Number 77 / 2012
Working Paper Series
by the University of Applied Sciences bfi Vienna

A practical guide to the use of the chain-ladder method for determining technical provisions for outstanding reported claims in non-life insurance

Oktober 2012

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Gefördert vom BMVIT und vom BMWFJ.
Die Abwicklung erfolgt durch die Österreichische Forschungsförderungsgesellschaft im Rahmen der Programmlinie COIN "Aufbau".

ISSN 1995-1469

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Abstract

The Solvency II directive establishes a revised set of capital adequacy rules for insurance and reinsurance undertakings in the EEA. The starting point for assessing the available capital of an undertaking is to value its assets and liabilities. The liabilities of insurance undertakings include the technical provisions which constitute a significant proportion of their balance sheets. Under Solvency II the projection of run-off triangles is one of the allowed methods for valuing the technical provisions for non-life insurance business. This paper demonstrates how the chain-ladder method, a simple form of run-off triangle methods, can be used by a non-life insurer in determining the technical provisions for outstanding claims.

1 Introduction

When assessing the solvency position of any company it is necessary to analyse its assets and its liabilities and to understand how these are valued. For insurance companies the technical provisions are a significant part of the liabilities. At the end of 2011 Austrian non-life insurers (excluding health insurance) reported technical provisions which constituted about 44% of their total liabilities (see FMA 2011). The valuation of these technical provisions is the responsibility of the non-life actuary.

The technical provisions of a non-life insurer consist of provisions for unearned premiums and provisions for claims losses. The technical specifications for Solvency II state that traditional actuarial techniques for valuing the best estimate for provisions for non-life insurance obligations include the projection of run-off triangles (see EIOPA 2012: 62). This paper describes the simplest of these run-off triangle methods, the chain-ladder method (CLM).

The aim of this paper is to demonstrate the CLM in a simple and intuitive way. Complicated formulae are avoided. The idea behind the approach is demonstrated through a worked example using fictitious claims loss data. The notation and the formulae used in run-off triangle techniques are formally described in Appendix A.

In Section 2 the paper describes the minimum data requirements for using run-off triangle techniques. This section also introduces the concept behind the techniques and demonstrates how run-off triangles are constructed for both incremental and cumulative claims loss amounts (using fictitious claims loss data). Section 3 demonstrates how the CLM is used in reserving for outstanding reported claims.

2 Data required for using run-off triangle methods

2.1 Claims occurring

An insurance undertaking promises its policyholders to pay out benefits if certain events occur. For a non-life insurer such events include, for example, the following:

- A car accident where the insurer's policyholder is at fault and a third party's car is damaged (motor vehicle liability insurance).
- A fire occurs in a policyholder's house causing damage to the kitchen (property damage/fire insurance).
- A tanker runs aground resulting in a loss of cargo and damage to the environment (marine insurance).
- A policyholder is sued by a client for providing poor professional advice and the case is ruled in favour of the plaintiff, resulting in the policyholder having to compensate his or her client (professional indemnity insurance).

A *claim event* is an event that gives rise to a claim against an insurer by a policyholder. The ultimate cost to the insurer of a claim event, including the benefit payments and claims handling expenses, is called the ultimate gross *claim loss*. The ultimate net claim loss allows for the deduction of any reinsurance recoveries and other recoveries.

For any claim event there may be a delay between the occurrence of the event and the date on which the claim is reported to the insurer *(reporting delay)* and another delay between the reporting date and the date on which the claim loss is finally settled *(settlement delay).* Any amount paid or expense incurred in respect of a claim event is called a *claim loss settlement amount* or *a claim loss settled*.

In this paper the use of run-off triangles is demonstrated through a worked example using claims loss data for a fictitious portfolio of non-life insurance business[1]. The insurance undertaking wishes to determine the technical provisions required for claims that have already been reported but not yet fully settled[2].

The insurer has data for all claims that have been reported in the past. For our purposes we consider only the claims benefit payments and the associated claims handling expenses[3].

The first step in creating the claims loss settlement run-off triangle is to group the claims loss settlement amounts by the year in which the associated claims events occurred; this is called the *claims occurrence year*[4] (or the *accident year).* On 1 January 2013, the insurer might have total claims losses settled (benefit payment amounts plus claims handling expenses) in respect of the last eight claims occurrence years as shown in Figure 1[5].

