STK4540: Non-life Insurance Mathematics

Exercise list 1

Exercise 1

Suppose that the random variable N follows a geometric distribution with parameter $p \in (0,1)$, that is for $k \geqslant 1$

$$P(N = k) = (1-p)^{k-1} p.$$

(a) Show that the geometric distribution defines a probability distribution on $\mathbb{R}$.

(b) Let $n \in \mathbb{N} \setminus \{0\}$. Calculate $P(N \geqslant n)$.

(c) Calculate $E[N]$.

(d) Let $t < -\log(1-p)$. Calculate $M_N(t) = E[e^{tN}]$.

(e) Calculate $\frac{d}{dt} M_N(t)\big|_{t=0}$.

Exercise 2

Suppose that a random variable X follows an exponential distribution with parameter $\lambda > 0$, i.e. the density f_X of X is given by

$$f_X(x) = \lambda e^{-\lambda x}, \quad x \geqslant 0.$$

(a) Show that the exponential distribution defines a probability distribution on $\mathbb{R}$.

(b) Let $0 < x_1 < x_2$. Calculate $P(x_1 < X < x_2)$.

(c) Calculate $E[X]$ and $Var[X]$.

(d) Let $t < \lambda$. Calculate $\log M_X(t) = \log E[e^{tX}]$.

(e) Calculate $\frac{d^2}{dt^2} M_X(t)\big|_{t=0}$.

Exercise 3

Suppose that an insurance company distinguishes between small and large claims, where a claim is called large if it exceeds a fixed threshold $\theta > 0$. We use a random variable I to indicate whether a claim is small or large, i.e. we have $I = 0$ for a small claim and $I = 1$ for a large claim. In particular, we get $P(I = 0) = 1 - p$ and $P(I = 1) = p$ for some $p \in (0, 1)$. Finally, we model the size of a claim that belongs to the large claims section by the random variable Y, where, given that we have a small claim, Y is equal to 0 almost surely and, given that we have a large claim, Y follows a Pareto distribution with threshold $\theta > 0$ and tail index $\alpha > 0$, i.e. the density $f_{Y|I=1}$ og $Y|I = 1$ is given by

$$f_{Y|I=1}(x) = \frac{\alpha}{\theta} \left(\frac{x}{\theta}\right)^{-(\alpha+1)}, \quad x \geqslant \theta.$$

(a) Let $y > \theta$. Calculate $P(Y \geqslant y)$.

(b) Calculate $E[Y]$.