STK4540: Non-life Insurance Mathematics

Exercise list 7

Exercise 1

We consider the ith policy in the heterogeneity model and suppress the dependence on i in the notation. Assume we have one claim number X in the policy which is $Poiss(\theta)$-distributed, given some positive random variable θ. Assume that the moments $m_k = E[\theta^k] < \infty$, $k = 1, 2, 3, 4$, are known.

(a) Determine the linear Bayes estimator $\widehat{\theta}$ for $\mu(\theta) = E[X|\theta] = \theta$ based on X only in terms of X, m_1,m_2. Express the minimal Bayes risk $\rho(\widehat{\mu})$ as a function of m_1 and m_2.

(b) Now we want to find the best estimator $\widetilde{\theta}_{LB}$ of θ with respect to the quadratic risk $\rho(\widetilde{\mu}) = E[(\theta - \widetilde{\theta})^2]$ in the class of linear functions of X and $X(X-1)$:

$$\widetilde{\theta} = a_0 + a_1 X + a_2 X(X-1), \quad a_0, a_1, a_2 \in \mathbb{R}.$$

This means that $\widetilde{\theta}_{LB}$ is the linear Bayes estimator of θ based on the data $\vec{X} = (X, X(X-1))'$. Apply the normal equations to determine a_0, a_1 and a_2. Express the relevant quantities by the moments m_k. Hint: Use the well-known identity $E[Y^{(k)}] = \lambda^k$ for the factorial moments $E[Y^{(k)}] = E[Y(Y-1)\cdots(Y-k+1)]$, $k \geqslant 1$, of a random variable $Y \sim Poiss(\lambda)$.

Exercise 2

In exercise 1 from list 6, calculate the linear Bayes estimator of $p(\theta) = E[Y_1|\theta]$ based on the data $X_1, \ldots, X_n$ and the corresponding linear Bayes risk. Compare the Bayes and the linear Bayes estimators and their risk.

Exercise 3

In exercise 3 from list 6, calculate the linear Bayes estimator of $E[X_1|\theta]$ and the corresponding linear Bayes risk. Compare the Bayes and the linear Bayes estimators and their risk.

Exercise 4

In exercise 4 from list 6, calculate the linear Bayes estimator of $E[X_1|\theta]$ and the corresponding linear Bayes risk. Compare the Bayes and the linear Bayes estimators and their risk.

Exercise 5

Consider a portfolio with n independent policies.

(a) Assume that the claim numbers $X_{i,t}$, $t = 1, 2, \ldots$, in the ith policy are independent and $Pois(p_{i,t}\theta_i)$-distributed, given θ_i. Assume that $p_{i,t} \neq p_{i,s}$ for some $s \neq t$. Are conditions of th Bühlmann-Straub model satisfied?

(b) Assume that the claim sizes $X_{i,t}$, $t = 1, 2, \ldots$ in the ith policy are independent and $\Gamma(\gamma_{i,t}, \beta_{i,t})$-distributed, given θ_i. Give conditions on $\gamma_{i,t}$ and $\beta_{i,t}$ under which the Bühlmann-Straub model is applicable. Identify the parameters μ, φ, λ and $p_{i,t}$.