STK4540: Non-life Insurance Mathematics

Exercise list 2

Exercise 1

Consider S with Poisson distributed claim number process with mean value function $\mu(t)$ and Pareto distributed individual claim amounts with scale and shape parameters $\theta > 0$ and $\alpha > 2$, respectively. Compute $E[S(t)]$ and $Var[S(t)]$.

Exercise 2

Let

$$S_j(t) = \sum_{i=1}^{N_i(t)} X_i, \quad j = 1, 2$$

independent with $N_i(t)$ independent Poisson distributed claim number processes with mean value functions $\mu_1(t)$ and $\mu_2(t)$, respectively. Define $S(t) = S_1(t) + S_2(t)$, $t \geqslant 0$. Show that the distribution of $S(t)$ is equal to the distribution of

$$\mathcal{S}(t) := \sum_{i=1}^{N_1(t)+N_2(t)} X_i.$$

Exercise 3

Under an excess of loss reinsurance arrangement, a claim is shared between the insurer and the reinsurer only if the claim exceeds a fixed amount called the retention level. Otherwise, the insurer pays the claim in full. Let M denote the retention level, and let Y and Z denote the amounts paid by the insurer and the reinsurer respectively. Mathematically, this arrangement can be represented as the insurer pays $Y = \min(X, M)$ and the reinsurer pays $Z = \max(0, X - M)$. Observe that $Y + Z = X$.

Now, let W denote the amount of a non-zero payment by the reinsurer under an excess of loss reinsurance arrangement with retention level M. Find the distribution of W in terms of the distribution, say F, of X.

Exercise 4

Assume that the aggregate claim amount S is shared by the insurer and reinsurer regardless of the type of reinsurance arrangement. We can write $S = S_I + S_R$ where S_I denotes the insurer's aggregate claims, net of reinsurance, and S_R denotes the reinsurer's aggregate claim amount. Let us assume that the insurer has effected excess of loss reinsurance with retention level M. Then we can write

$$S_I = \sum_{i=1}^{N} \min(X_i, M)$$

with $S_I = 0$ when $N = 0$, and

$$S_R = \sum_{i=1}^{N} \max(0, X_i - M)$$

with $S_R = 0$ when $N = 0$. Observe that $S_R = 0$ even if $N > 0$. This situation would arise if $N > 0$ claims occurred and each of these claims was for an amount less than M.

Let N_R denote the number of non-zero claims for the reinsurer. Find the distribution of N_R when N is Poisson distributed.

Exercise 5

An insurance company decides to offer a no-claims bonus to good car drivers, namely

- a 10% discount after three years of no claim, and
- a 20% discount after six years of no claim.

If the base premium is defined as the expected claim amount, how does it need to be adjusted so that this no-claims bonus can be financed? For simplicity, we consider one car driver who has been insured for at least six years. Answer the question in the following two situations:

(a) The claim counts of the individual years of the considered car driver are independent, identically Poisson distributed random variables with frequency parameter $\lambda = 0.2$.

(b) Suppose Θ follows a gamma distribution with shape parameter $\gamma = 1$ and scale parameter $c = 1$, i.e. the density f_Θ of Θ is given by

$$f_\Theta(x) = e^{-x}, \quad x \geqslant 0.$$

Now, conditionally given Θ, the claim counts of the individual years of the considered car driver are independent, identically Poisson distributed random variables with frequency parameter $\Theta\lambda$, where $\lambda = 0.2$ as above.