Delayed claims

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1 Introduction

Settling claims in general insurance is not an instantaneous process. Consider a big industrial fire, a hurricane, a flood or as an extreme case the World Trade Centre affair in 2001 where two skyscrapers in New York City were hit by aeroplanes. That lead to their collapse with huge loss of human life and with other buildings in the neighbourhood having to be demolished as well. It takes time to sort the cost of such things out, and the insurance industry has to reserve money to deal with them even though a substantial part of the expenses may belong to the future. Incidents such as those are known as Reported-But-Not-Settled or **RBNS** for short. They may be less exotic than big storms or aeroplanes having destoryed skyscrapers. One case is personal injuries following a fire or a motor accident; it may well be a slow process to decide on what a company should eventually pay, especially if there are legal disputes involved. An insurance portfolio has at any point in time unliquidated claims that are pending, and those are by definition of the RBNS type.

Time delays have a second reason too. Sometimes the events that have occured, simply haven't been discovered or brought to the attention of an insurance company. A prominent example of such Incurred-But-Not-Reported (**IBNR**) incidents are whiplash injuries in motor insurance where neck or back ailments following a car crash only show up several or even many years after the event. Yet by experience they do appear, and again insurance companies are required to reserve money to deal with them.

The purpose of this note is to introduce the **chain ladder**, the most commonly used method for dealing with delayed claims. For a huge number of other methods and ideas on this rather limited problem you may consult the monograph by Wütrich and Merz (2008) which are pretty heavy-handed mathematically; see also Dahl (2003) for a reference with the same flavour. Mathematical notation and indexing below differ slightly from what you will find in these references.

2 The approach

Virtually all methods in practical use are in terms of aggregates of portfolio losses rather than the breakdown on claim numbers and sizes used elsewhere in general insurance. One of the reasons is that an IBNR or RBNS claim may involve several payments which may be cumbersome to handle in the usual manner. Suppose we are at the end of year 0 with the past as $-j$ for $j = 0, 1, \ldots, J$, and let X_{jk} be the the sum of claims originating in year $-j$ which are settled in year $-j + k$ for $k = 0, \ldots, K$ where K is the maximum delay. There is then partition

$$\underset{\textit{Observed}}{X_{j0}, \ldots, X_{jj}} \qquad \underset{\textit{Belongs to the future}}{X_{jj+1}, \ldots, X_{jK}}$$

where those on the left are known at the end of year 0. Out aim is to predict the future ones on the right, in particular the total losses in year k which is for $k = 1, \ldots, K$

$$X_{+k} = \sum_{j=0}^{j_k} X_{jj+k} \qquad \text{where} \qquad j_k = \min(J, K - k). \tag{1}$$

To understand the details here note that X_{jj+k} is settled in year $-j + k$, and we should not for given k follow the sequence further back than to year $-j = -K + k$ (which corresponds to the maximum delay K) or to year $-J$, whichever the smaller. Estimates for all X_{+k} are developed below, and the present values of the total outstanding losses can be found by invoking some rule of discounting; see Section 15.3 in Bølviken (2014) for such schemes.

3 The chain ladder

The chain ladder (the most commonly used method) was originally invented as a purely mechanical device. It *is* possible to proceed model-based as in the much cited paper by Mack (1993), but the construction there is not convincing, and we shall below follow the original line of thought where kick-off is the accumulated portfolio losses

$$C_{jk} = X_{j0} + \ldots + X_{jk}, \qquad k = 0, \ldots, K. \tag{2}$$

Note that C_{jk} is the sum of claims from year $-j$ settled in year $-j + k$ or earlier which means that C_{jk} has been observed at the end of year 0 if $k \leq j$. Introduce the coefficients

$$\hat{f}_k = \frac{C_{kk} + \ldots + C_{Jk}}{C_{kk-1} + \ldots + C_{Jk-1}}, \qquad k = 1, \ldots, K. \tag{3}$$

which is the average portfolio payout accumulated after k years divided on the same quantity one year earlier (divide on $J - k + 1$ in numerator and denominator). We have to assume $k \leq J$ for this to be feasible; we can't by this method deal with delays longer than the time span the portfolio has been recorded.

