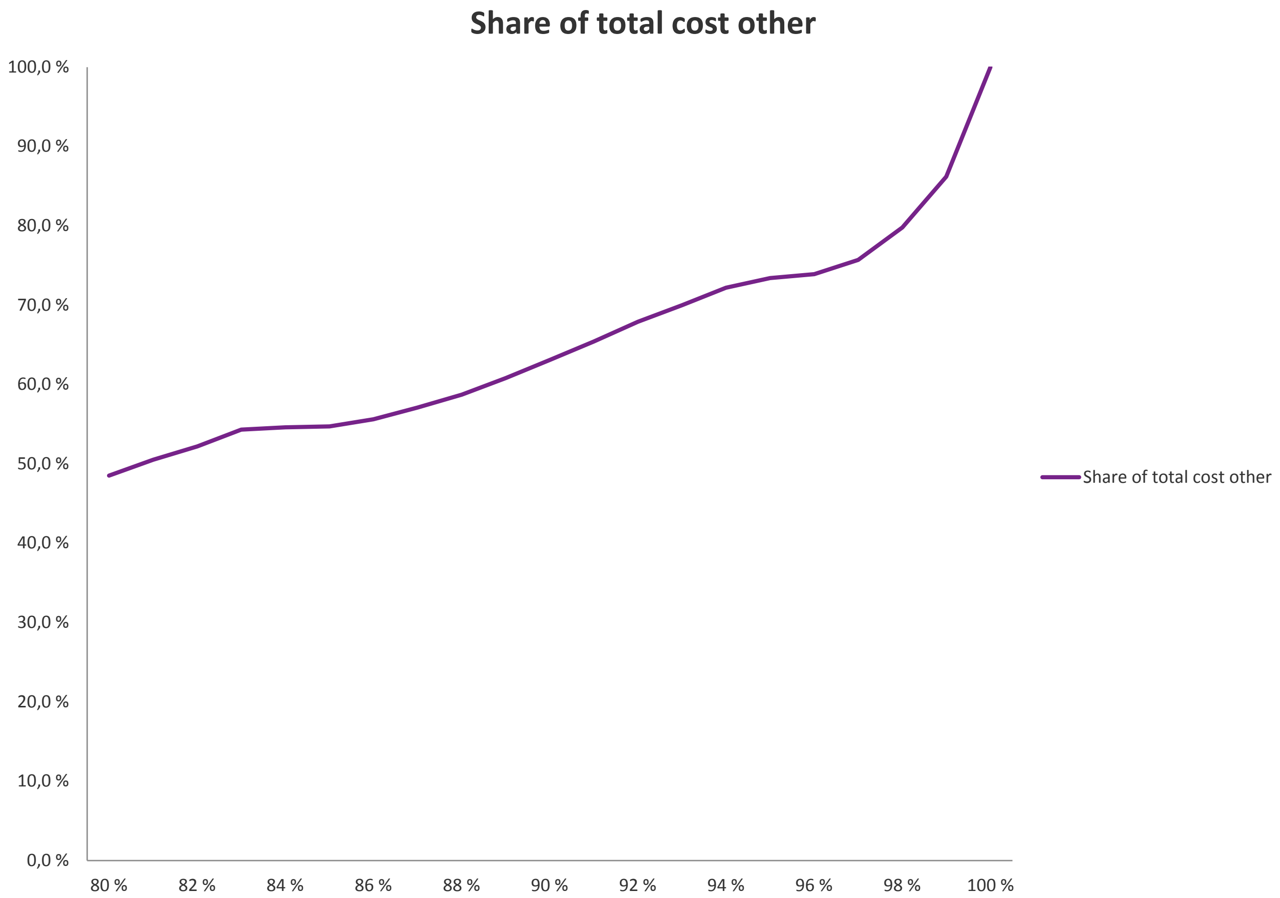
Log-normal, Gamma

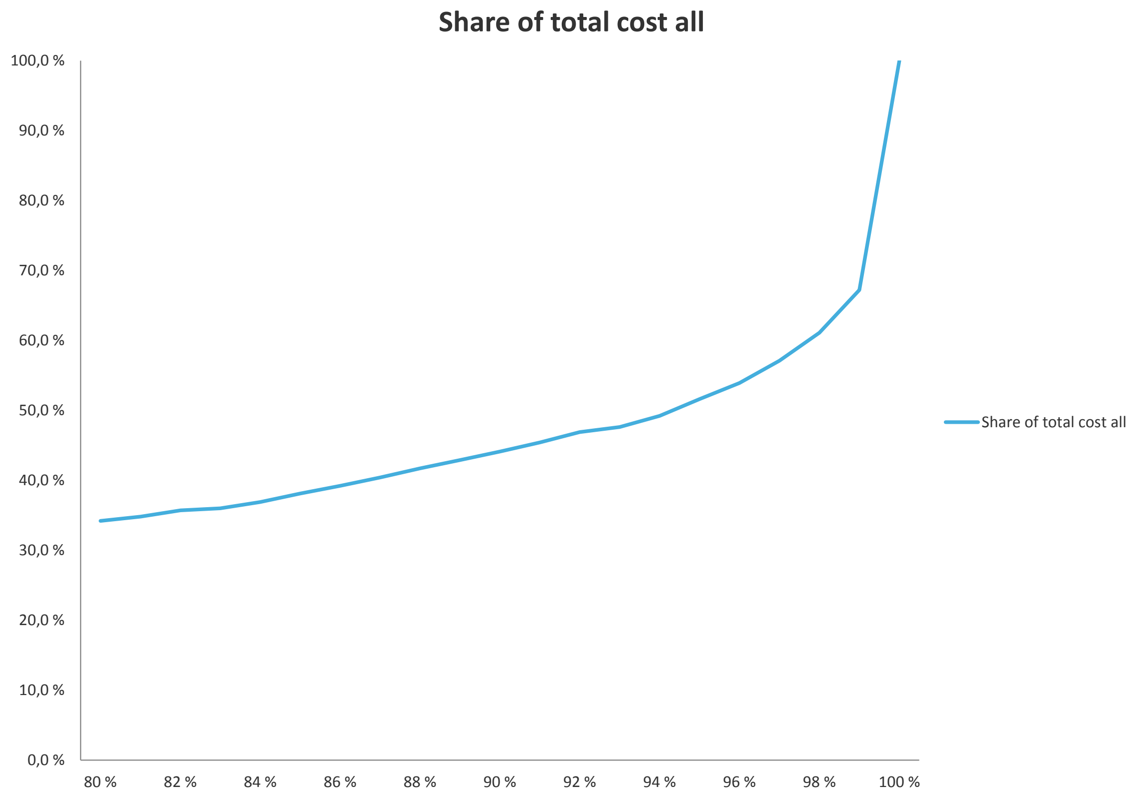
The Pareto

Extreme value

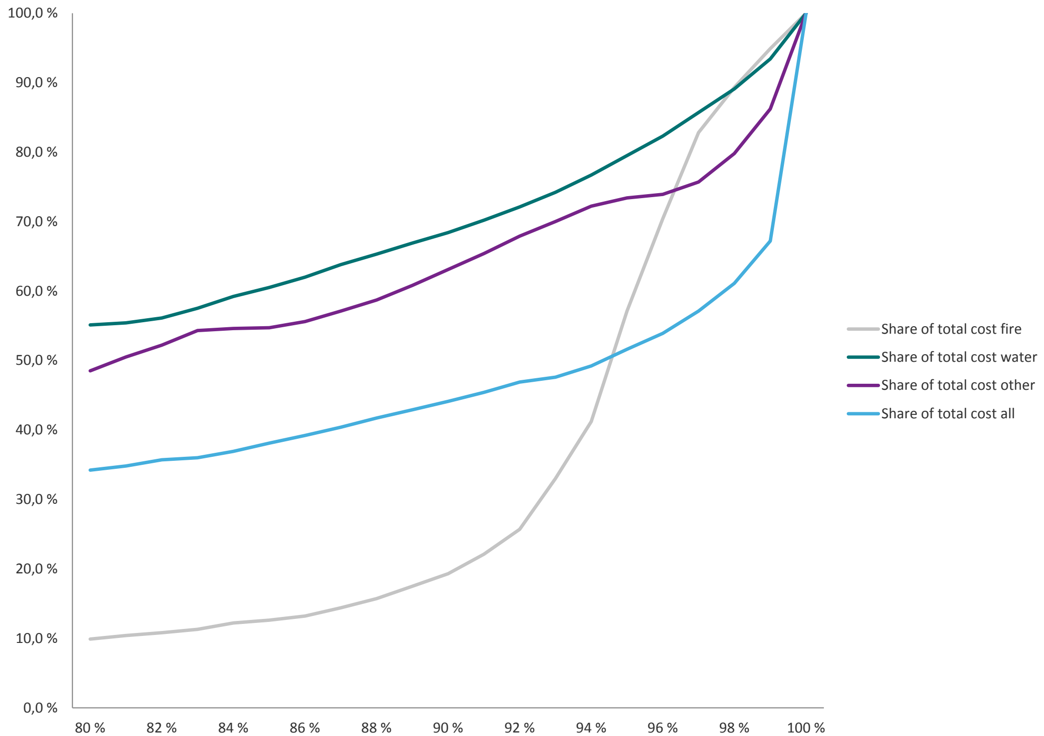
Searching




Share of total cost other
100,0 %
90,0 %
80,0 %
70,0 %
60,0 %
50,0 %
40,0 %
30,0 %
20,0 %
10,0 %
0,0 %
80 %
82 %
84 %
86 %
88 %
90 %
92 %
94 %
96 %
98 %
100 %
Share of total cost other


Share of total cost all
100,0 %
90,0 %
80,0 %
70,0 %
60,0 %
50,0 %
40,0 %
30,0 %
20,0 %
10,0 %
0,0 %
80 %
82 %
84 %
86 %
88 %
90 %
92 %
94 %
96 %
98 %
100 %
Share of total cost all

Non parametric
Log-normal, Gamma
The Pareto
Extreme value
Searching



	fire	water	other	all
99,1 %	4 927 015	377 682	300 000	656 972
99,2 %	5 021 824	406 859	307 666	726 909
99,3 %	5 226 985	424 013	344 006	871 552
99,4 %	5 332 034	464 769	354 972	1 044 598
99,5 %	5 576 737	511 676	365 925	1 510 740
99,6 %	6 348 393	576 899	409 618	2 330 786
99,7 %	6 647 669	663 382	462 719	3 195 813
99,8 %	7 060 421	740 187	724 682	3 832 486
99,9 %	7 374 623	891 226	1 005 398	4 733 953
100,0 %	9 099 312	2 490 558	4 100 000	9 099 312

