

Oblig STK4540: BSCR in Solvency II

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1 Introduction

The balance of any insurance company consists of assets A and liabilities L . Their difference

$$\text{BOF} = A - L \tag{1}$$

is known as the *Basic Own Funds*. The company is bankrupt if $\text{BOF} < 0$, and things must be organized so that this outcome is a remote one. Recall how general insurance is run. Premia is typically collected by the company in the beginning or at the early stages of a contract and are then entered the assets A . In return for these fees policy holders receive compensations for accidents, injuries and other insurable events that may or may not materialize. These are the company liabilities L . There is both a *gross* and *net* version depending on whether the insurer has employed risk mitigating techniques, such as reinsurance. ‘Gross’ then means liabilities with no reinsurer protection and ‘net’ the opposite where the responsibilities of the cedent company is reduced by compensation received from reinsurer, but only after a reinsurer premium having been deducted from the assets.

Solvency II is a bold attempt by the European Union to regulate these activities for the entire European insurance industry, and the regime came into force on January 1, 2016. A corner stone is the valuation of A and L being based on what the market is willing to pay. With A that depends on how the funds have been invested, say whether they are in equity, bonds, property or most likely a mixture of all three, but they all have a liquid market from which values can be read off. Such valuation for the liabilities L may not be equally obvious. Many of them are future ones that haven’t yet materialized and it may also be uncertain what those that are in our books will eventually cost. However, if the underlying models for the insurance process is known, we can compute the expectation $E(L)$ and add overhead costs and a price for taking risk on top of it where the latter must depend on the uncertainty perceived in the different branches of insurance. Solvency II envisages that such a market valuation for L does indeed exist.

In precise terms the regulation comes down to demanding that

$$\Pr(\text{BOF} < 0) = \epsilon \quad \text{with} \quad \epsilon = 0.005. \tag{2}$$

BOF is known today (at least in principle), but not in the future as the values of both A and L then becomes uncertain. The Solvency II regimes takes an annual view and seeks the business to be arranged at the start of the year so that (2) is satisfied at the end of it. Note that the entire activity of a company goes into this, possibly a huge undertaking. How it should be done is summarized in EIOPA (2014) and in the Commission Delegated Regulation (2015) or in books such as Sandström (2011), Cadoni (2014) and Doff (2014). The oblig below is radically simplified with many aspects of

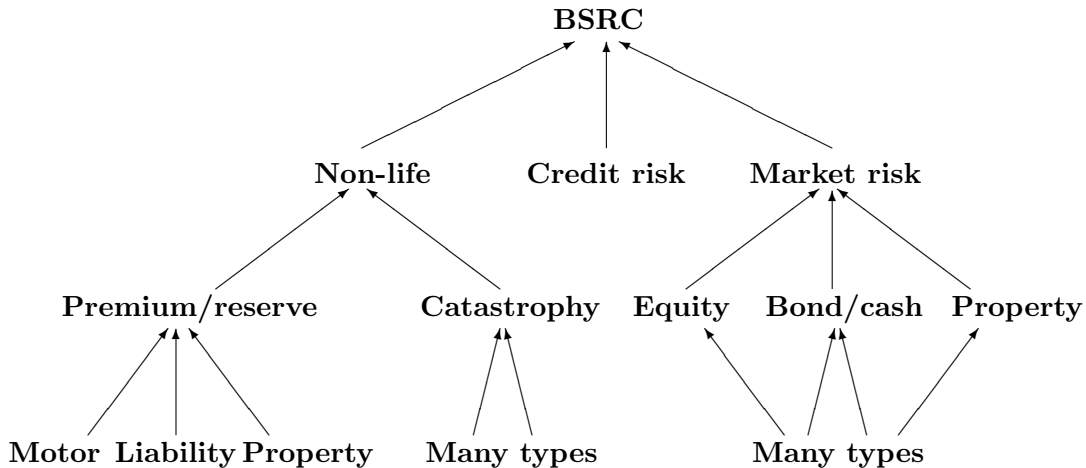


Figure 1 A simplified tree structure for the basic solvency capital requirement BSRC.

company risk ignored, yet it captures the central ingredients in how the Solvency II regime has been set up. Detailed specifications and clauses referred to in the following have been taken from EIOPA (2014).

