Non-Life Insurance Mathematics
Exercises 7, 9.11.2023

Problem 1 Consider Mack's model. The following *run-off triangle* in the figure shows the cumulative paid claims:

Year i	$S_{i,i}$	$S_{i,i+1}$	$S_{i,i+2}$	$S_{i,i+3}$	$S_{i,i+4}$	$S_{i,i+5}$
2011	52546	81275	90461	98277	103162	106264
2012	62285	98495	110096	118346	123682	
2013	72173	113299	124340	132883		
2014	86135	127359	138409			
2015	97068	150476				
2016	128982					

(i) Compute the predictors $\widehat{S}_{i,m+k}$ of the total claim amounts

$$\begin{aligned}
&S_{2012,m+1},\\
&S_{2013,m+1}, S_{2013,m+2},\\
&S_{2014,m+1}, ..., S_{2014,m+3},\\
&S_{2015,m+1}, ..., S_{2015,m+4},\\
&S_{2016,m+1}, ..., S_{2016,m+5}
\end{aligned}$$

for $m = 2016$ based on the data of the run-off triangle.

(ii) Calculate the technical provisions for the year 2018, that is

$$\sum_{j=0}^{3}(\widehat{S}_{2013+j,2018} - \widehat{S}_{2013+j,2017})$$

and for the year 2021, that is

$$\widehat{S}_{2016,2021} - \widehat{S}_{2016,2020}.$$

Hint: Recall that

$$\widehat{S}_{i,m+k} = \widehat{f}^{(m)}_{m-i+k-1} \cdot ... \cdot \widehat{f}^{(m)}_{m-i} \cdot S_{i,m},$$

where

$$\widehat{f}^{(m)}_j = \frac{\sum_{i=1}^{m-j-1} S_{i,i+j+1}}{\sum_{i=1}^{m-j-1} S_{i,i+j}}$$

is the chain ladder estimator of f_j.

Problem 2 Assume Mack's model. Show that

$$Var[\widehat{f}^{(m)}_j] \leq \frac{\sigma_j^2}{m-j-1}(1 - \int_1^\infty y^{-2}(P(y \leq N_{1,1+j})^{m-j-1}dy),$$

where σ_j^2 corresponds to the constant in condition (v) of Mack's model (see relation (11.2.15) in T. Mikosch).