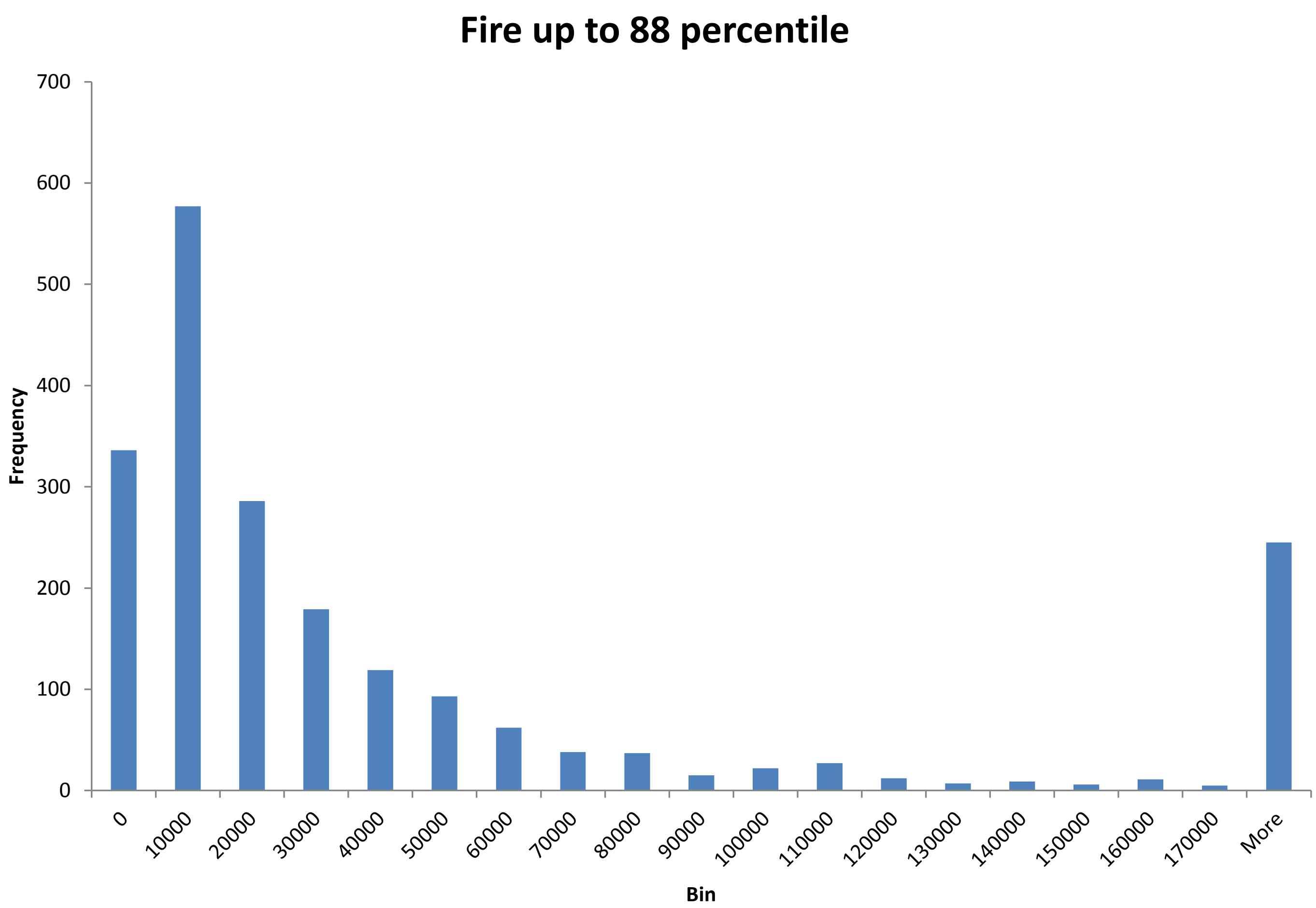
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



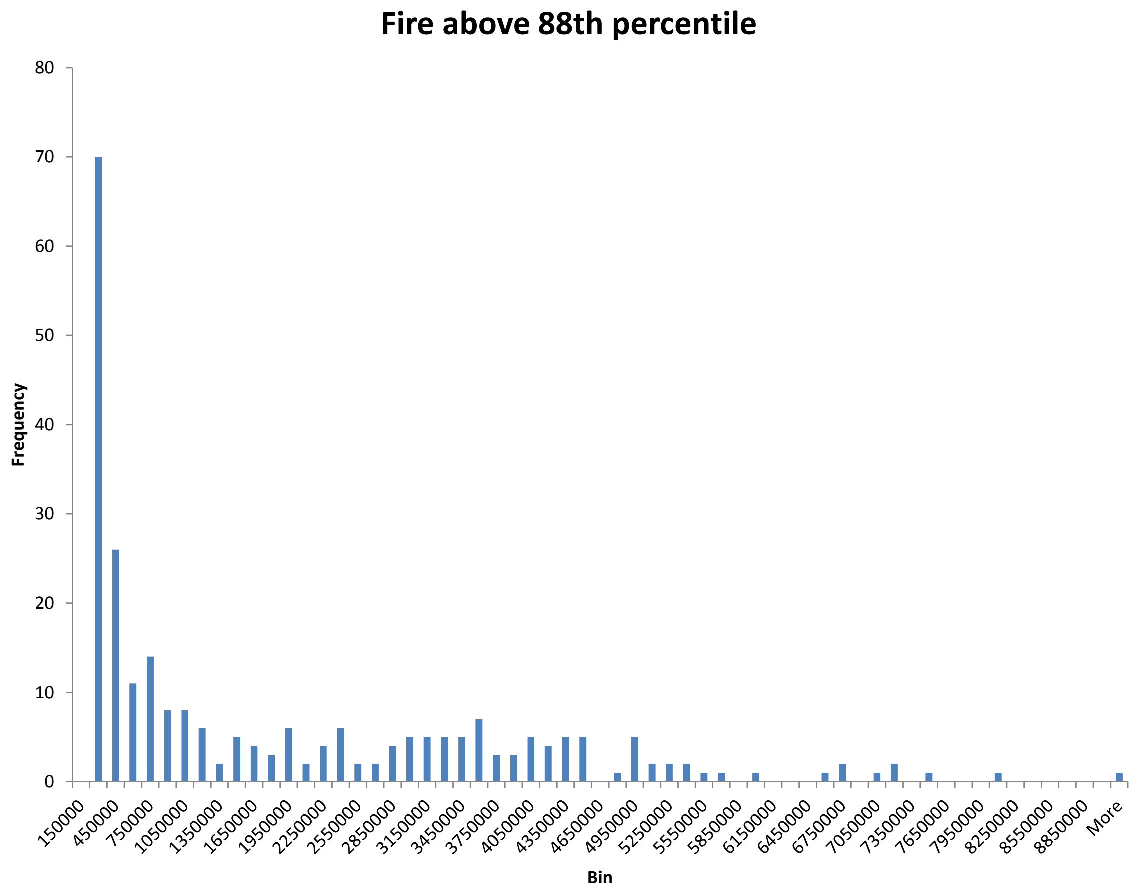
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



