Non-Life Insurance Mathematics

Exercises 3, 21.09.2023

Problem 1 Consider the Danish fire insurance data in the period from 09/01/1983 until 09/25/1983 in Exercises 1, Problem 4.

Assume that the claim sizes are Pareto distributed, that is

$$\overline{F}(x) = \gamma^{\alpha} x^{-\alpha}, x > \gamma$$

for $\gamma > 0$ and $\alpha > 1$.

(i) Compute the maximum likelihood estimator of the parameters.

(ii) Sketch the graph of the QQ-plot.

(iii) Sketch the graph of the ME-plot.

Problem 2 Compute the mean excess function $e_F(u)$ for the estimated Pareto distribution in Problem 1.

Problem 3 Give a (rough) estimate of the ruin probability $\Psi(u)$ by using the exact asymptotics of ruin probabilities in the case of large claims based on the estimated Pareto distribution in Problem 1.

Here the safety loading is $\rho = 10\%$ and the initial capital is $u = 4$ and 12 (in millions of Danish Krone).

Problem 4 Let $f(x) = x^{\delta} L(x)$ be a regularly varying function. Here L is a slowly varying function and $\delta \in \mathbb{R}$.

A classic result from the theory of regularly varying functions is the following *theorem of Karamata*:

For any $y_0 > 0$, we have that

$$\lim_{y \longrightarrow \infty} \frac{\int_y^{\infty} f(x)dx}{y f(y)} = -(1+\delta)^{-1}, \text{ if } \delta < -1$$

and

$$\lim_{y \longrightarrow \infty} \frac{\int_{y_0}^{y} f(x)dx}{y f(y)} = (1+\delta)^{-1}, \text{ if } \delta > -1.$$

Use this result to show that the integrated tail distribution of any regularly varying distribution with index $\alpha > 1$ is subexponential.