Non-Life Insurance Mathematics

Exercises 4, 28.09.2023

Problem 1 Assume the heterogeneity model and require that in the i-th policy of a car insurance the claim numbers $X_t, t \geq 1$ given the heterogeneity parameter θ are Poisson distributed with intensity θ. Suppose that θ is Gamma distributed with parameters $\gamma, \beta > 0$.

Find an explicit representation of the Bayes estimator $\widehat{\mu}_B$ with respect to the claim numbers of the policy and calculate $\widehat{\mu}_B$ and its corresponding risk for the claim number data $X_1 = 1$, $X_5 = 2$, $X_7 = 1$ and $X_j = 0$ for $j = 2, 3, 4, 6, 8$ (in years) and parameters $\gamma = 2, \beta = 1$.

Problem 2 A direct consequence of Lemma 5.2.2 in the book of T. Mikosch for $\mathcal{A} = \sigma(X_1, ..., X_r)$ (entirety of claim size information) and the definition of Bayes estimators $\widehat{\mu}_B$ in connection with the heterogeneity model is that

$$\widehat{\mu}_B = E[\mu(\theta_i) \,|X_1, ..., X_r] \,.$$

Show that

$$\widehat{\mu}_B = E[\mu(\theta_i) \,|X_i] \,.$$

Problem 3 Consider the heterogeneity model for the i-th policy of an insurance. Given $\theta \sim \Gamma(\gamma, \beta)$ (Gamma distribution) the claim sizes $X_1, ..., X_n$ in the policy are assumed to be Pareto distributed with parameters (λ, θ), that is

$$P(X_i > x \,|\theta) = (\lambda/x)^\theta, x > \lambda.$$

(i) Find the conditional density of θ given $X_1 = x_1, ..., X_n = x_n$.

(ii) A reinsurance company takes into account only the values X_i exceeding a known high treshold K. They "observe" the counting variables $Y_i := 1_{(K,\infty)}(X_i)$ for a known $K > \lambda$. The company is interested in estimating $P(X_1 > K \,|\theta)$.

Show that Y_i given θ has the distribution $Bin(1, p(\theta))$ (Binomial distribution) with $p(\theta) := E[Y_1 \,|\theta]$. Further, find an explicit expression for the Bayes estimator of $p(\theta)$ based on the claim size data $X_1, ..., X_n$.

Problem 4 Consider the i-th policy in a heterogeneity model. Assume that the claim size variables $X_j, j \geq 1$ given θ are log-normally distributed, that is $X_j = \exp(\theta + \tau Z_j), j \geq 1$. Here θ is normally distributed with mean μ and variance σ^2, $\tau > 0$ a constant and $Z_j, j \geq 1$ is an $i.i.d.$-sequence of standard normally distributed random variables being independent of θ.

Find a formula for the Bayes estimator with respect to the premium of a car insurance and calculate that for the claim size data $X_1 = 10$, $X_5 = 20$, $X_j = 1$ (in 1000 NOK) for $j = 2, 3, 4, 6, 7, 8$ (in years) and the parameters $\mu = 0, \sigma^2 = \tau = 1$.