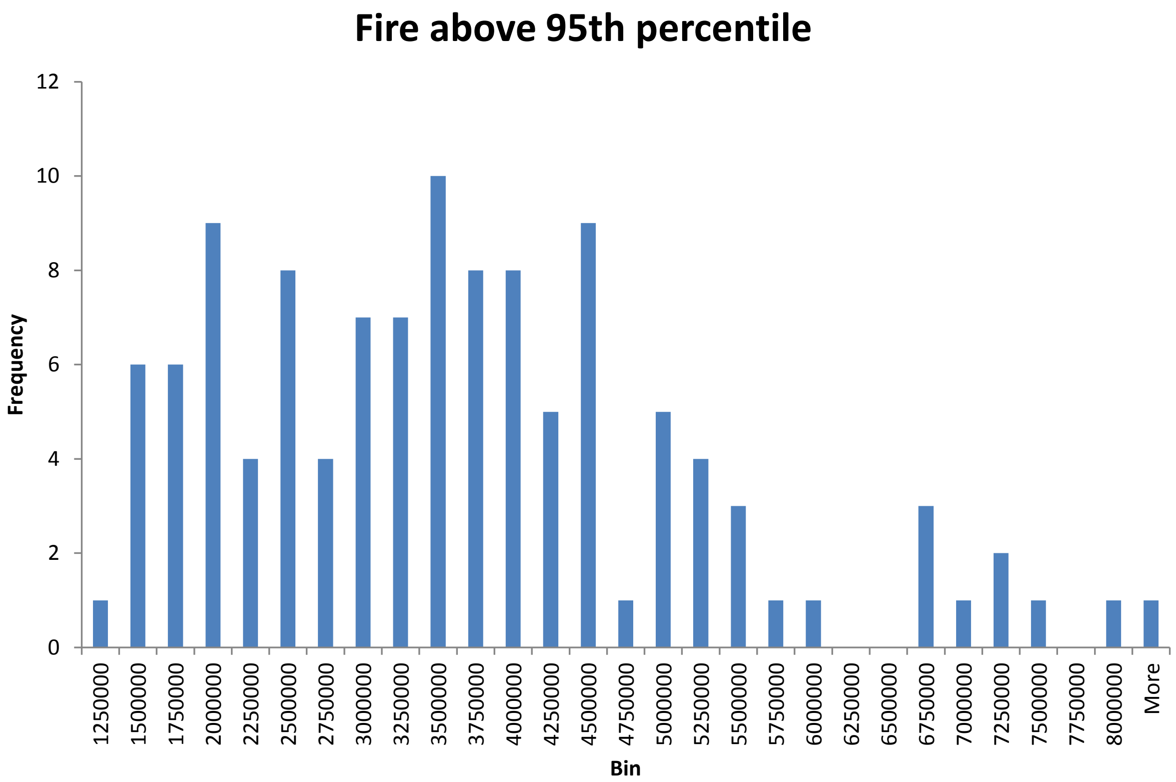
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



Searching for the model

- How is the final model for claim size selected?
- Traditional tools: QQ plots and criterion comparisons
- Transformations may also be used (see Erik Bølviken’s material)

Descriptive Statistics for Variable Skadeestimat	
Number of Observations	185
Number of Observations Used for Estimation	185
Minimum	331206.17
Maximum	9099311.62
Mean	2473661.54
Standard Deviation	1892916.16

Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching

Model Selection Table

Distribution	Converged	-2 Log Likelihood	Selected
Burr	Yes	5808	No
Logn	Yes	5807	No
Exp	Yes	5817	No
Gamma	Yes	5799	No
Igauss	Yes	5804	No
Pareto	Yes	5874	No
Weibull	Yes	5799	Yes

Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching

All Fit Statistics Table

Distribution	-2 Log Likelihood		AIC		AICC		BIC		KS		AD		CvM	
Burr	5808		5814		5814		5823		1,41053		3,46793		0,56310	
Logn	5807		5811		5812		5818		1,72642		4,17004		0,69457	
Exp	5817		5819		5819		5822		1,71846		4,02524		0,54964	
Gamma	5799		5803		5803		5809		1,20219	*	2,81879		0,45146	
Igauss	5804		5808		5808		5814		2,05008		5,23274		0,94893	
Pareto	5874		5878		5878		5884		2,49278		11,82091		1,91891	
Weibull	5799	*	5803	*	5803	*	5809	*	1,28745		2,64769	*	0,41281	*