2 The approach

2.1 Introduction

Solvency II starts by separating expectations $E(A)$ and $E(L)$ from the rest. The latter is known as the *Best Estimate* and is in Solvency II literature denoted BE; for details regarding its computation consult EIOPA (2014). When these quantities are subtracted, we obtain random variables with mean 0, i.e. $A - E(A)$ and $L - E(L)$, and the uncertainty for the coming year is computed from those. It is common in insurance to switch signs so that losses is counted positive. If that is done, we are lead to

$$-\{\text{BOF} + E(\text{BOF})\} = \{L - E(L)\} - \{A - E(A)\} \quad (3)$$

and a central quantity in the Solvency II regulation is the so-called *Basic Solvency Capital Requirement* (denoted **BSRC**) which is the $1 - \epsilon$ percentile of $-\text{BOF} + E(\text{BOF})$ or in mathematical terms

$$\Pr\{-\text{BOF} + E(\text{BOF}) \geq \text{BSRC}\} = \epsilon. \quad (4)$$

You are in this oblig asked to compute BSRC for a grossly simplified insurance company.

2.2 Network and recursion

An insurance company is a network of activities which can be depicted hierarchically as in the example in Figure 1. There are several layers and sub-layers on top of each other, yet the representation shown is only a gross simplification of the real thing. Note the layer immediately beneath BSRC where ‘Non-life’ and ‘Market risk’ correspond to $L - E(L)$ and $A - E(A)$ in Section 2.1 whereas ‘Credit risk’ is due to defaults by other companies. Reinsurance is a typical example. There is always *some* risk that a reinsurer becomes insolvent and unable to honour its obligations. Solvency

II has set up rules for how this risk should be taken into account, but that is ignored in the following, and ‘Credit risk’ is 0 from now on.

‘Non-life’ and ‘Market risk’ have both several components under them which in turn, depend on other components. Their meaning and interpretation will be presented in Section 3; here the topic is how the total risk coming out of the network is evaluated. Perhaps the most obvious way would be through a model of the entire company which could be simulated in the computer. This would lead to BSCR, but it is *not* the Solvency II way. Why is that? It could be that the approach has been regarded as too advanced for small and medium-sized companies to live up to or too complicated for the supervisor to oversee properly. Whatever the explanation the Solvency II evaluations proceed as recursions starting at the bottom nodes of networks such as the one in Figure 1 and working their way upwards. The central concept here is that of the *Solvency Capital Requirement* or (*SCR*) which applies to all nodes. Each one, whether ‘Non-life’, ‘Motor’, or ‘Equity’, is associated with some risk variable X with its $1 - \epsilon$ percentile defined as its SCR; i.e.

$$\Pr(X \geq \text{SCR}) = \epsilon. \quad (5)$$

This is the same type of quantity as BSCR above.

Consider $Y = -\text{BOF} + E(\text{BOF})$ in (3) and note that it can be written

$$Y = X_1 + X_2, \quad \text{where} \quad X_1 = L - E(L), \quad X_2 = -A + E(A), \quad (6)$$

and the Solvency II set-up is to deduce the $1 - \epsilon$ percentile of Y from the same percentiles for X_1 and X_2 . If the latter are denoted $\text{SCR}_1 = \text{SCR}_{\text{non-life}}$ and $\text{SCR}_2 = \text{SCR}_{\text{market}}$, we seek a relationship

$$(\text{SCR}_{\text{non-life}}, \text{SCR}_{\text{market}}) \longrightarrow \text{BSCR} \quad (7)$$

where you should recall that ‘Credit risk’ in Figure 1 has been ignored. Most of the relationships in Figure 1 (i.e. the arrows) are handled in this manner, but not the breakdown of ‘Non-life’ on their underlying lines of business ‘Motor’, ‘Liability’ and ‘Property’ which is treated differently as will emerge in Section 3. There are many such exceptions and ad-hoc rules in Solvency II.

