Non-life insurance mathematics

Nils F. Haavardsson, University of Oslo and DNB Skadeforsikring

Risk premium expresses cost per policy and is important in pricing

- Up to now we have been busy trying to answer the question:
  - What is our prediction of the risk premium into the near future, say the next 12 months?
- Risk premium is defined as P(Event)*Consequence of Event

$$\text{Risk premium} = P(\text{event}) * \text{Consequence of event}$$

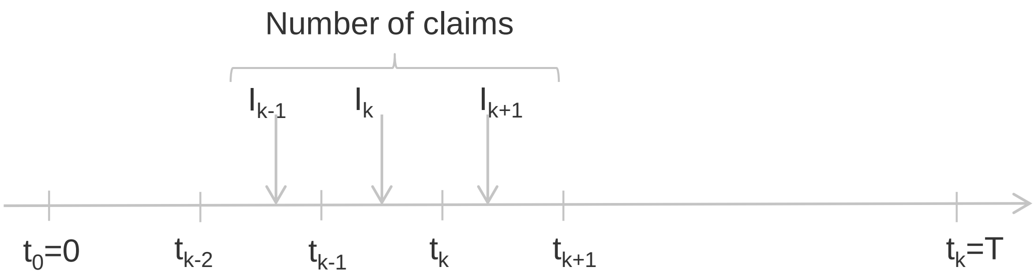
$$= \text{Claim frequency} * \text{Claim severity}$$

$$= \frac{\text{Number of claims}}{\text{Number of risk years}} * \frac{\text{Total claim amount}}{\text{Number of claims}}$$

$$= \frac{\text{Total claim amount}}{\text{Number of risk years}}$$

- From above we see that risk premium expresses cost per policy

The world of Poisson (Chapter 8)



- Divide a given time period T into K small pieces of equal length h=T/K

$$\Rightarrow \Pr(N=n) \underset{K\to\infty}{\to} \frac{(\mu T)^n}{n!} e^{-\mu T}$$

- Muh above is the claim intensity per time unit
- There are two ways of modelling variation in muh:
    - Assume muh stochastic then N is no longer Poisson
    - Assume log muh can be expressed as a linear predictor of covariates and covariate effects

Repetition claim size

- The concept
- Non parametric modelling
- Scale families of distributions
- Fitting a scale family
- Shifted distributions
- Skewness
- Non parametric estimation
- Parametric estimation: the log normal family
- Parametric estimation: the gamma family
- Parametric estimation: fitting the gamma

The ultimate goal for calculating the pure premium is pricing

Pure premium = Claim frequency x claim severity

$$Claim\ severity = \frac{total\ claim\ amount}{number\ of\ claims}$$

$$Claim\ \ frequency = \frac{number\ of\ claims}{number\ of\ \ policy\ \ years}$$

Parametric and non parametric modelling (section 9.2 EB)

The log-normal and Gamma families (section 9.3 EB)

The Pareto families (section 9.4 EB)

Extreme value methods (section 9.5 EB)

Searching for the model (section 9.6 EB)

Overview

Important issues	Models treated	Curriculum	Duration (in lectures)
What is driving the result of a non-life insurance company?	insurance economics models	Lecture notes	0,5
How is claim frequency modelled?	Poisson, Compound Poisson and Poisson regression	Section 8.2-4 EB	1,5
How can claims reserving be modelled?	Chain ladder, Bernhuetter Ferguson, Cape Cod,	Note by Patrick Dahl	2
How can claim size be modelled?	Gamma distribution, log-normal distribution	Chapter 9 EB	2
How are insurance policies priced?	Generalized Linear models, estimation, testing and modelling. CRM models.	Chapter 10 EB	2
Credibility theory	Buhlmann Straub	Chapter 10 EB	1
Reinsurance		Chapter 10 EB	1
Solvency		Chapter 10 EB	1
Repetition			1

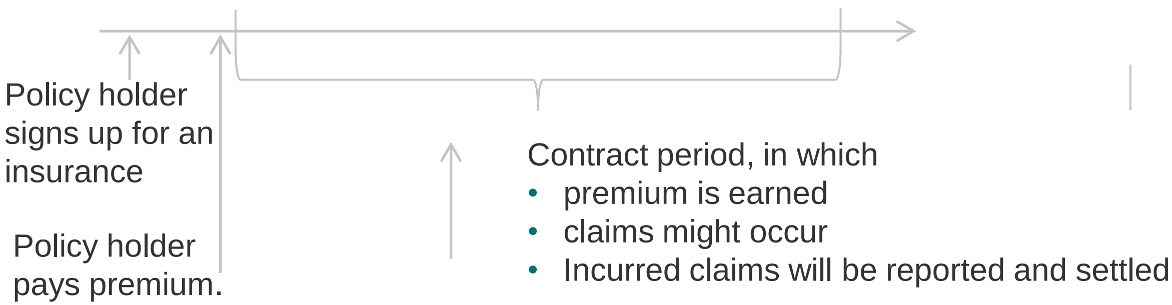
Overview of this session

- Introduction to reserving
- The chain ladder model (Patrick Dahl note)
- An example
- The naive loss ratio method (Patrick Dahl note)
- How can loss ratio be predicted?

Introduction to reserving

Non-life insurance from a financial perspective:

for a premium an insurance company commits itself to pay a sum if an event has occured



Issues that need to be solved:

- How much premium is earned?
- How much premium is unearned?
- How do we measure the number and size of unknown claims?
- How do we know if the reserves on known claims are sufficient?