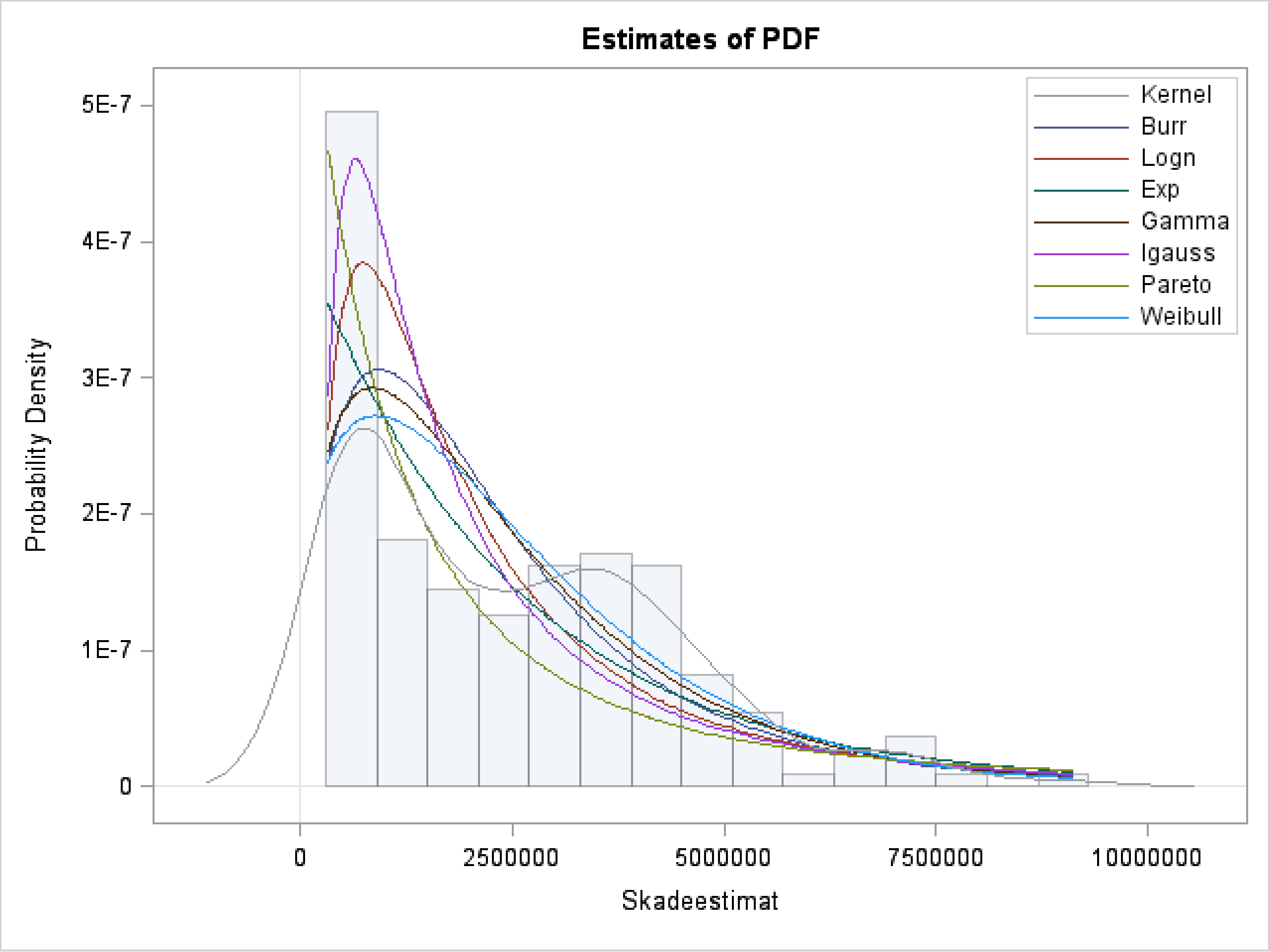
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



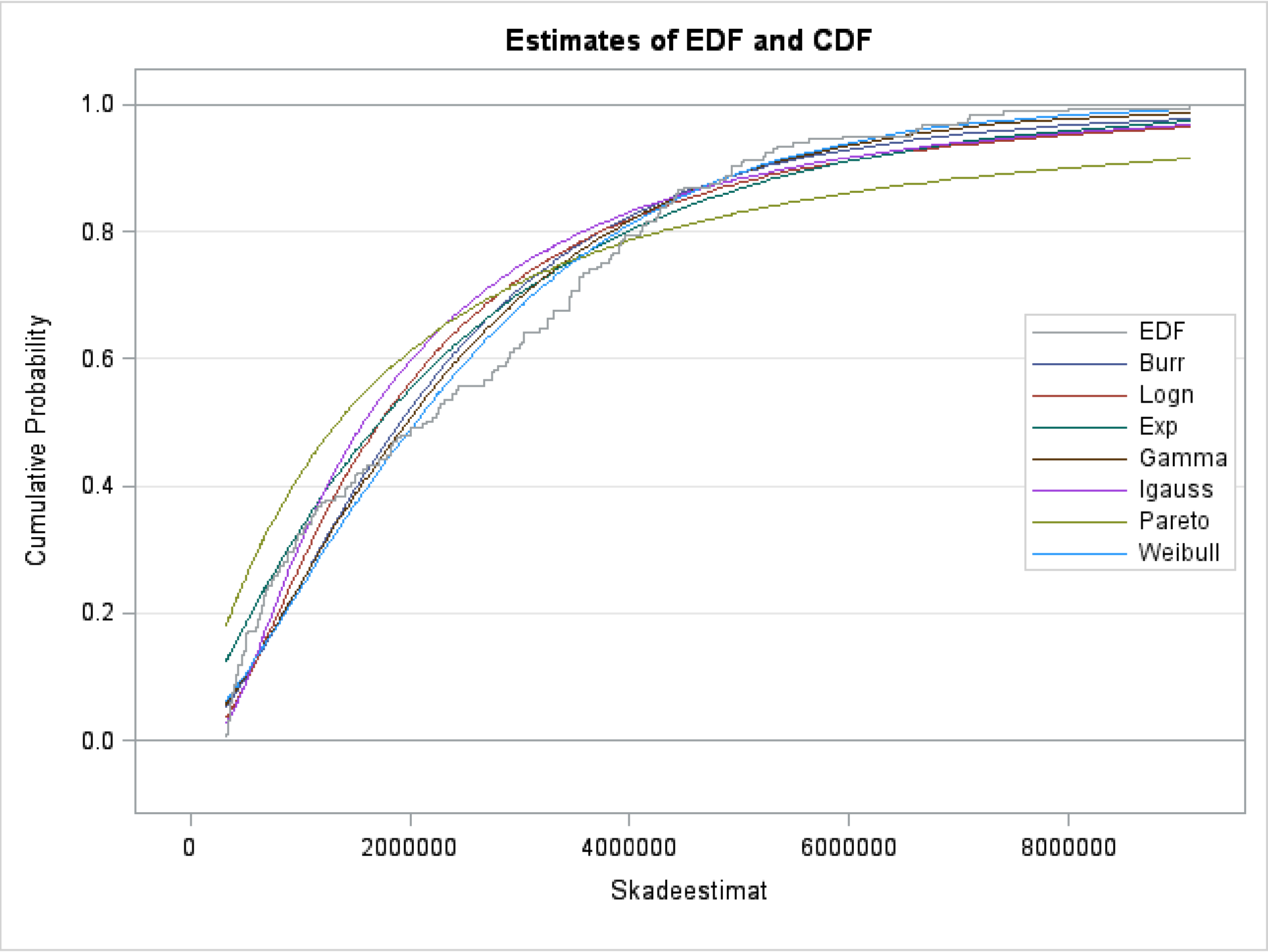
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