Figure 1: Claims loss settlement data grouped by claims occurrence year

Claims occurrence year	Claims losses settled
2005	3963
2006	4975
2007	5873
2008	6401
2009	6563
2010	6358
2011	6918
2012	3072

Source: own illustration

[1] Typically, when calculating technical provisions, the actuary should consider homogeneous groups of insurance business. The claims data should be segmented by line of business. The business should be further segmented if it is known that there are differences in, for example, the claims handling process, claim sizes or reporting/settlement delays.
[2] The worked example considers the settlement delays only. The process for estimating future reported claims amounts is similar.
[3] See section I.1 in Appendix A for more information on claims data that may be used.
[4] The claims occurrence year is denoted by y; see Definition A2 in Appendix A. In this paper the claims occurrence year is represented by the calendar year of occurrence (…; 1900; 1901; …; most recent completed year) rather than by cardinal numbers (0; 1; …; n). The use of calendar years allows for easier understanding. The CLM method described in this paper is, however, the same as that described in Schmidt 2006 and GDV 2011.
[5] The numbers shown are fictitious and are designed for demonstration purposes only.

2.2 The development of claims losses settled

Typically, claims losses settled for each claims occurrence year are not paid on one date but rather over a number of years (or time periods). The insurer's data for the claims loss settlement amounts shown in Figure 1 might be expanded to show the years[6] in which the amounts were settled as in Figure 2.

Figure 2: Incremental claims loss settlement data presented as a run-off triangle

Incremental claims loss settlements	Development year							
Claims occurrence year	0	1	2	3	4	5	6	7
2005	1232	946	520	722	316	165	48	14
2006	1469	1201	708	845	461	235	56	
2007	1652	1416	959	954	605	287		
2008	1831	1634	1124	1087	725			
2009	2074	1919	1330	1240				
2010	2434	2263	1661					
2011	2810	4108						
2012	3072							

Source: own illustration

The sum of each row in Figure 2 is equal to the amount shown for that row in Figure 1.

Figure 2 incorporates some features that should be commented on or defined. The *development year*[7] for a claims settlement amount reflects how long after the claims occurrence year the amount was settled. An amount settled during the claims occurrence year is considered to be settled in development year 0, an amount settled in the following year is settled in development year 1, and so on. (In general, an amount settled in the k^{th} calendar year after the claims occurrence year is settled in development year k).

In the data used for the example it is assumed that the largest development year observed for any claims occurrence year is 7[8].

The data shown in each of the cells in Figure 2 represents the *incremental claims losses settled*[9] in the development year (column) for the given claims occurrence year (row).

The representation of the data in a table requires it to be shown as a triangle, the so-called *run-off triangle*. For any cell in the table, the value shown represents the incremental claims loss amount that was settled in

[6] The time period used (months or years) depends on the observed settlement delays for the line of business being considered.
[7] The development year is denoted by k; see Definition A3 in Appendix A.
[8] The largest development year observed in the data is denoted by n; see Definition A3 in Appendix A. In our worked example n equals 7.
[9] The incremental claims losses settled are denoted by Z(y;k); see Definition A4 in Appendix A.

calendar year (claims occurrence year + claims development year). Each diagonal set of data represents the amounts settled in a single calendar year. In particular, each green cell along the blue diagonal line represents a claims loss amount settled in 2012, the latest completed calendar year.

Finally, it can be noted that all green cells represent observed data (amounts settled in the past) and all red cells represent time periods in the future for which we wish to estimate the expected claims settlement amounts.

2.3 Cumulative claims losses settled

The data in Figure 2 can be presented as cumulative claims losses settled. For each claims occurrence year the incremental claims loss settled for a particular development year is the amount settled in that development year. The *cumulative claims losses settled*[10] is the total amount settled up to that development year, i.e. it is the sum of the incremental claims losses settled up to that date. The cumulative claims losses settled for the worked example are presented in Figure 3.

Figure 3: Cumulative claims loss settlement data presented as a run-off triangle

Cumulative claims loss settlements		Development year							
		0	1	2	3	4	5	6	7
Claims occurrence year	2005	1232	2178	2698	3420	3736	3901	3949	3963
	2006	1469	2670	3378	4223	4684	4919	4975	
	2007	1652	3068	4027	4981	5586	5873		
	2008	1831	3465	4589	5676	6401			
	2009	2074	3993	5323	6563				
	2010	2434	4697	6358					
	2011	2810	6918						
	2012	3072							

Source: own illustration

By definition the values along the diagonal blue line must equal the total amounts settled to date for each claims occurrence year. Accordingly, the amounts shown in the last diagonal of Figure 3 correspond with those in Figure 1.

[10] The cumulative claims losses settled are denoted by S(y;k); see Definition A5 in Appendix A.

3 The chain-ladder method

Once the raw data for claims losses settled is collected and segmented as in Figure 3 the insurer is ready to estimate the values that can be expected in the red cells, i.e. in the future.

3.1 Assumptions underlying the CLM

Intuitively, it seems reasonable to assume that patterns of claims loss settlement observed in the past will continue in the future. All run-off triangle methods rely on this assumption, i.e. they all assume that the development of claims loss settlement over the development years follows an identical pattern for every claims occurrence year[11].