Premium reserves

The premium reserve is split in two parts:

- Provision for unearned premiums
- Provisions for unexpired risks

Earned and unearned premium:

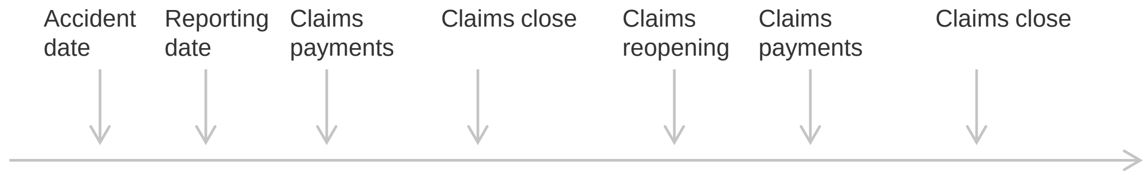
- Written premium is earned evenly/uniformly over the cover period
- The share of the premium that has been earned is the past time's proportion of the total period
- If a larger premium has been received the difference is the unearned premium

Example:
An insurance policy starts on September 1 2012 and is valid until August 31 2013.
The premium for the entire period is 2400.
At 31 December we have received two quarterly premiums or 1200.
We have then earned (4/12)*2400 = 800.
Unearned is 1200-800=400

Unexpired risk reserve

- Regard entire period covered by the insurance
- From a point in time, say 31/12-2012, we look forward to all the claims and expenses that could occur after this point. Call them FC_{3112}
- If FC_{3112}>Future premiums yet not due (FP)+unearned premium reserve (UP) the difference is accounted as unexpired risk reserve
- In example assume FC_{3112} = 1800>FP+UP=1200+400=1600, so unexpired risk reserve is 200

Claims reserves



Claims reserving issues:

- How do we measure the number and size of unknown claims? (IBNR reserve, i.e., Incurred Bot Not Reported)
- How do we know if the reserves on known claims are sufficient? (RBNS reserve, i.e., Reserved But Not Settled)

Claims reserves

Claims occuring:

- A *claim event* is an event that gives rise to a claim against an insurer by a policy holder
- *Gross claim loss*: the ultimate cost to the insurer of a claim event, including benefit payments and claims handling expenses
- *Net claim loss*: deduction of reinsurance recoveries

- Example: settlement delays are considered (RBNS). The process for estimating future reported claim amounts (IBNR) is similar.

- Step 1: Group the claims loss settlement amounts by the year in which the associated claims events occured

Claims occurence year	Claims losses settled
2005	3963
2006	4975
2007	5873
2008	6401
2009	6563
2010	6358
2011	6918
2012	3072

Claim payments plus claims handling expenses

The development of claims losses settled

- Claims losses settled for each claims occurence year are often not paid on one date but rather over a number of years

Incremental claims loss settlement data presented as a run-off triangle

Incremental claims loss settlements		Development year							
		0	1	2	3	4	5	6	7
Claims occurence year	2005	1232	946	520	722	316	165	48	14
	2006	1469	1201	708	845	461	235	56	
	2007	1652	1416	959	954	605	287		
	2008	1831	1634	1124	1087	725			
	2009	2074	1919	1330	1240				
	2010	2434	2263	1661					
	2011	2810	4108						
	2012	3072							

Comments:

- *The development year* for a claims settlement amount reflects how long after the claims occurence year the amount was settled.
  - An amount settled during the claims occurence year was settled in development year 0
- In the example the largest development year for any claims occurence years is 7
- The data shown represents the incremental claims losses settled in the development year
- For any cell in the table, the value shown represents the incremental claims loss amount that was settled in calendar year
- Each diagonal set of data represents the amounts settled in a single calendar year
- Green cells represent observed data – all red represent time periods in the future for which we wish to estimate the expected claims settlements amounts

The development of claims losses settled

- Claims losses settled for each claims occurence year are not paid on one date but rather over a number of years

Incremental claims loss settlement data presented as a run-off triangle

Cumulative claims loss settlements		Development year							
		0	1	2	3	4	5	6	7
Claims occurence year	2005	1232	2178	2698	3420	3736	3901	3949	3963
	2006	1469	2670	3378	4223	4684	4919	4975	
	2007	1652	3068	4027	4981	5586	5873		
	2008	1831	3465	4589	5676	6401			
	2009	2074	3993	5323	6563				
	2010	2434	4697	6358					
	2011	2810	6918						
	2012	3072							

Comments:

- The data can be presented as cumulative losses settled
- For each claims occurence year the incremental claims loss settled for a particular development year is the amount settled in that development year
- *The cumulative claims losses settled* is the total amount settled up to that development year, i.e., the sum of the incremental claims losses settled up to that date.

Assumptions underlying the CLM

- Patterns of claims loss settlement observed in the past will continue in the future
- The development of claims loss settlement over the development years follows an identical pattern for every claims occurence year
- But the observed claims loss settlement patterns may change over time:
  - Changes in product design and conditions
  - Changes in the claims reporting, assessment and settlement processes (example: different owners)
  - Change in the legal environment
  - Abnormally large or small claim settlement amounts
  - Changes in portfolio so that the history is not representative for predicting the future (example: strong growth)

Development patterns and development factors

- The insurer may tend towards using any of the following patterns for estimation purposes:
- The proportion of the ultimate cumulative claims losses that is settled in a particular development year (development pattern for incremental claims losses settled)
- The proportion of the ultimate cumulative claims losses that is settled by a particular development year (development pattern for cumulative claims losses settled)
- The ratio of the cumulative losses settled by a particular year to the cumulative claims losses settled by the previous development year (cumulative loss factor)
- The three patterns are equivalent
- CLM relies on the last pattern above holds for all claims occurence years. For any development year the quotient
  - *Expected cumulative claims losses settled up to and including the development year/Expected cumulative claims losses settled up to and including the previous development year*

  is called the *cumulative claims loss settlement factor* for that development year
- Example: the cumulative claims settlement factor for development year 4 is derived from the cumulative settlement amounts for development years 3 and 4.