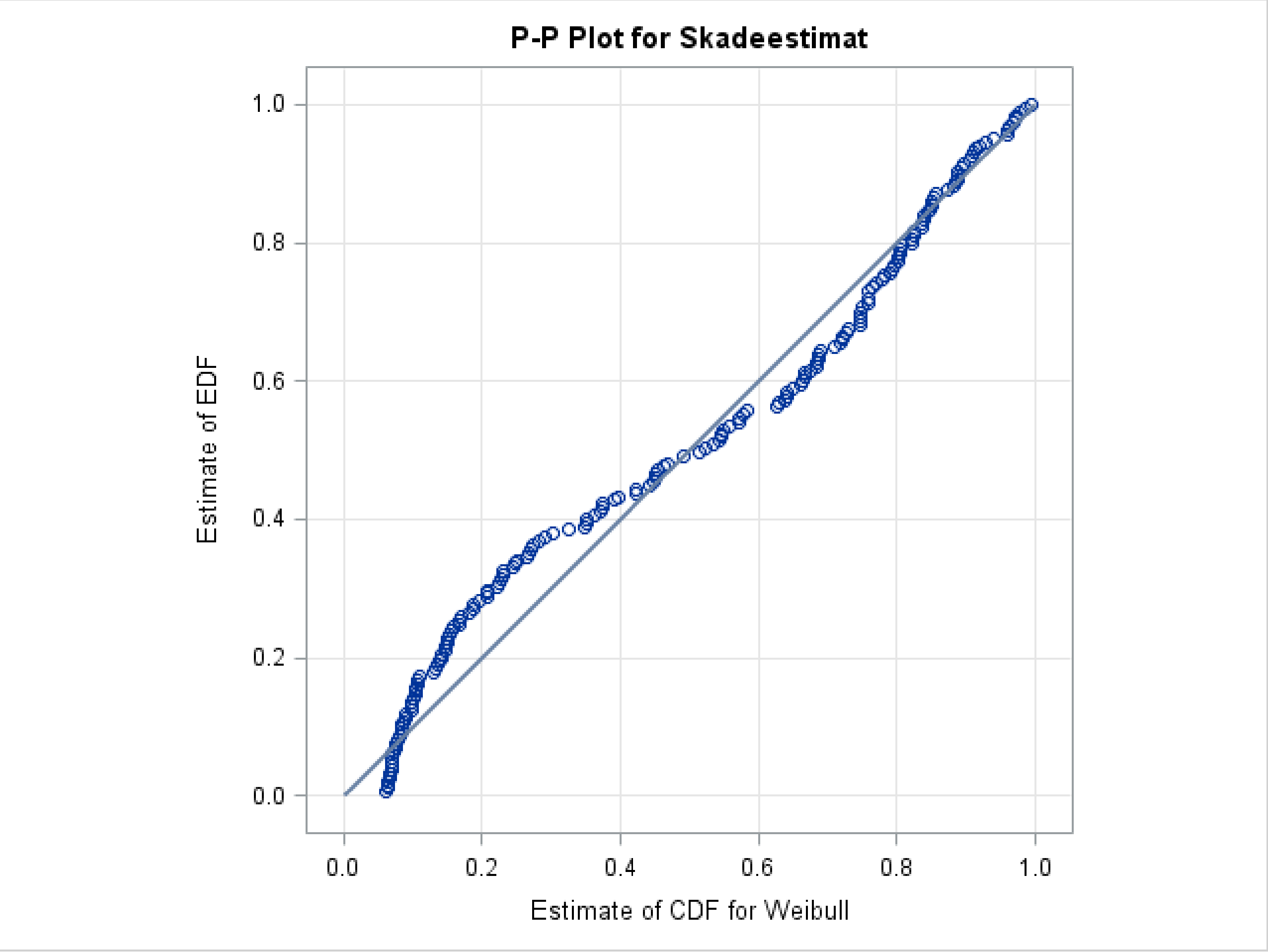
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching

Descriptive Statistics for Variable Skadeestimat	
Number of Observations	115
Number of Observations Used for Estimation	115
Minimum	1323811.02
Maximum	9099311.62
Mean	3580234.13
Standard Deviation	1573560.35

Non parametric
Log-normal, Gamma
The Pareto
Extreme value
Searching

Model Selection Table			
Distribution	**Converged**	**-2 Log Likelihood**	**Selected**
Burr	Yes	3590	No
Logn	Yes	3586	No
Exp	Yes	3701	No
Gamma	Yes	3587	No
Igauss	Yes	3586	Yes
Pareto	Yes	3763	No
Weibull	Yes	3598	No

- Non parametric
- Log-normal, Gamma
- The Pareto
- Extreme value
- Searching

All Fit Statistics Table

Distribution	-2 Log Likelihood		AIC		AICC		BIC		KS		AD		CvM	
Burr	3590		3596		3596		3604		0,60832		0,46934		0,05441	
Logn	3586		3590		3591		3596		0,78669		0,59087		0,08734	
Exp	3701		3703		3703		3706		3,41643		17,71328		3,44525	
Gamma	3587		3591		3591		3597		0,51905	*	0,41652	*	0,04250	*
Igauss	3586	*	3590	*	3590	*	3595	*	0,82086		0,61707		0,09611	
Pareto	3763		3767		3767		3773		4,11264		25,12499		5,09728	
Weibull	3598		3602		3602		3607		0,88778		1,12241		0,10294	