It also seems reasonable to expect that the estimates for settlement amounts in the future will be more accurate if all of the available data is used in the estimation. The CLM is a technique that follows this expectation. It relies on the use of the cumulative claims loss settlement data for all claims occurrence years rather than considering, for example, only the latest claims occurrence year (see Schmidt 2006: 284).

The assumption of identical settlement patterns should not be made lightly. A basic requirement for using past experience to estimate future developments is that the data is accurate and free of errors. An insurer should also ensure that its own experience does not contradict the assumptions underlying the estimation method. Reasons why the observed claims loss settlement patterns may change over time include:

- Changes in product design and conditions.
- Changes in the claims reporting, assessment and settlement processes.
- Changes in the legal environment.
- Abnormally large or small claim settlement amounts.

If it is found that the past experience contradicts the assumptions of the method to be used then either the data must be appropriately adjusted or an alternative method must be used.

A closer inspection of Figure 2 suggests that the settlement amount for development year 1 of claims occurrence year 2011 is particularly large. A query to the claims handling department yields that there was a significantly large settlement amount paid in 2012 in respect of a single claim event occurring in 2011. The insurer subsequently decides to exclude this claim loss outlier from the data in the run-off triangle[12]. The incremental settlement amount in this cell reduces from 4108 to 2108. If the insurer believes that the claim is not yet fully settled it will have to determine the technical provisions required for the claim on an individual basis.

[11] See Assumption A1, Assumption A2, Assumption A3 and Assumption A4 (in Appendix A).
[12] There is no single correct way to adjust the data. The insurer could instead reduce the claim settlement amount for the relevant claim event to a level that is more representative of its average claim loss amount. The insurer must, however, ensure that it provisions separately for the amount that is not included in the run-off triangle.

3.2 Development patterns and development factors

The insurer may intuitively tend toward using any of the following patterns for estimation purposes:

- The proportion of the ultimate cumulative claims losses that is settled *in* a particular development year (development pattern for incremental claims losses settled)[13].
- The proportion of the ultimate cumulative claims losses that is settled *by* a particular development year (development pattern for cumulative claims losses)[14].
- The ratio of the cumulative claims losses settled *by* a particular development year to the cumulative claims losses settled *by* the previous development year (cumulative claims loss factor)[15].

If the assumption regarding identical development patterns for all claims occurrence years holds for any one of these patterns then it holds for the other two patterns as well. The three patterns are equivalent; if one of them is observed then the other two also hold true. So, if an actuary or the reader is comfortable with working with any one of the three development patterns described above, then he or she may rest assured that the other two patterns are a natural consequence of the first.

The CLM explicitly relies on the assumption that the last pattern described above holds for all claims occurrence years. For any development year the quotient

> *Expected cumulative claims losses settled up to and including the development year / Expected cumulative claims losses settled up to and including the previous development year*

is called the *cumulative claims loss settlement factor* for that development year[16]. For example, the cumulative claims loss settlement factor for development year 4 is derived from the cumulative settlement amounts for development years 3 and 4.

3.3 Estimating future claims settlement amounts

The underlying assumption for the CLM is that the cumulative claims loss settlement factor for a specific development year is assumed to be the same for all claims occurrence years[17]. The CLM estimator for each of the factors is based on the cumulative settlement data for as many claims occurrence years as possible. This is demonstrated in Figure 4 for the factor for development year 4. The data for claims occurrence year 2011 and development year 1 has been adjusted as described in section 3.1.

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[13] This proportion is denoted by $\vartheta(k)$; see Definition A6 in Appendix A.
[14] This proportion is denoted by $\gamma(k)$; see Definition A7 in Appendix A.
[15] This ratio is denoted by $\varphi(k)$; see Definition A8 in Appendix A.
[16] See Definition A8 in Appendix A.
[17] See Assumption A4 in Appendix A.

Figure 4: Determining the CLM estimator for the cumulative claims loss settlement factor

Cumulative claims loss settlements		Development year							
		0	1	2	3	4	5	6	7
Claims occurrence year	2005	1232	2178	2698	3420	3736	3901	3949	3963
	2006	1469	2670	3378	4223	4684	4919	4975	
	2007	1652	3068	4027	4981	5586	5873		
	2008	1831	3465	4589	5676	6401			
	2009	2074	3993	5323	6563				
	2010	2434	4697	6358					
	2011	2810	4918						
	2012	3072							
CLM estimator for claims loss settlement factor			1,8508	1,3140	1,2422	1,1151	1,0491	1,0118	1,0035

3736+4684+5586+6401 = 20407

3420+4223+4981+5676 = 18300

20407/18300 = 1,1151

Source: own illustration

These CLM estimators for the cumulative claims loss settlement factors are used to estimate the cumulative claims loss settlement amounts in the future. For each claims occurrence year the last historical observation is used together with the appropriate CLM estimator for the development factor to estimate the cumulative settlement amount in the next development year. This value is, in turn, multiplied by the estimator for the development factor for the next development year and so on. Figure 5 demonstrates the calculations and the resulting CLM estimators for the cumulative claims loss settlement amounts for claim events that occurred in 2008.