Estimating future claims settlement amounts

- Underlying assumption (CLM):
  - the cumulative claims loss settlement factor for a specific development year is assumed to be the same for all claims occurence years
- The CLM estimator for each of the factors is based on the cumulative settlement data for as many claims occurence years as possible

CLM in practice

Determining the CLM estimator for the cumulative claims loss settlement factor

Cumulative claims loss settlements		Development year							
		0	1	2	3	4	5	6	7
Claims occurence year	2005	1232	2178	2698	3420	3736	3901	3949	3963
	2006	1469	2670	3378	4223	4684	4919	4975	
	2007	1652	3068	4027	4981	5586	5873		
	2008	1831	3465	4589	5676	6401			
	2009	2074	3993	5323	6563				
	2010	2434	4697	6358					
	2011	2810	6918						
	2012	3072							
CLM estimator for claims loss settlement factor			1,9989	1,3140	1,2422	1,1151	1,0491	1,0118	1,0035

3736+4684+5586+6401 =20407

3736+4684+5586+6401 =20407

20407/18300=1,1151

Comments:

- These CLM estimators for the cumulative claims loss settlement factors are used to estimate the cumulative claims loss settlement amount in the future
- For each claims occurence year the last historical observation is used together with the appropriate CLM estimator for the development factor to estimate the cumulative settlement amount in the next development year
- This value is, in turn, multiplied by the estimator for the development factor for the next development year and so on.

CLM in practice

Determining the estimated cumulative claims loss settlements in future periods

Cumulative claims loss settlements		Development year							
		0	1	2	3	4	5	6	7
Claims occurence year	2005	1232	2178	2698	3420	3736	3901	3949	3963
	2006	1469	2670	3378	4223	4684	4919	4975	4993
	2007	1652	3068	4027	4981	5586	5873	5942	5963
	2008	1831	3465	4589	5676	6401	6401 *1,0491 = 6715	6715 *1,0118 = 6794	6794 *1,0035 = 6818
	2009	2074	3993	5323	6563	7319	7678	7768	7796
	2010	2434	4697	6358	7898	8807	9239	9348	9381
	2011	2810	4918	6462	8027	8952	9391	9502	9535
	2012	3072	5686	7471	9280	10349	10857	10985	11024
CLM estimator for claims			1,8508	1,3140	1,2422	1,1151	1,0491	1,0118	1,0035

Comments:

- The values shown in the red cells are the estimators for future cumulative claims settled
- These estimates are always based on the latest available cumulative claims settlement amounts for the relevant claims occurence year, i.e., the estimated future cumulative claims settlements are always based on the last green diagonal of data
- It is now simple to derive the estimated incremental claims settlement amounts for the future periods
- An incremental settlement amount is the difference between tow consecutive cumulative settlement amounts

CLM in practice

Determining the estimated incremental settlement amounts from the estimated cumulative amounts

Incremental claims loss settlements		Development year							
		0	1	2	3	4	5	6	7
Claims occurence year	2005	1232	946	520	722	316	165	48	14
	2006	1469	1201	708	845	461	235	56	18
	2007	1652	1416	959	954	605	287	69	21
	2008	1831	1634	1124	1087	725	**6715-6401** = 314	**6794-6715** = 79	**6818-6794** = 24
	2009	2074	1919	1330	1240	756	359	91	28
	2010	2434	2263	1661	1540	909	432	109	33
	2011	2810	4108	1544	1565	924	439	111	34
	2012	3072	2614	1785	1810	1069	508	128	39

CLM in practice

Determining the estimated incremental settlement amounts from the estimated cumulative amounts

Incremental claims loss settlements	Development year							
Claims occurence year	0	1	2	3	4	5	6	7
2005								
2006								18
2007							69	21
2008						314	79	24
2009					756	359	91	28
2010				1540	909	432	109	33
2011			1544	1565	924	439	111	34
2012		2614	1785	1810	1069	508	128	39

Calendar year	Estimated claims loss settlement amounts
2013	6855
2014	4718
2015	2181
2016	1069+439+109+28=1645
2017	652
2018	162
2019	39

Comments:

- Group the estimated incremental claims loss settlement amounts by the year in which they will be settled
- These cash flows can then be discounted to determine the technical provisions
- Norwegian State Treasury Bonds (Statsobligasjoner in Norwegian) may be used as discount factor
  - Example: a cash flow due in 2017 is discounted with a 4 year old Norwegian State Treasury Bond etc. Why do we hope that the development year does not exceed 10 ??

Claims reserves

- Claims reserving is not only about statistical models:
- What is the purpose of the reserving?
- Know your data:
  - Is the history consistant? (relevant when a company has had several owners)
  - Is the history representative when future predictions are to be made? (relevant during strong growth)
  - Has there been significant events that affect settlement practice? (relevant for agencies)
- Should the reserves be calculated gross or net?
- Adjust for inflation
- Know your claims department:
  - How are provisions set?
  - When do they set the provision?
  - How will the lead time be affected by the size of the claims?
  - Are there any backlogs?
  - When do they change the reserves?