The general case deals with J variables X_1, \dots, X_J underlying some node with risk Y at the level immediately above. Clearly

$$Y = X_1 + \dots + X_J, \quad (8)$$

and the extension of (7) is

$$(\text{SCR}_1, \dots, \text{SCR}_J) \longrightarrow \text{SCR}_y \quad (9)$$

where $\Pr(X_j \geq \text{SCR}_j) = \epsilon$ for $j = 1, \dots, J$ and $\Pr(Y \geq \text{SCR}_y) = \epsilon$. Suppose a rule for such relationships has been worked out. We can then specify SCR’s for the bottom nodes and climb the tree in Figure 1 by deducing SCR’s higher and higher up. For example, SCR’s for ‘Equity’, ‘Bond/cash’ and ‘Property’ yield $\text{SCR}_{\text{market}}$ which is then combined with $\text{SCR}_{\text{non-life}}$ (itself accumulating several contributions under it) for the final BSCR at the top.

2.3 The standard formula

How the SCR’s at the bottom of the network are found is dealt with in Section 3, and we now

concentrate on the specification of the relationship (9). Solvency II offers the so-called *standard formula*

$$\text{SCR}_y = \left(\sum_{i=1}^J \sum_{j=1}^J \rho_{ij} \times \text{SCR}_i \times \text{SCR}_j \right)^{1/2} \quad (10)$$

where $\rho_{ij} = \text{cor}(X_i, X_j)$ are correlations specified by the regulatory regime. The rationale behind this rule comes from the ordinary formula for the variance of sums of dependent random variables; i.e.

$$\text{sd}(Y) = \left(\sum_{i=1}^J \sum_{j=1}^J \rho_{ij} \times \text{sd}(X_i) \times \text{sd}(X_j) \right)^{1/2}, \quad (11)$$

consult Section 5.4 in Bølviken (2014) if you are not familiar with it. Suppose X_1, \dots, X_J are Gaussian risks. We are assuming that they have expectation 0 which implies that

$$\text{SCR}_j = \text{sd}(X_j)\phi_\epsilon$$

where ϕ_ϵ is the $1 - \epsilon$ percentile of the standard normal random variable¹. In the same manner $\text{SCR}_y = \text{sd}(Y) \times \phi_\epsilon$, and (10) follows when (11) is multiplied by ϕ_ϵ on both sides.

2.4 Normal or lognormal risks?

The standard formula is exact for Gaussian risks, but is this assumption valid? Large portfolios may become Gaussian as a consequence of the central limit theorem, but only if policy risks are independent which they are not if there are underlying common random factors such as weather conditions influencing all of them jointly. Nor is the cost of settling catastrophic events and other forms of big claims insurance likely to be Gaussian. Such discrepancies induce error in the standard formula, and SCR_y 's at higher level no longer correspond to $1 - \epsilon$ percentiles even if $\text{SCR}_1, \dots, \text{SCR}_J$ do. The issue was investigated by Pfeifer and Strassburger (2008) with the rather gloomy conclusion that the standard formula might underestimate risk substantially with SCR_y a good deal smaller than it should.

Yet, this is how the Solvency II regulation works, and the error is at least to some extent mitigated by its strategy of starting recursions higher than the $1 - \epsilon$ percentiles of Gaussian variables. Indeed, much of the documentation describes a risk variable X as log-normal rather than normal which captures skewness much better. You are required to analyse this issue in Section 4.1. Standard formulae in terms of log-normal or other families of distributions allowing skewness are constructed in Bølviken and Guillen (2016).

3 Risk in a simplified company

3.1 BSCR from non-life and market risk

The first step is to link BSRC to the solvency capital requirements $\text{SCR}_{\text{non-life}}$ and $\text{SCR}_{\text{market}}$ through the standard formula. We need their correlation which is specified as 0.25 in the Solvency II documentation. It follows that (10) yields

$$\text{BSCR} = \left((\text{SCR}_{\text{non-life}})^2 + (\text{SCR}_{\text{market}})^2 + 0.5 \times \text{SCR}_{\text{non-life}} \times \text{SCR}_{\text{market}} \right)^{1/2}, \quad (12)$$

¹When $\epsilon = 0.005$ as in Solvency II, $\phi_\epsilon = 2.576$.

and the next step is how $\text{SCR}_{\text{non-life}}$ and $\text{SCR}_{\text{market}}$ are computed from their sub-components.

3.2 Non-life insurance

Non-life insurance risk is in Solvency II broken down on

- *premium* risk,
- *reserve* risk,
- *catastrophy* risk (or *cat* risk for short)

where *premium* risk applies to *new* incidents that will appear during the coming year and *reserve risk* is uncertainty due to the cost to settling *existing claims*. *Cat* risk is the responsibility for big (catastrophic) events. They can be man-made (for example big industrial fires or collapse of an oil rig) or natural events such as earthquakes, storms or floods. The latter are also broken down on the regions on which they appear in a very complex system of inter-dependent, big risks. Our treatment here is radically simplified in that only a single type of catastrophe can occur.