Figure 5: Determining the estimated cumulative claims loss settlements in future periods

Observed and estimated cumulative claims loss settlements		Development year							
		0	1	2	3	4	5	6	7
Claims occurrence year	2005	1232	2178	2698	3420	3736	3901	3949	3963
	2006	1469	2670	3378	4223	4684	4919	4975	4993
	2007	1652	3068	4027	4981	5586	5873	5942	5963
	2008	1831	3465	4589	5676	6401	*6401 •1,0491* = 6715	*6715 •1,0118* = 6794	*6794 •1,0035* = 6818
	2009	2074	3993	5323	6563	7319	7678	7768	7796
	2010	2434	4697	6358	7898	8807	9239	9348	9381
	2011	2810	4918	6462	8027	8952	9391	9502	9535
	2012	3072	5686	7472	9281	10350	10858	10986	11025
CLM estimator for claims loss settlement factor		-	1,8508	1,3140	1,2422	1,1151	1,0491	1,0118	1,0035

Source: own illustration

The values shown in the red cells are the estimators for the future cumulative claims settled. These estimates are always based on the latest available cumulative claims settlement amounts for the relevant claims occurrence year, i.e. the estimated future cumulative claims settlements are always based on the last green diagonal of data.

It is now a simple task to derive the estimated incremental claims settlement amounts for the future periods. An incremental settlement amount is the difference between two consecutive cumulative settlement amounts. This is demonstrated for claims occurrence year 2008 in Figure 6.

Figure 6: Deriving the estimated incremental settlement amounts from the estimated cumulative amounts

Observed and estimated incremental claims loss settlements		Development year							
		0	1	2	3	4	5	6	7
Claims occurrence year	2005	1232	946	520	722	316	165	48	14
	2006	1469	1201	708	845	461	235	56	18
	2007	1652	1416	959	954	605	287	69	21
	2008	1831	1634	1124	1087	725	*6715-6401* = 314	*6794-6715* = 79	*6818-6794* = 24
	2009	2074	1919	1330	1240	756	359	91	28
	2010	2434	2263	1661	1540	909	432	109	33
	2011	2810	2108	1544	1565	924	439	111	34
	2012	3072	2614	1785	1810	1069	508	128	39

Source: own illustration

The values in bold italics are taken from Figure 5, i.e. they are the estimated future cumulative claims loss settlements.

The final step in the use of run-off triangles is to group the estimated incremental claims loss settlement amounts by the years in which they will be settled. These predicted cash flows can then be discounted to determine the technical provisions. Each diagonal in the red section of Figure 6 represents a calendar year of claims settlements. Figure 7 demonstrates how this information is used to derive the expected claims settlement cash flows for each future calendar year.

Figure 7: Deriving the estimated claims loss settlement amounts for each future calendar year

Estimated future incremental claims loss settlement amounts	Development year							
Claims occurrence year	0	1	2	3	4	5	6	7
2005								
2006								18
2007							69	21
2008						314	79	24
2009					756	359	91	28
2010				1540	909	432	109	33
2011			1544	1565	924	439	111	34
2012		2614	1785	1810	1069	508	128	39

Calendar year	Estimated claims loss settlement amounts
2013	6855
2014	4718
2015	3281
2016	1069+439+109+28 = 1645
2017	652
2018	162
2019	39

Source: own illustration

Before determining its total technical provisions the insurer must ensure that claims data that is not represented in the run-off triangles is considered using some other provisioning method. For example, the technical provisions for the claim settlement amount that was excluded from the run-off triangle data (see section 3.1) must be determined separately if the insurer believes that there may still be amounts outstanding in respect of the relevant claim event(s).

4 Literature

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Appendix A: The theory behind the chain-ladder method for determining technical provisions for outstanding reported claims

I. Data requirements

I.1 Claims loss data

As a starting point for using run-off triangle techniques to estimate future claims losses from existing insurance policies an insurance undertaking requires information regarding its past claims experience. Specifically, it will need data for the total amounts of claims that were settled in the past. These claims losses include:

a) The claims benefit payments (gross of reinsurance recoveries).

b) Any indirect expenses that were allocated to the claims.

c) Any direct claims handling expenses.

d) The reinsurance recoveries (or other recoveries).

Reinsurance recoveries result in a reduction in claims losses (or no change). In order to simplify the explanations these potential negative claims losses are ignored in this paper.

In order to represent the claims loss data in run-off triangles the following information is required for each individual claim loss settlement (see GDV 2011: 14):

a) The claim event in respect of which the claims loss is settled.

b) The date of the claim event (e.g. the date on which storm damage was caused to an insured property).

c) The date on which the claim event was reported to the insurer.

d) The date on which the claim loss was settled. The benefits and the expenses in respect of a claim may be paid or allocated over a number of reporting periods.