Notation

Accident year	Reporting year 1	2	3
1998	C_{11}	C_{12}	C_{13}
1999	C_{21}	C_{22}	
2000	C_{31}		

C_{ij} Cumulative claims from accident year i, reported through the end of period j

D_{ij} $= C_{ij} - C_{ij-1}$ Incremental claims from accident year i, reported in period j

C_{im} Ultimate claims for accident year i, where

m is the last development period that is known and

R_i $= E[C_{im}] – C_{ij}$ is the reserve for accident year i

f_i one period loss development factor. Also called age-to-age factor or link ratio

F_{ij} Development factor from accident year i, period j, to ultimate

L_i Claims relative to an exposure for accident year i

P_i A measure of exposure for accident year i

A_k Experience up to development period k

Chain ladder

Chain ladder builds on that cumulative claims in a period are proportional to the claims in the preceding period. The proportionality factor depends on the number of periods since outset, but is expected to be the same for all periods. It is assumed that:

$$(CL1)\quad E[C_{ij+1} \mid C_{i1}, C_{i2}, \ldots, C_{ij}] = C_{ij} * f_j$$

Observe that f_j does not depend on accident year.

$$(CL2)\quad \text{The vectors } \{C_{i1}, C_{i2}, \ldots, C_{im}\}$$
$$\text{and } \{C_{k1}, C_{k2}, \ldots, C_{km}\} \text{ are independent if } i \neq k$$

CL1 just brings us one step ahead, whereas we want to get to the end. To get there we are going to utilize the rule of double expectation:

$$(\mathit{Lemma}\ 6.1)\quad \text{If } \mathrm{E[Z]} \text{ is finite, then } \mathrm{E[E[Z \mid X]]}$$

Using this lemma and CL1, we find

Chain ladder

$$E[C_{ij+k} \mid C_{i1}, C_{i2}, ..., C_{ij}] =$$
$$= E[E[C_{ij+k} \mid C_{i1}, C_{i2}, ..., C_{ij+k-1}] \mid C_{i1}, C_{i2}, ..., C_{ij}] =$$
$$= E[C_{ij+k-1} * f_{j+k-1} \mid C_{i1}, C_{i2}, ..., C_{ij}] =$$
$$= E[C_{ij+k-1} \mid C_{i1}, C_{i2}, ..., C_{ij}] * f_{j+k-1} =$$
$$= E[E[C_{ij+k-1} \mid C_{i1}, C_{i2}, ..., C_{ij+k-2}] \mid C_{i1}, C_{i2}, ..., C_{ij}] * f_{j+k-1} =$$
$$= E[C_{ij+k-2} * f_{j+k-2} \mid C_{i1}, C_{i2}, ..., C_{ij}] * f_{j+k-1} =$$
$$= E[C_{ij+k-2} \mid C_{i1}, C_{i2}, ..., C_{ij}] * f_{j+k-1} * f_{j+k-2} =$$
$$= ... = C_{ij} * f_j * f_{j+1} * f_{j+2} * .... * f_{j+k-1} \qquad (6.2)$$

Chain ladder

We could rewrite CL1 on the following form:

$$(CL1')\ \ E[C_{ij+1} / C_{ij} \mid C_{i1}, C_{i2}, ..., C_{ij}] = f_{ij}$$

Thus, we could use observed ratios C_{ij+1}/C_{ij} as unbiased estimators of f_j. Before combining estimates of the same f_j we make a further assumption,

$$(CL3)\ \ Var[C_{ij+1} \mid C_{i1}, C_{i2}, ..., C_{ij}] = C_{ij} * \sigma_j^2$$

Observe that the last factor in the variance is no depending on accident year. We also need the following Lemma

Chain ladder

(*Lemma* 6.2) Suppose X_i are uncorrelated random variables with the same mean, but with variances σ_i^2. Then the best linear unbiased estimator of the mean is given by

$$\sum_{i=1}^{n} w_i X_i \text{ where } w_i = \frac{C_{ij}}{\sum_{k=1}^{m-j} C_{kj}} \text{ and } \sum_{i=1}^{n} w_i = 1$$

From Lemma 6.2 it turns out that

$$\hat{f}_j = \frac{\sum_{i=1}^{m-j} C_{ij+1}}{\sum_{i=1}^{m-j} C_{ij}}$$

Chain ladder

Proof: Form the Lagrangian

Constraint

$$L(w_1,\ldots,w_n,\lambda)=\underbrace{\sum_{i=1}^{n} w_i^2\sigma_i^2}_{\mathrm{Var}\sum_{i=1}^{n} w_i X_i\text{, which we want to minimize}}+\lambda(\overbrace{1-\sum_{i=1}^{n} w_i}^{\text{Constraint}})$$

Var $\sum_{i=1}^{n} w_i X_i$, which we want to minimize

Solve the system

$$\begin{cases} \dfrac{\partial}{\partial w_k} L(w_1,\ldots,w_n,\lambda)=0, \quad \mathrm{k}=1,\ldots,\mathrm{n} & (1) \\ \dfrac{\partial}{\partial \lambda} L(w_1,\ldots,w_n,\lambda)=0 & (2) \end{cases}$$