Lines of Business (or *LOB*'s) that do not belong to the cat type (for example ‘Motor’, ‘Liability’, and ‘Property’ in Figure 1) have a premium part (new incidents) and a reserve part (existing claims). Their Solvency II treatment in Figure 1 aggregates them into one single $\text{SCR}_{\text{preres}}$ for both without defining individual SCR_{pre} and SCR_{res} . If SCR_{cat} represents the catastrophe SCR, then $\text{SCR}_{\text{non-line}}$ at the next level becomes

$$\text{SCR}_{\text{non-life}} = \left((\text{SCR}_{\text{preres}})^2 + (\text{SCR}_{\text{cat}})^2 + 1.5 \times \text{SCR}_{\text{preres}} \times \text{SCR}_{\text{cat}} \right)^{1/2} \quad (13)$$

which corresponds to correlation 0.75 between premium/risk and cat.

The next step is to calculate $\text{SCR}_{\text{preres}}$ and SCR_{cat} . Most work in this oblig lies with the former. Suppose the premium/reserve knode aggregates L different lines of business (or *LOB*'s). A company is required to keep track on the expected premia P_1, \dots, P_L for the coming year for all of them. Certain contracts may run over several years with premium contributions coming in later. In that case F_1, \dots, F_L are that part of the premia that are received later than one year. The company has also assessed what the current, unsettled claims are going to cost, say $\text{CL}_1, \dots, \text{CL}_L$. The *volum* of the *LOB*'s are then defined as

$$V_i = \underbrace{P_i + F_i}_{\text{premium part}} + \underbrace{\text{CL}_i}_{\text{reserve part}}, \quad i = 1, \dots, L. \quad (14)$$

We also need standard deviation factors $\sigma_{i,\text{pre}}$ and $\sigma_{i,\text{res}}$ for premium and reserve risk which apply per mony unit under risk. Evaluations of them for each *LOB* have been provided by the Solvency II regime after statistical studies of past events. They are aggregated to a single number σ_i for the i 'th *LOB* through

$$\sigma_i = \left((\sigma_{i,\text{pre}})^2 \times (1 - f_i)^2 + \sigma_{i,\text{pre}} \times \sigma_{i,\text{res}} \times (1 - f_i) \times f_i + (\sigma_{i,\text{res}})^2 \times f_i^2 \right)^{1/2} \quad (15)$$

where²

$$f_i = \text{CL}_i / V_i, \quad (16)$$

²Formula (15) resemble the standard formula (10), but with the second term in the sum on the right differently (it must be multiplied by 2 for complete analogy). There is no misprint; it is exactly like that on p. 257 in EIOPA (2014).

and $\sigma_1, \dots, \sigma_L$ are in turn aggregated to an overall assessment σ for the entire premium/reserve component through

$$\sigma = \left(\sum_{i=1}^L \sum_{j=1}^L \rho_{ij} \times \sigma_i \times \sigma_j \times \frac{V_i}{V} \times \frac{V_j}{V} \right)^{1/2}, \quad \text{where} \quad V = V_1 + \dots + V_L. \quad (17)$$

Here ρ_{ij} is a correlation between LOB i and j which is specified in EIOPA (2014). The final link to $\text{SCR}_{\text{preres}}$ is then

$$\text{SCR}_{\text{preres}} = q \times \sigma \times V \quad \text{where} \quad q = 3. \quad (18)$$

Where does the coefficient $q = 3$ come from? It was pointed out in Section 2.4 that Solvency II envisages risk variables to be lognormal. Their standard deviation is in this case $\sigma \times V$, and it is claimed that $q = 3$ makes $\text{SCR}_{\text{preres}}$ correspond to a 99.5% percentile. The oblig discusses in Section 4.1 whether this is a realistic proposition.