Using all of the above data allows an insurer to model delays between the occurrence of the claim event and the reporting of the claim event (reporting delay, used in provisioning for "incurred but not reported claims") and the delays between the reporting and the settlement of the claim (settlement delays). For simplicity purposes, this paper considers only the settlement delays, i.e. only reported claims are considered. However, the approach for determining reporting delays for incurred but not reported (IBNR[18]) claims is similar.

I.2 Basic terminology and definitions

For the purposes of this paper the following terms are defined as shown below (Schmidt 2006: 270):

Calendar year

A period of 12 months running from January to December. **_Definition A1_**

[18] IBNR refers to those claims which have not yet been reported to the insurer even though the claim events have already occurred, i.e. the insurer is not yet aware of these claim liabilities.

Claims occurrence year, y [19]

The calendar year in which a claim event occurred; y ∈{1900; 1901; …; Y} where Y represents the most recent complete calendar year. Also called: accident year. ***Definition A2***

Development year, k

The complete number of years that have elapsed between the end of the claims occurrence year and the end of the year in which a claims loss amount is settled (partly or in full) [20]*; k ∈{0; 1; …; n} where n represents the maximum number of development years observed for any claims occurrence year.* ***Definition A3***

Incremental claims loss settled, Z(y;k)

The claims loss amount, for claims occurrence year y, that is settled in development year k.

Definition A4

It should be clear that y+k represents the calendar year in which the incremental claims loss settlement amount is paid. The incremental claims loss settlement information for the claims occurrence years Y-n to Y can be represented as in Figure A1.

Figure A1: Incremental claims loss settlement data represented as a run-off triangle

		Development year k						
	Z(y;k)	**0**	**1**	**…**	**i**	**…**	**n-1**	**n**
Claims occurrence year y	**Y-n**	**Z(Y-n;0)**	**Z(Y-n;1)**	**…**	**Z(Y-n;i)**	**…**	**Z(Y-n;n-1)**	**Z(Y-n;n)**
	Y-n+1	**Z(Y-n+1;0)**	**Z(Y-n+1;1)**	**…**	**Z(Y-n+1;i)**	**…**	**Z(Y-n+1;n-1)**	**Z(Y-n+1;n)**
	…	**…**	**…**	**…**	**…**	**…**	**…**	**…**
	Y-i	**Z(Y-i;0)**	**Z(Y-i;1)**	**…**	**Z(Y-i;i)**	**…**	**Z(Y-i;n-1)**	**Z(Y-i;n)**
	…	**…**	**…**	**…**	**…**	**…**	**…**	**…**
	Y-1	**Z(Y-1;0)**	**Z(Y-1;1)**	**…**	**Z(Y-1;i)**	**…**	**Z(Y-1;n-1)**	**Z(Y-1;n)**
	Y	**Z(Y;0)**	**Z(Y;1)**	**…**	**Z(Y;i)**	**…**	**Z(Y;n-1)**	**S(Y;n)**

Source: own illustration

In any cell in Figure A1 Z(y;k) represents the claims loss amount settled in calendar year y+k in respect of claims events which occurred in year y. For example, the amount represented by the value Z(Y-n+1;1) was settled during the calendar year Y-n+2. It should be clear that in each green cell on the diagonal blue line

[19] Schmidt 2006 and GDV 2011 denote the „accident year“ by i where i ∈{0; 1; 2; …; n}. In this paper the accident year denotes actual calendar years (e.g. 2009) as the author feels that this is a more intuitive representation. The difference is, however, purely one of presentation. The method demonstrated in this paper is the same as that described by Schmidt 2006 and GDV 2011.

[20] When considering reporting delays rather than settlement delays the definitions in this paper would refer to “claims losses reported” rather than “claims losses settled”.

y+k always equals Y. This diagonal, therefore, represents the amounts settled in the most recent completed calendar year, Y. The upper triangle (green cells) represents settlement payments made before the end of calendar year Y, i.e. in the past (observed data). The lower triangle (red cells) represents the settlement payments that will be made in the future, i.e. after the most recent calendar year, Y. The red cells are therefore the settlement payments which must be estimated or predicted.

Based on the definition of n, claims occurrence year Y-n is the only year for which claims are (expected to be) fully settled. For all other claims occurrence years it is expected that there will be further claims settlements in the future.

Cumulative claims losses settled, S(y;k)

The claims loss amount, for claims occurrence year y, that is settled by development year k, i.e. in or before development year k. ***Definition A5***

It should be clear that:

$$S(y;k) = \sum_{i=0}^{k} Z(y;i)$$ ***Formula A1***

From Formula A1 it follows that

$S(y;0) = Z(y;0)$ and

$Z(y;k) = S(y;k) - S(y;k-1)$ and

the ultimate claims loss for claims occurrence year y is $S(y;n) = \sum_{i=0}^{n} Z(y;i)$.