Chain ladder

Proof: Starting with (1) yields

$$\frac{\partial}{\partial w_k} L(w_1,\ldots,w_k,\lambda) = 2w_k\sigma_k^2 - \lambda = 0$$

$$\Leftrightarrow \lambda = 2w_k\sigma_k^2 \Leftrightarrow w_k = \frac{\lambda}{2\sigma_k^2}$$

Continuing with (2) yields

$$\frac{\partial}{\partial \lambda} L(w_1,\ldots,w_k,\lambda) = 1 - \sum_{i=1}^{n} w_i = 0$$

$$\Leftrightarrow \sum_{i=1}^{n} w_i = 1 \overset{(3)}{\Leftrightarrow} \sum_{i=1}^{n} \frac{\lambda}{2\sigma_i^2} = 1 \Leftrightarrow \lambda = 2\frac{1}{\sum_{i=1}^{n}\sigma_i^{-2}} \qquad (4)$$

Chain ladder

Combining (3) and (4) yields

$$w_k = \frac{2\frac{1}{\sum_{i=1}^{n}\sigma_i^{-2}}}{2\sigma_i^2} = \frac{\sigma_i^{-2}}{\sum_{i=1}^{n}\sigma_i^{-2}} \qquad (5)$$

Rewriting CL3 gives

$$(CL3')\ Var[C_{ij+1}/C_{ij} \mid C_{i1}, C_{i2}, \ldots, C_{ij}] = \frac{1}{C_{ij}^{\ 2}} C_{ij} * \sigma_j^2 = \frac{\sigma_j^2}{C_{ij}}$$

and the weights are thus by (5)

$$w_i = (\sigma_j^2 / C_{ij})^{-1} / \sum_{k=1}^{m-j} (\sigma_j^2 / C_{kj})^{-1} = C_{ij} / \sum_{k=1}^{m-j} C_{kj}$$

Chain ladder

and

$$\hat{f}_j = \sum_{i=1}^{m-j} w_i \hat{f}_{ij} = \sum_{i=1}^{m-j} \frac{C_{ij}}{\sum_{k=1}^{m-j} C_{kj}} \frac{C_{ij+1}}{C_{ij}} = \frac{\sum_{i=1}^{m-j} C_{ij+1}}{\sum_{i=1}^{m-j} C_{ij}} \qquad (6)$$

To be able to use the algorithm suggested by formula (6.2) with the estimators from (6) above we need to prove that estimates are uncorrelated. Define the set of experience up to development period k by

$$A_k = \{C_{ij} \mid j \leq k, i \leq m\}$$

Then we have

Chain ladder

$$E[\hat{f}_j * \hat{f}_k] = E[E[\hat{f}_j * \hat{f}_k \mid A_k]] = E\ \hat{f}_j *[E[\hat{f}_k \mid A_k]]$$

$$= E\ \hat{f}_j *[E[(\sum_{v=1}^{m-k} C_{vk+1}) / (\sum_{v=1}^{m-k} C_{vk}) \mid A_k]] =$$

$$= E[\hat{f}_j *[E[(\sum_{v=1}^{m-k} C_{vk+1}) \mid A_k] / (\sum_{v=1}^{m-k} C_{vk})] =$$

$$= E[\hat{f}_j *[\sum_{v=1}^{m-k} E[C_{vk+1} \mid C_{vk}] / (\sum_{v=1}^{m-k} C_{vk})] =$$

$$= E[\hat{f}_j *[f_k * (\sum_{v=1}^{m-k} C_{vk}) / (\sum_{v=1}^{m-k} C_{vk})] =$$

$$= E[\hat{f}_j] * f_k = E[\hat{f}_j] * E[\hat{f}_k]$$

Chain ladder

Thus we have shown that the estimates of f_j and f_k are uncorrelated. If we combine this with (6.2) it shows that the following ultimate estimator is unbiased,

$$E[C_{im} \mid C_{ij}] = C_{ij} * \hat{f}_j * \ldots * \hat{f}_{m-1}$$

Chain ladder - example

- Example: one fire/combined product (fire/combined in Norway includes home, contents and cabin)
- The triangle shows the payments for the different years
- How do we fill out the blanks?

	1	2	3	4	5
2008	7 008 148	25 877 313	31 723 256	32 718 766	33 019 648
2009	30 105 220	65 758 082	76 744 305	79 560 296	
2010	89 181 138	171 787 015	201 380 709		
2011	109 818 684	198 015 728			
2012	97 250 541				

- We start by estimating f_1, f_2, f_3, f_4

f_1_hat	1,954
f_2_hat	1,176
f_3_hat	1,035
f_4_hat	1,009

$$\hat{f}_1 = \frac{\sum_{i=1}^{5-1} C_{i1+1}}{\sum_{i=1}^{5-1} C_{i1}} = \frac{C_{12}+C_{22}+C_{32}+C_{42}}{C_{11}+C_{21}+C_{31}+C_{41}} =$$

$$= \frac{25.9+65.8+171.8+198.0}{7.0+30.1+89.2+109.8} = 1.954$$

Chain ladder - example

- Then we use the formula $E[C_{im} | C_{ij}] = C_{ij} * f_j * \ldots * f_{m-1}$ to fill out the blanks