The catastrophe solvency capital SCR_{cat} with less historical data to lean on is handled simpler although several types of specifications have been put forward in the Solvency II documentation. A common choice is to link them to the maximum loss S that arises when the maximum insured sums are added over all policies³. The simplest version (and the only one considered here) is

$$\text{SCR}_{\text{cat}} = c \times S \quad (19)$$

where c is a coefficient specified by Solvency II for each type of catastrophe. We shall use $c = 0.3$ in Section 4.

3.3 Enter reinsurance

Companies routinely cede parts of their risks to reinsurers, and this clearly influences the net solvency capital that should be demanded. It also creates the additional risk that the reinsurer may not be able to pay (because of insolvency), but this is ignored here. Reinsurance agreements enter the specifications in Section 3.2 in two ways. Firstly, the premia P_i in (14) that defined the volume V_i is replaced by a *net* version $P_{i,\text{net}}$ where the reinsurer premium $P_{i,\text{re}}$ is subtracted. In other words, words, if $P_{i,\text{gross}}$ is what the company receives from its policy holders, then

$$P_{i,\text{net}} = P_{i,\text{gross}} - P_{i,\text{re}} \quad (20)$$

is inserted for P_i in (14). The standard deviations factor $\sigma_{i,\text{pre}}$ may be also affected. Suppose the reinsurance contract is of the so-called *non-proportional* type (which means that reinsurer responsibility is *not* a fixed percentage of the claim). Then the former value of $\sigma_{i,\text{pre}}$ is reduced by a factor denoted NP (for ‘non-proportional’) so that

$$\sigma_{i,\text{pre}} \rightarrow \text{NP} \times \sigma_{i,\text{pre}}, \quad (21)$$

and the right hand side takes place of $\sigma_{i,\text{pre}}$ in (15).

How catastrophe risk should be modified by reinsurance has a somewhat troubled history in Solvency II, and EIOPA (2014) specifies that it should not be taken into account at all; consult its

³In practice such insurance contracts specify such a maximum responsibility.

Section 9. This seems to be changing; see McHugh and Moormann (2014). We shall in the next section allow the expected compensation obtained from the reinsurer to be subtracted SRC_{cat} in 19) while the reinsurer premium must be subtracted the assets.

3.4 Financial risk

Then comes risk due to financial markets behaving unfavourably. Our simplified company holds asses in equity and in bonds with one year to maturity. Their solvency capital requirements $\text{SCR}_{\text{equity}}$ and SCR_{bond} are aggregated through

$$\text{SCR}_{\text{market}} = \left((\text{SCR}_{\text{equity}})^2 + (\text{SCR}_{\text{bond}})^2 + \text{SCR}_{\text{equity}} \times \text{SCR}_{\text{bond}} \right)^{1/2} \quad (22)$$

which corresponds to correlation 0.5 between equity and bonds, a value taken from EIOPA (2014). The specifications of $\text{SCR}_{\text{equity}}$ and SCR_{bond} are linked to how much the basic own funds BOF may go down when the financial markets exhibit something close to worst-case scenarios. This entails almost a collapse in the stock market, and in our situation that the rate of interest goes down. In cases where the liabilities extend many years ahead interest rate movements affect BOF both through the valuation BE of the liabilities (which are subtracted the assets for BOF) *and* the values of the bond portfolios, but in our simplified company there are only short-term liabilities so that the effect on the liabilities are ignored.

The specification of $\text{SCR}_{\text{equity}}$ and SCR_{bond} reflect the values of equity and bonds today, say V_{equity} and V_{bond} which are entered the balance sheet of the company. Firstly,

$$\text{SCR}_{\text{equity}} = c \times V_{\text{equity}} \quad \text{where} \quad c = 0.465. \quad (23)$$

Again the coefficient c which allows the value of stock falling more than 50% in a worst case scenario, is taken from Solvency II documentation. Our bond component is a simplified version of money market investments which in practice consist of securities with different times to maturity. With r the current rate of interest the SCR at the end of the year is specified as

$$\text{SCR}_{\text{bond}} = (1 - c) \times r \times V_{\text{bond}} \quad \text{where} \quad c = 0.75. \quad (24)$$