The cumulative claims loss settlement information for an insurer can be represented as in Figure A2.

Figure A2: Cumulative claims loss settlement data represented as a run-off triangle

		Development year k						
	S(y;k)	0	1	...	i	...	n-1	n
Claims occurrence year y	Y-n	S(Y-n;0)	S(Y-n;1)	...	S(Y-n;i)	...	S(Y-n;n-1)	S(Y-n;n)
	Y-n+1	S(Y-n+1;0)	S(Y-n+1;1)	...	S(Y-n+1;i)	...	S(Y-n+1;n-1)	S(Y-n+1;n)
	...	...	...	...	...	...	...	...
	Y-i	S(Y-i;0)	S(Y-i;1)	...	S(Y-i;i)	...	S(Y-i;n-1)	S(Y-i;n)
	...	...	...	...	...	...	...	...
	Y-1	S(Y-1;0)	S(Y-1;1)	...	S(Y-1;i)	...	S(Y-1;n-1)	S(Y-1;n)
	Y	S(Y;0)	S(Y;1)	...	S(Y;i)	...	S(Y;n-1)	S(Y;n)

Source: own illustration

In Figure A2 the S(y;k) values along the blue diagonal line represent the claims loss amounts, for claims occurrence years Y-n to Y, that were settled up to and including the most recent completed calendar year, Y.

II. Development factors

The use of run-off triangles for estimating future claims loss payments requires the assumption that the development of settled claims losses follows the same pattern for every claims occurrence year (see Schmidt 2006: 273 and GDV 2011: 36). ***Assumption A1***

Furthermore, it is assumed that this pattern will hold for future payments, i.e. the pattern will continue into the future.

Development pattern for incremental claims losses settled, ϑ

The parameter vector (ϑ(0), ϑ(1), …, ϑ(n)), with $\sum_{k=0}^{n} \vartheta(k) = 1$, where, for any claims occurrence year y and for any development year k ∈{0; 1; …; n}, ϑ(k) = E[Z(y;k)]/E[S(y;n)] ***Definition A6***

Implicit in Definition A6 is the assumption that, for any claims development year k, E[Z(y;k)]/E[S(y;n)] is identical for every claims occurrence year y. ***Assumption A2***

Development pattern for cumulative claims losses settled, γ

The parameter vector (γ(0), γ(1), …, γ(n)), with γ(n)=1, where, for any claims occurrence year y and for any development year k ∈{0; 1; …; n}, γ(k) = E[S(y;k)]/E[S(y;n)] ***Definition A7***

Implicit in Definition A7 is the assumption that, for any claims development year k, E[S(y;k)]/E[S(y;n)] is identical for every claims occurrence year y. ***Assumption A3***

Development pattern for cumulative claims loss settlement factors, φ

The parameter vector (φ(0), φ(1), …, φ(n)) where, for any claims occurrence year y and for any development year k ∈{1; …; n}, φ(k) = E[S(y;k)]/E[S(y;k-1)] ***Definition A8***

Implicit in Definition A8 is the assumption that, for any claims development year k≥1, E[S(y;k)]/E[S(y;k-1)] is identical for every claims occurrence year y. ***Assumption A4***

Assumption A2, Assumption A3 and Assumption A4 are equivalent with one another and with Assumption A1.

III. Estimation using the chain-ladder method

The chain-ladder method (CLM) is a simple method for estimating future claims loss settlement amounts. This run-off triangle technique uses only the φ(k) parameters and therefore explicitly uses Assumption A4. Other run-off triangle techniques such as the Bornhuetter-Ferguson method require the use of prior estimators of the ultimate claims losses and prior estimators of the γ(k) parameters. These prior estimators may be based on information contained in the run-off triangle itself or on information obtained from external observations or experience (see Schmidt: 278). The use of appropriate prior estimators may improve the reliability of the estimations. The choice of appropriate prior estimators required for other run-off triangle techniques is an important actuarial decision and is not considered in this paper.

In order to estimate the Z(y;k) for those cells in the run-off triangle where y+k>Y (i.e. the incremental claims loss amounts that will be settled in the future) the CLM is used to estimate firstly the claims loss settlement factors for every k ∈ {1; …; n}. These factors are then used to derive the estimators of the S(y;k) and, ultimately, the Z(Y;k).