EksempelChainLadder - Microsoft Excel

B4 | 2010

	A	B	C	D	E	F	G	H	
1	Tidsperioden	Tidsperioden	Periode + 0	Periode + 1	Periode + 2	Periode + 3	Periode + 4	(Tidsperioden)	Utb
2	0	2008	7008147,76	25877312,92	31723256,17	32718766,17	33019648,17		0
3	1	2009	30105219,65	65758082,38	76744304,57	79560296,33	80291933,365	731637,03533	0,91960
4	2	2010	89181137,64	171787014,81	201380708,65	208457137,03	210374110,31	8993401,6569	4,46587
5	3	2011	109818684,47	198015727,55	232914240,17	241098742,8	243315888,5	45300160,954	22,8770
6	4	2012	97250541,11	190057610,37	223553575,54	231409148,58	233537188,71	136286647,6	140,139
9								=G4-E4	=F3*D
10			f_1_hat	=SUMMER(D2:D5)/SUMMER(C2:C5)				=G5-D5	=E4*D
11			f_2_hat	=SUMMER(E2:E4)/SUMMER(D2:D4)				=G6-C6	=D5*[
12			f_3_hat	=SUMMER(F2:F3)/SUMMER(E2:E3)					
13			f_4_hat	=G2/F2					
24									
25		1	2	3	4	5	skadereserver		
26	2008	7008147,76	25877312,92	31723256,17	32718766,17	33019648,17	=0		
27	2009	30105219,65	65758082,38	76744304,57	79560296,33	=D13*E27	=F27-E27		
28	2010	89181137,64	171787014,81	201380708,65	=D28*D12	=E28*D13	=F28-D28		
29	2011	109818684,47	198015727,55	=D5*D11	=D29*D12	=E29*D13	=F29-C29		
30	2012	97250541,11	=B30*D10	=C30*D11	=D30*D12	=E30*D13	=F30-B30		
31	totale reserver						=SUMMER(G26:(		

ChainLadder | Ark1 | BonhuetterFerguson

The naive loss ratio method

- This method assumes that we a priori know the ultimate losses share of the premium, *Pi* . This is
- usually referred to as the ultimate loss ratio, *Li* .
- Li could come from
  - the pricing calculations or from
  - “guesstimates” by e.g. account executives or
  - fire engineers according to their experience (the infamous “underwriting judgment”),

$$E[\hat{C}_{im}] = L_i P_i$$

- Thus, the necessary IBNR reserve will be the difference between the ultimate losses and the reported claims

$$R_{ij} = L_i P_i - C_{ij}$$

- This method does not presupppose anything about the claims location in time, nor does it differentiate between actual claims or expected claims, it simply sees them as communicating vessels.
- This method is simplistic and have its most proponents among the “practical men”.
- It has limited value outside the case in the early life of an accident year when just a few and small claims are known.

How can loss ratio be predicted?

- Example: One fire/combined product
- Individual losses adjusted for
  - inflation with appropriate index (FNO's index for home and cabin, CPI for contents)
  - portfolio development measured with indexed Earned Premium adjusted for rate changes