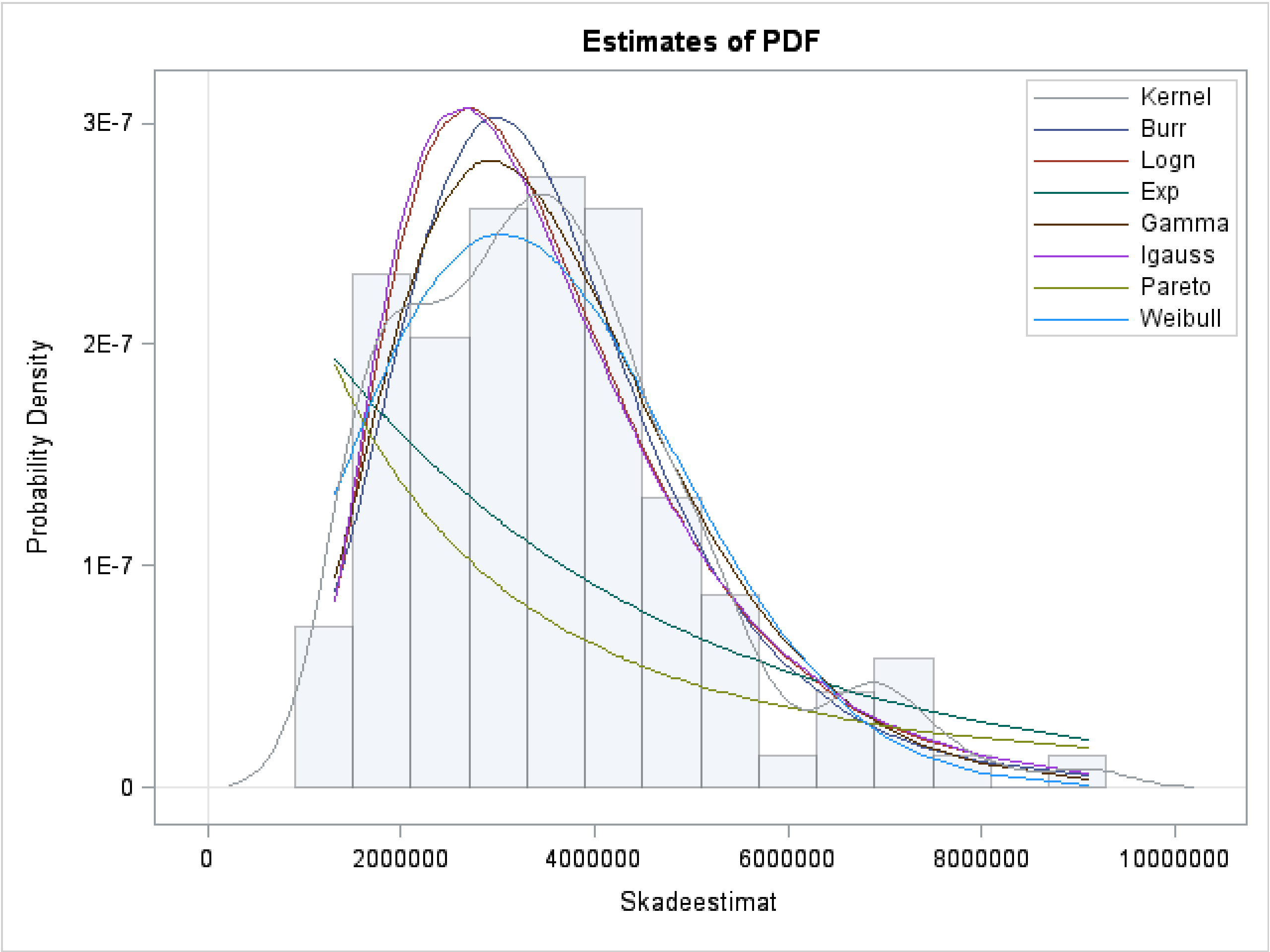
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



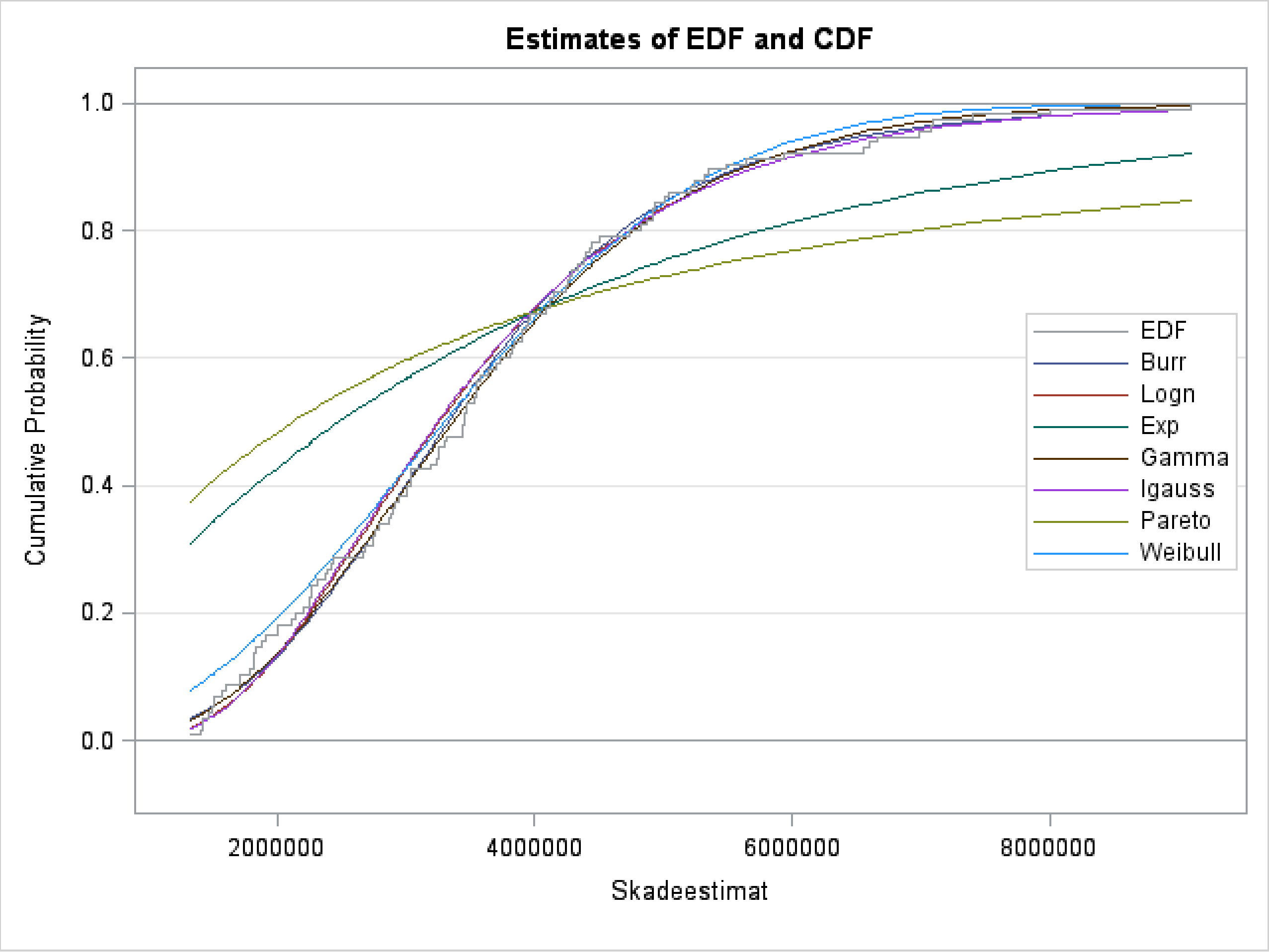
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