CLM estimator for φ(k), $\varphi^{CL}(k)$

The estimator for φ(k) derived by using the CLM ***Definition A9***

CLM estimator for S(y;k), $S^{CL}(y;k)$

The estimator for S(y;k) derived by using the CLM ***Definition A10***

CLM estimator for Z(y;k), $Z^{CL}(y;k)$

The estimator for Z(y;k) derived by using the CLM ***Definition A11***

The CLM estimator for φ(k) is

$\varphi^{CL}(k) = \sum_{y=Y-n}^{Y-k} S(y;k-1) \,/\, \sum_{y=Y-n}^{Y-k} S(y;k)$ for any $k \in \{1; \ldots; n\}$ ***Formula A2***

The CLM estimator for S(y;k), where y+k>Y, is

$S^{CL}(y;k) = S(y;Y-y) \bullet \varphi^{CL}(Y-y+1) \bullet \varphi^{CL}(Y-y+2) \bullet \ldots \bullet \varphi^{CL}(k)$ ***Formula A3***[21]

Formula A3 is equivalent to

$S^{CL}(y;k) = S(y;Y-y) \bullet \prod_{j=Y-y+1}^{k} \varphi^{CL}(j)$ ***Formula A4***

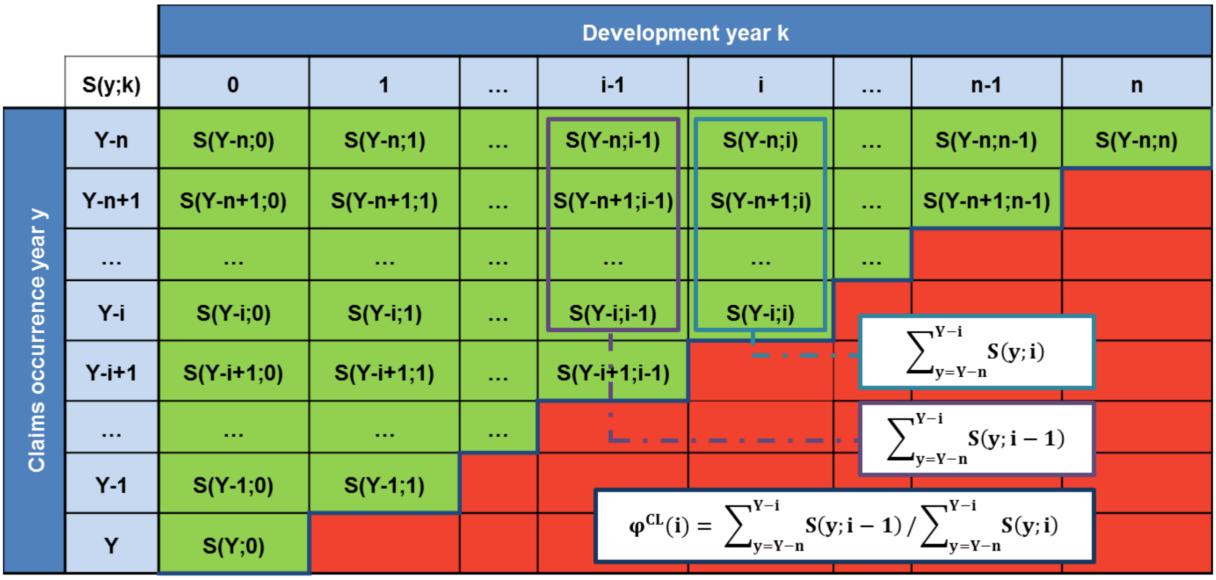
It can also be derived (and is intuitively obvious) that

$S^{CL}(y;k) = S^{CL}(y;k-1) \bullet \varphi^{CL}(k)$ ***Formula A5***

Figure A3 demonstrates how elements of the observed historical data are used for determining the CLM estimators for the cumulative claims loss factors, $\varphi^{CL}(k)$ using Formula A2.

[21] Schmidt 2006 demonstrates on pages 284-285 that the CLM predictors are equivalent to the Bornhuetter-Ferguson method with the prior estimators of the ultimate claims losses and the γ(k) parameters relying solely on information contained in the run-off triangle.

Figure A3: Determining the CLM estimator for the cumulative claims loss settlement factor, $\varphi^{CL}(k)$, using run-off triangle information



Source: own illustration

Figure A4 demonstrates how the derived $\varphi^{CL}(k)$ factors are used together with the S(y; Y-y) values along the diagonal (the most up-to-date cumulative claims settlement data) to calculate the estimated future cumulative claims settlement amounts using Formula A3 and Formula A4.