One fire/combined product

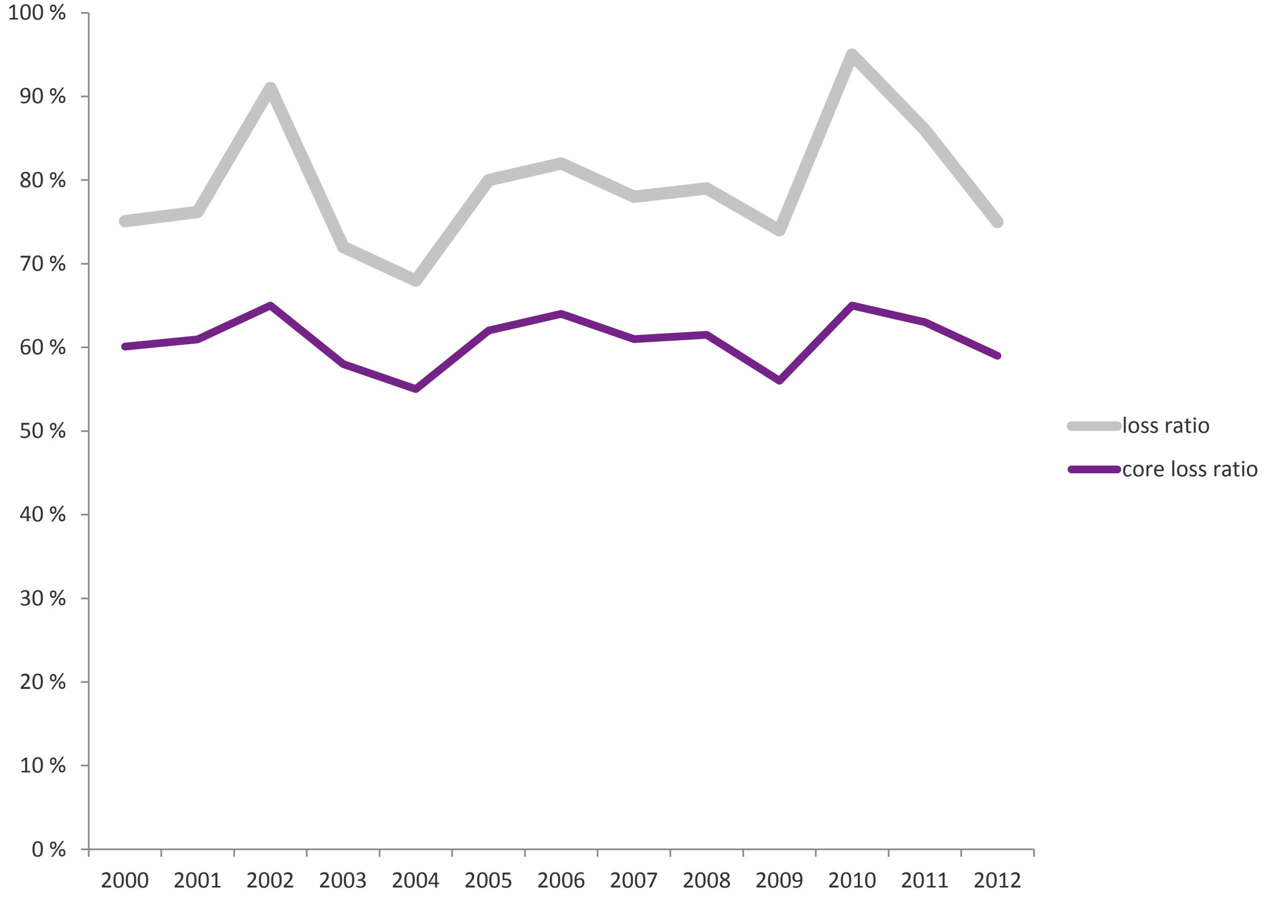
Portfolio development overview

- Portfolio development measured with indexed EPI adjusted for rate increases

Year	Original	Rate changes	inflation 2013	trend
2004	50,3	65,4	98,1	4,1
2005	67,2	87,4	126,7	3,2
2006	84,3	109,6	153,4	2,6
2007	106,7	138,7	187,3	2,1
2008	127,2	165,4	215,0	1,9
2009	225,3	292,9	366,1	1,1
2010	254,1	330,3	396,4	1,0
2011	296,7	311,5	358,3	1,1
2012		347,0	364,4	1,1
2013			400,8	1,0

Loss Ratios

Average adj. Core L/R = 54%, CV = 9.5% (increased to 12.5%)



