Non-Life Insurance Mathematics
Exercises 1, 07.09.2023

Problem 1 Consider the Danish fire insurance data collected at Copenhagen Reinsurance in the period from 02/09/1987 until 02/22/1987:

Date	Loss in DKM
02/10/1987	4.128015
02/11/1987	1.076994
02/12/1987	1.502783
02/13/1987	1.021336
02/15/1987	4.500000
02/16/1987	3.107607
02/18/1987	2.254174
02/19/1987	1.757885
02/21/1987	1.398887
02/22/1987	1.252319

The above data comprise fire losses expressed in millions of Danish Krone. Assume that the claim sizes and claim arrival times are described by the Cramér-Lundberg model.

Calculate the maximum likelihood estimator of the (jump) intensity λ of the claim numbers and determine the distribution, the mean and the variance of the arrival times of the fire losses.

Problem 2 Consider a portfolio where the claims arrive according to the Cramér-Lundberg model. The *backward recurrence time* given by

$$B(t) := t - T_{N(t)}$$

is the time span between t and the last claim arrival at time $T_{N(t)}$. The process

$$F(t) := T_{N(t)+1} - t$$

is called *forward recurrence time* and measures the period of time between the time point t and the next claim arrival at time $T_{N(t)+1}$.

Show that $B(t)$ and $F(t)$ are independent.

Problem 3 Consider the following real-life fire insurance data collected at Copenhagen

Reinsurance in the period from 09/01/1990 until 10/01/1990:

Date	Loss in DKM
09/08/1990	14.851485
09/11/1990	1.596535
09/15/1990	3.877888
09/19/1990	2.021452
09/20/1990	1.650165
09/27/1990	1.010726
09/27/1990	1.485149
09/30/1990	1.278878
10/01/1990	1.023102
10/01/1990	4.702970

The fire losses in the above table are expressed in millions of Danish Krone. Assume that the dynamics of the total claim amount follows the renewal model.

Employ the above data to calculate for "large" t

(i) the net premium $p_{Net}(t)$ based on the equivalence principle.

(ii) the expected value premium $p_{EV}(t)$ for the safety loading $\rho = 4\%$. How must be the "safety loading" α of the variance principle based premium $p_{Var}(t) = E[S(t)] + \alpha Var[S(t)]$ chosen such that $p_{Var}(t)$ and $p_{EV}(t)$ become equivalent in the sense of

$$\frac{p_{Var}(t)}{p_{EV}(t)} \longrightarrow 1$$

for $t \longrightarrow \infty$.

Problem 4 Once more let us cast a glance at the following Danish fire insurance data collected at Copenhagen Reinsurance in the period from 09/01/1983 until 09/25/1983:

Date	Loss X_i in DKM	Date	Loss X_i in DKM
09/03/1983	3.876529	09/16/1983	3.061179
09/03/1983	3.857319	09/17/1983	1.436040
09/04/1983	1.437954	09/18/1983	2.279199
09/05/1983	1.175275	09/18/1983	1.491766
09/08/1983	1.501669	09/19/1983	6.916554
09/09/1983	1.506003	09/21/1983	1.604229
09/13/1983	5.925473	09/25/1983	1.668521
09/16/1983	12.631813	09/25/1983	1.157211

The fire losses in the above table are given in millions of Danish Krone. Suppose that the claim size distribution is modelled by an exponential distribution with intensity $\lambda > 0$.

Compute the maximum likelihood estimator $\widehat{\lambda}$ of the intensity of the claim sizes. Sketch the graph of the QQ-plot of the above data against $F \sim Exp(\widehat{\lambda})$.

Hint: Apply the following convention: Claims $X_1^{(m)}, ..., X_n^{(m)}$ arriving at day $m \geq 0$ are interpreted as 1 claim $X_m := \sum_{i=1}^{n} X_i^{(m)}$ arriving at day m.

Problem 5 Consider the fire insurance data of Problem 3 with respect to the period from 09/01/1990 to 10/01/1990.

Assume that the dynamics of the total claim amount follows the renewal model. Further require that the insurer's premium income is given by

$$p(t) = c \cdot t,$$

where c is the premium rate.

Use the sample mean as a (rough) estimator for the expected fire losses and inter-arrival times and give the net profit condition for the premium rate c.

Problem 6 Assume in Problem 1 a safety loading given by $\rho = 3\%$ with respect to the premium income $p(t) = (1 + \rho)E[S(t)]$. Further, require that the net profit condition holds and that claim sizes are exponentially distributed.

Derive a formula for the non-ruin probability $\varphi(u) = 1 - \Psi(u)$, when the initial capital u is zero, and calculate its value.