Non-Life Insurance Mathematics
Exercises 6, 2.11.2023

Problem 1 Consider the Bühlmann model with the claims $X_j, t \geq 1$ and the heterogeneity parameter θ with respect to the i-th insurance policy.

Show that
$$E[Var[\overline{X}\,|\theta]] = n^{-1}E[Var[X_1\,|\theta]],$$
where $\overline{X} := \frac{1}{n}\sum_{j=1}^{n} X_i$.

Problem 2 Explain how a homogeneous Poisson process can be obtained from a Poisson random measure $PRM(\mu)$.

Problem 3 The *Laplace functional* of a point process N is defined as
$$\Psi_N(g) = E[\exp(-\int_E g(z)N(\omega, dz))],$$
where g is any non-negative bounded measurable function g on the state space E.

Show that two point processes N_1 and N_2 are equal in distribution if and only if their Laplace functionals coincide.

Problem 4 Consider a continuous distribution F on $\mathbb{R}$ and let $Y_i, i \geq 0$ be a sequence of *i.i.d.* random variables with common distribution $F^* := \sqrt{F}$. Define the random variables $X_i := \max(Y_{i-1}, Y_i), i \geq 1$

(i) Show that X_i and X_{i+k} are independent for all $k \geq 2$, whereas X_i and X_{i+1} are dependent for all i.

(ii) Verify that
$$F_{X_i}(x) \stackrel{def}{=} P(X_i \leq x) = F(x), x \in \mathbb{R}.$$

(iii) Show for $n \geq 2$ that N_n given by
$$N_n(A) = \sum_{i=1}^{n} \varepsilon_{X_i}(A), A \subset \mathbb{R} \text{ Borel set}$$
is a non-simple point process for the state space E (ε_x Dirac measure at x).