This creates a basis for forecasting how losses may evolve. Indeed, $\hat{f}_k$ defines a typical, *relative* growth in the accumulated losses from year $k-1$ to year k. Consider year $-j$ for which C_{jj} is known whereas $C_{jj+1}, \ldots, C_{jK}$ belong to the future and are unknown. One way to predict them is to start at $\widehat{C}_{jj} = C_{jj}$ and then enter the recursion

$$\widehat{C}_{jk} = \hat{f}_k \widehat{C}_{jk-1}, \qquad k = j + 1, \ldots K, \tag{4}$$

and $\widehat{C}_{jj+1}, \ldots, \widehat{C}_{jK}$ are the chain ladder predictions. We get hold of the individual losses per year through

$$\widehat{X}_{jk} = \widehat{C}_{jk} - \widehat{C}_{jk-1} \tag{5}$$

and their sum yields prediction of the total amount that will be liquidated in year k through an analogy to (1), i.e.

$$\hat{X}_{+k} = \sum_{j=0}^{j_k} \widehat{X}_{jj+k} \qquad \text{where} \qquad j_k = \min(J, K-k). \tag{6}$$

This is the chain ladder in IBNR situations. It *could* be used for RBNS too, but when claims have been reported, there are preliminary assessments of how much they will eventually cost. If those are considered reliable, we may prefer to adapt the start of the recursion (4) correspondingly. Let E_j be the company assessment of $X_{jj+1} + X_{JK}$ which are the sum of the remaining claims from development year $-j$. The final predicted, accumulated loss $\widehat{C}_{jK}$ should then be forced to coincide with $C_{jj} + E_j$ which is obtained when

$$\widehat{C}_{jj} = \frac{C_{jj} + E_j}{f_{j+1} \cdots f_K} \tag{7}$$

is initializing the recursion (4).

4 An example

The example comes from a Norwegian insurance company though masked a bit by simulation to hide business secrets. This apart the loss data in Table 1, due to fire, are completely realistic. Note that there is for each column a growing, albeit irregular trend downwards which was due to the underlying portfolio increasing in size over the period in question from 2010 to 2015. The maximum delay is $K = 5$ years which means that everything from 2010 has been settled at the end of 2015 whereas there are outstanding claims from the ensuing years.

Year	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
2010	213.6	117.2	43.2	10.4	4.9	0.8
2011	266.9	111.7	11.9	16.5	7.7	
2012	253.1	232.2	24.2	13.7		
2013	319.8	134.2	16.8			
2014	604.2	222.6				
2015	402.7					

Table 1 *Historical portfolio losses broken down on delay.*

Imagine that we are at the start of 2016 and want to predict the payments in the years up to 2020 where the last claims from 2015 will be finalized. Start by converting the data in Table 1 to the cumulative ones in Table 2.

Year	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
2010	213.6	330.8	374.0	384.4	389.3	390.1
2011	266.9	378.6	390.5	407.0	414.7	
2012	253.1	485.3	509.5	523.2		
2013	319.8	454.0	470.8			
2014	604.2	826.8				
2015	402.7					

Table 2 *Cumulative portfolio losses broken down on the delay.*

Next the growth factors $\hat{f}_k$, estimated through the method in (3) have been recorded in Table 3. Note that most of the claims are settled within two years.

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$\hat{f}_k$	1.493	1.058	1.033	1.016	1.002

Table 3 *Estimated growth factors f_k.*

The example below has for simplicity been treated as a case of IBNR rather than RBNS (as it of course is) with the the prediction (4) run from the accumulated historical losses C_{jj} at the the end of year 2015. Each development year has in Table 4 been continued into the future which leads to

Year	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
2010						**390.1**
2011					**414.7**	415.6
2012				**523.2**	531.6	532.7
2013			**470.8**	485.8	493.6	494.6
2014		**826.8**	875.0	902.9	917.3	919.3
2015	**402.7**	601.4	636.5	656.8	667.3	668.7

Table 4 *Predicted cumulative losses $\widehat{C}_{jk}$ with observed ones in bold face.*

These cumulative losses must then be converted to predictions of what is paid each year; i.e. Table 4 yields

Year	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
2010					0
2011				0	0.9
2012			0	8.4	1.1
2013		0	15.0	7.8	1.0
2014	0	48.2	27.9	14.4	2.0
2015	198.7	35.1	20.3	10.5	1.4

Table 5 *Predicted future losses per year $\widehat{X}_{jk}$.*

Finally, invoke the method (6) which for the year 2016 amounts to adding the contributions for all previous years; i.e $198.7+48.2+15.0+8.4+0.9 = 271.2$. For 2017 there is one year less and the assessment is now $35.1+27.9+7.8+1.1 = 71.9$, and similar for the remaining years 2018, 2019 and 2020. The final summary is

Year	2016	2017	2018	2019	2020
$\widehat{X}_{+k}$	271.2	71.9	35.8	12.4	1.4

Table 6 *Estimated remaining losses in case of IBNR.*

from which a present value for the total expected cots can be calculated using a suitable scheme of discounting.

5 References

Bølviken, E. (2014). *Computation and modelling in insurance and finance.* Cambridge University Press, Cambridge.

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Mack, T. (1993). Distribution-free calculation of standard error of chain-ladder reserve estimates. *Astin Bulletin*, 23, 213-225.

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