Exercises 1

Prob. 6 : $\varphi(0)$?

Recall from L. 3.3.3 that

$$\varphi(u) = \varphi(0) + \frac{1}{(1+\rho)E[X_1]} \int_0^u \underbrace{\overline{F}_{X_1}(y)}_{=P(X_1>y)}\, \varphi(u-y)\, dy \qquad (*)$$

safety loading

On the other hand, we know that

$$\varphi(u) = P\Big(\sup_{n\geq 1} S_n \leq u\Big) \qquad \text{(see the proof of Th. 3.1.6)}$$

$$\text{of prob. meas.} \overset{\text{Property (iii)}}{\Longrightarrow} \varphi(u) \uparrow P\Big(\sup_{n\geq 1} S_n < \infty\Big) \text{ for } u \to \infty$$

However $S_n \overset{\text{def.}}{=} \sum_{i=1}^n Z_i \xrightarrow[n\to\infty]{} -\infty$, since

$$\frac{1}{n} S_n \xrightarrow[n\to\infty]{} E[Z_1] \overset{NPC}{<} 0 \quad \text{(due to the SLLN)}$$

$$\Longrightarrow \lim_{u\to\infty} \varphi(u) = 1$$

$\lim_{n\to\infty}(\cdot)$ on both sides of $(*)$, mon. converg. $\Longrightarrow$

$$1 = \varphi(0) + \frac{1}{(1+\rho)E[X_1]} \int_0^\infty \overbrace{\Big(\lim_{u\to\infty} \mathbb{1}_{[0,u]}(y)\varphi(u-y)\Big)}^{=1} \overline{F}_{X_1}(y)\, dy$$

$$= \varphi(0) + \frac{1}{(1+\rho)E[X_1]} \overbrace{\int_0^\infty P(X_1 > y)\, dy}^{=E[X_1]} = \varphi(0) + \frac{1}{1+\rho}$$

$$\Longrightarrow \varphi(0) = \frac{\rho}{1+\rho} \overset{\rho = 0.03}{\Longrightarrow} \varphi(0) \approx 2.9\,\%.$$