Loss Ratios

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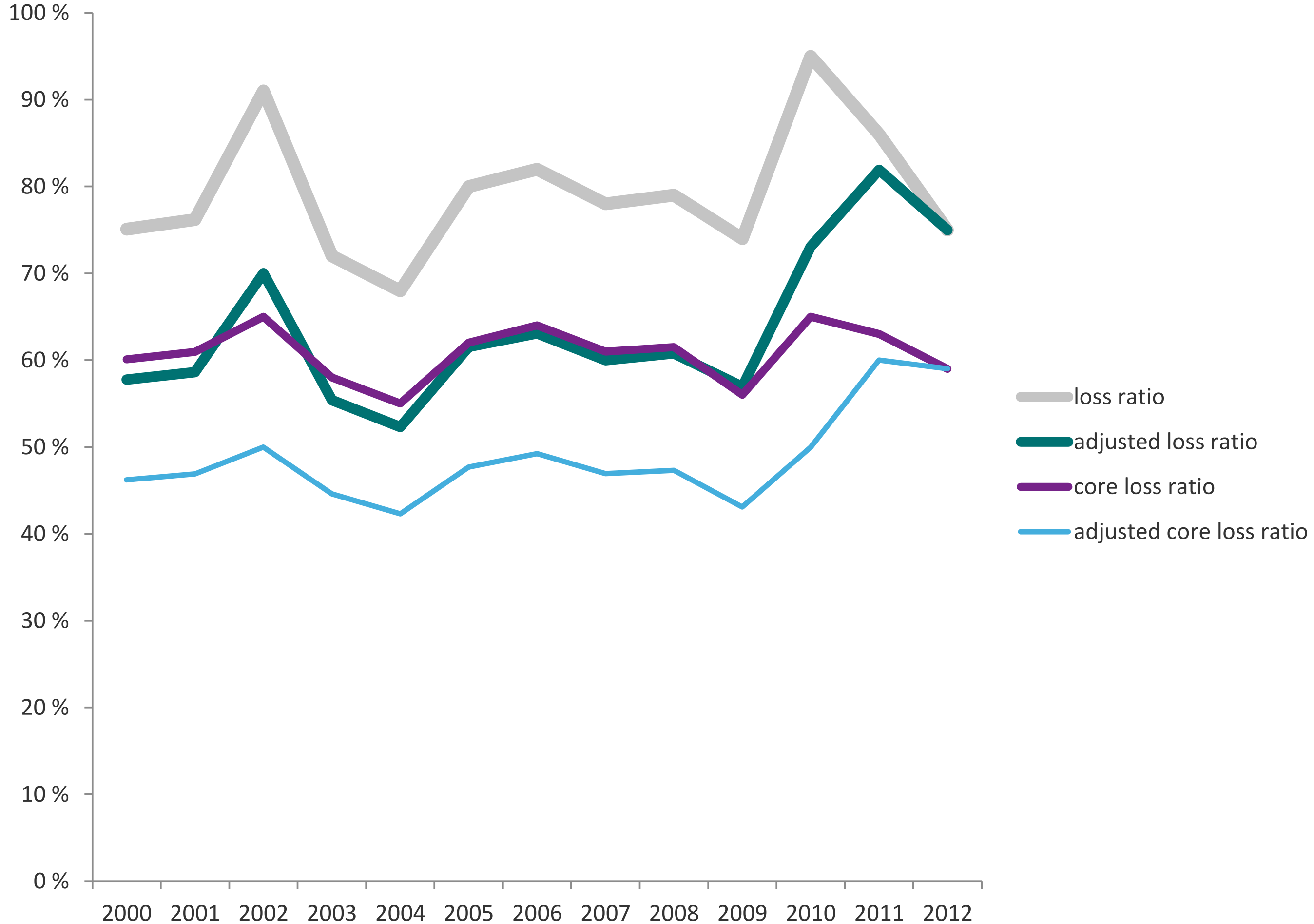
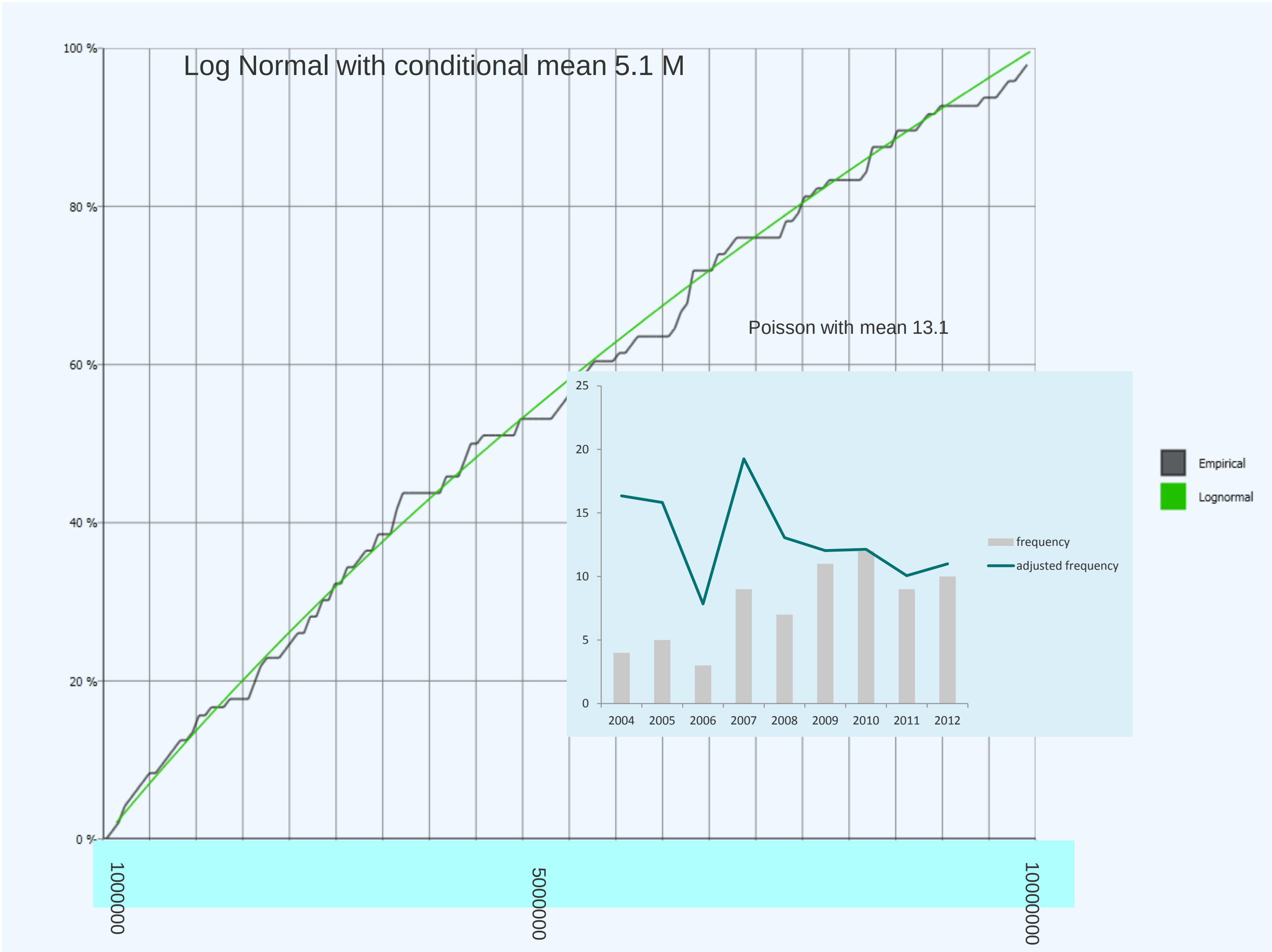


Individual losses between NOK 1 M and NOK 10 M Frequency and Severity fit

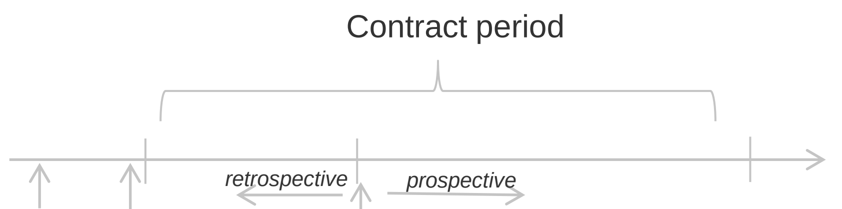


Fire/combined summary

	Frequency	Severity	Loss ratio
Core	1	220	55,0 %
Large	13,1	5,1	16,7 %
			71,7 %

Introduction to reserving

Non-life insurance from a financial perspective:
for a premium an insurance company commits itself to pay a sum if an event has occured



Policy holder signs up for an insurance

Policy holder pays premium. Insurance company starts to earn premium

During the duration of the policy, some of the premium is earned, some is unearned
- How much premium is earned?
- How much premium is unearned?
- Is the unearned premium sufficient?

Premium reserve, *prospective*

During the duration of the policy, claims might or might not occur:
- How do we measure the number and size of unknown claims?
- How do we know if the reserves on known claims are sufficient?

Claims reserve, *retrospective*