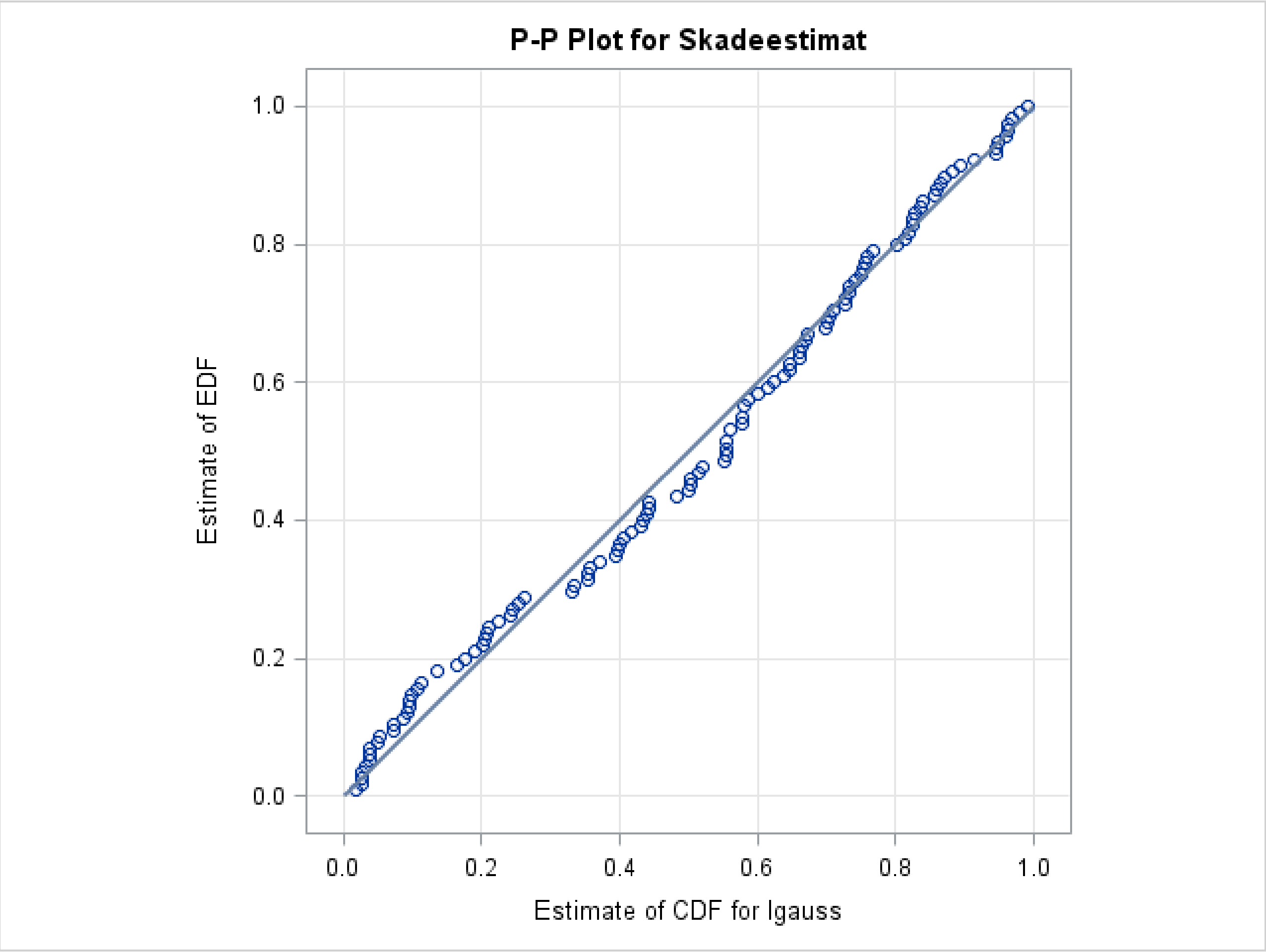
Non parametric

Log-normal, Gamma

The Pareto

Extreme value

Searching



Searching for the model

- Can we do better?
- Does it exist a more generic class of distribution with these distributions as special cases?
- Does this generic class of distributions outperform the selected model in the two examples (fire above 90th percentile and fire above 95th percentile)?