4 Oblig calculations

4.1 The lognormal assumption

It was pointed out in Section 3.2 that Solvency II documentation often refers to the lognormal distribution as model for the risk variables at the bottom of networks such as the one in Figure 1. In (18) that lead to $\text{SCR}_{\text{preres}} = \sigma \times V \times q$ with $q = 3$ which is claimed in EIOPA (2014) to be a robust value that corresponds to the 99.5% percentile over a wide range of log-normal shapes. The first part of the Oblig is to investigate this assertion numerically for the premium/reserve risk component. The underlying log-normal model is then

$$X_{\text{preres}} = \sigma V \frac{e^{-\tau^2/2 + \tau\varepsilon} - 1}{\sqrt{e^{\tau^2} - 1}} \quad \text{where} \quad \varepsilon \sim N(0, 1). \quad (25)$$

Here τ is a parameter. The construction means that $E(X_{\text{preres}}) = 0$ and that $\text{sd}(X_{\text{preres}}) = \sigma V$ for all τ ; look up the log-normal model in Section 9.3 in Bølviken (2014) if you need it. You may also apply l'Hôpital's rule to the expression in (25) and convince yourself that X_{preres} becomes Gaussian as $\tau \rightarrow 0$.

Determine τ approximately when $\text{SCR}_{\text{preres}} = \sigma \times V \times 3$. You proceed by arguing that this leads to the equation

$$\frac{e^{-\tau^2/2+\tau\phi_\epsilon} - 1}{\sqrt{e^{\tau^2} - 1}} = 3 \quad \text{where} \quad \phi_\epsilon = 2.576 \quad (26)$$

which must be solved numerically. An accurate way is to use the ‘Optimize’ command in R⁴.

Look up the formula for skewness for log-normal variables in Section 9.3 in Bølviken (2014) and tabulate under the model (25) skewness and $\text{SCR}_{\text{preres}}$ when $\tau = 0.1 \times i$ for $i = 0, \dots, 10$. Try to take a position on the Solvency II strategy with $q = 3$ for the premium/reserve variable for all companies in Europe. Does it appear a reasonable one?

4.2 BSCR calculations

You are asked to tabulate BSCR for the the simplified insurance company in Section 3 when circumstances are varied as described below. There are three LOB’s under the premium/reserve component and one single catastrophe component. Assets are divided between equity and bonds that mature at the end of the year. Data for the company are as follows:

	Premium/reserve			Catastrophy	Assets
	P_i	F_i	CL_i	Maximum insured	Valued at 1 with fraction
Motor	0.40	0.00	0.08	sum S varied over	w in equity and the
Liability	0.30	0.00	0.12	0, 1.0, 2.0.	rest in bonds with w
Property	0.30	0.00	0.30		0 and 0.5

Table 1. Description of the insurance company.

Many Solvency II specifications were given above, but we also need some additional ones, notably standard deviation factors and correlations. Solvency II documentation offers

	Standard deviation factors for non-life		Correlations for non-life LOB’s			
	$\sigma_{i,\text{pre}}$	$\sigma_{i,\text{res}}$	Motor	Liability	Property	
Motor	$0.10 \times NP$	0.09	Motor	1	0.50	0.75
Liability	$0.14 \times NP$	0.11	Liability	0.50	1	0.25
Property	$0.09 \times NP$	0.10	Property	0.75	0.25	1

Table 2. Additional Solvency II specifications.

where on the left $NP = 1$ when there is no reinsurance and $NP = 0.8$ when non-proportional reinsurance has been bought. Annual rate of interest $r = 3\%$.

No reinsurance Suppose first there is no reinsurance. Compute the summarizing standard deviation factors σ in (17) and include their values in your report. Then tabulate BSCR for the company when the maximum insured sum S for catastrophe and the fraction w on equity are varied as in Table 1.

⁴But it is good enough to program the left hand side of (26) as τ is varied between, say $\tau = 0.0001$ in 10000 steps up to $\tau = 1$, plot it against τ and read off the solution approximately from the plot.

Enter reinsurance Introduce reinsurance in all the LOB's on the left in Table 2 and redo all the preceding computations based on the following information:

$P_{i, \text{re}}$ for premium/reserve	Cat reinsurance
Motor 0.10	Proportional contracts with 25%
Liability 0.05	covered by reinsurer. Premium
Property 0.20	for the reinsurance is 10% of S .

Table 3 Reinsurance assumptions.

Demonstrate how much σ now has changed and reduce the assets by the reinsurance premia that have now been paid. All other assumptions and conditions remain the same, in particular the maximum cat responsibility S and the weight w on equity which are to be varied in the same way as before.

5 References

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