Figure A4: Determining the CLM estimators for the future cumulative claims loss settlement amounts, $S^{CL}(y;k)$

S(y;k), S^{CL} (y;k)	Development year k							
Claims occurrence year y	**0**	**1**	**…**	**i-1**	**i**	**…**	**n-1**	**n**
Y-n	S(Y-n;0)	S(Y-n;1)	…	S(Y-n;i-1)	S(Y-n;i)	…	S(Y-n;n-1)	S(Y-n;n)
Y-n+1	S(Y-n+1;0)	S(Y-n+1;1)	…	S(Y-n+1;i-1)	S(Y-n+1;i)	…	S(Y-n+1;n-1)	$S^{CL}(Y-n+1;n) = S(Y-n+1;n-1) \bullet \varphi^{CL}(n)$
…	…	…	…	…	…	…	…	…
Y-i	S(Y-i;0)	S(Y-i;1)	…	S(Y-i;i-1)	S(Y-i;i)	…	…	$S^{CL}(Y-i;n) = S(Y-i;i) \bullet \prod_{j=i+1}^{n} \varphi^{CL}(j)$
Y-i+1	S(Y-i+1;0)	S(Y-i+1;1)	…	S(Y-i+1;i-1)	$S^{CL}(Y-i+1;i) = S(Y-i+1;i-1) \bullet \varphi^{CL}(i)$	…	…	$S^{CL}(Y-i+1;n) = S(Y-i+1;i-1) \bullet \prod_{j=i}^{n} \varphi^{CL}(j)$
…	…	…	…	…	…	…	…	…
Y-1	S(Y-1;0)	S(Y-1;1)	…	…	$S^{CL}(Y-1;i) = S(Y-1;1) \bullet \varphi^{CL}(2) \bullet \varphi^{CL}(3) \bullet ... \bullet \varphi^{CL}(i)$	…	…	$S^{CL}(Y-1;n) = S(Y-1;1) \bullet \prod_{j=2}^{n} \varphi^{CL}(j)$
Y	S(Y;0)	$S^{CL}(Y;1) = S(Y;0) \bullet \varphi^{CL}(1)$	…	$S^{CL}(Y;i-1) = S(Y;0) \bullet \varphi^{CL}(1) \bullet \varphi^{CL}(2) \bullet ... \bullet \varphi^{CL}(i-1)$	$S^{CL}(Y;i) = S(Y;0) \bullet \varphi^{CL}(1) \bullet \varphi^{CL}(2) \bullet ... \bullet \varphi^{CL}(i)$	…	$S^{CL}(Y;n-1) = S(Y;0) \bullet \prod_{j=1}^{n-1} \varphi^{CL}(j)$	$S^{CL}(Y;n) = S(Y;0) \bullet \prod_{j=1}^{n} \varphi^{CL}(j)$

Source: own illustration

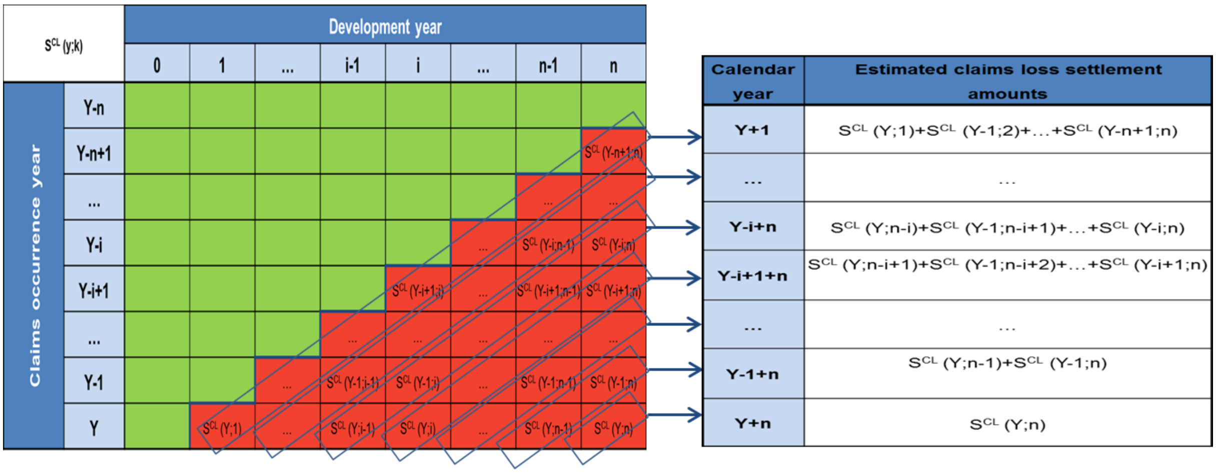
The CLM estimator for Z(y;k) is then simply calculated as

$$Z^{CL}(y;k) = S^{CL}(y;k) - S^{CL}(y;k-1) \qquad \textit{Formula A6}$$

Formula A4 is based on the relationship between incremental and cumulative claims losses settled as demonstrated in Formula A1.

Once the $Z^{CL}(y;k)$ are derived these estimated future incremental claims loss settlement amounts can be grouped by the expected year of settlement. These projected cash flows can then be discounted appropriately in order to determine the required technical provisions for these liabilities. Figure A5 demonstrates how the estimated incremental claims loss settlements can be allocated to future calendar years.

Figure A5: Allocating estimated future claims loss settlements to future calendar years



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