①

Exercises 7

Prob. 1 : We know that

$$\hat{S}_{i,m+k} = \hat{f}^{(m)}_{m-i+k-1} \cdots \hat{f}^{(m)}_{m-i} \cdot S_{i,m} \,,$$

where $\hat{f}^{(m)}_{j} = \dfrac{\sum_{i=1}^{m-j-1} S_{i,i+j+1}}{\sum_{i=1}^{m-j-1} S_{i,i+j}}$

$m = 2016$

$\longrightarrow$ $\hat{S}_{2016,2016+1} = \hat{f}^{(m)}_{0} \cdot S_{m,m}$, $\quad S_{m,m} = 128982$

$$\hat{f}^{(m)}_{0} = \frac{\sum_{i=2011}^{2015} S_{i,i+1}}{\sum_{i=2011}^{2015} S_{i,i}} = 1.542$$

$\Longrightarrow$ $\hat{S}_{2016,2016+1} = 198906$,

$$\hat{S}_{2016,2016+2} = \hat{f}^{(m)}_{1} \cdot \hat{f}^{(m)}_{0} \cdot S_{m,m} \,,$$

$$\hat{f}^{(m)}_{1} = \frac{\sum_{i=2011}^{2014} S_{i,i+2}}{\sum_{i=2011}^{2014} S_{i,i+1}} = 1.102$$

$\Longrightarrow$ $\hat{S}_{2016,2016+2} = 219192$

$$\hat{S}_{2015,2015+2} = \hat{f}^{(m)}_{1} S_{2015,2016} = 165823$$

$\vdots$

$\hat{S}_{i,m+k}$

$\longrightarrow$

Year i	$S_{i,i}$	$S_{i,i+1}$	$S_{i,i+2}$	$S_{i,i+3}$	$S_{i,i+4}$	$S_{i,i+5}$
2011	52546	81275	90461	98277	103162	106264
2012	"	"	"	"	123682	127401
2013	"	"	"	"	139155	143537
2014	"	"	"	148893	155918	160606
2015	"	"	165823	178383	186799	192416
2016	128982	178906	217192	235794	246920	254344

$\hat{S}_{2016,2016+1}$ $\quad$ $\hat{S}_{2016,2016+2}$ $\quad$ $\hat{S}_{2016,2016+5}$

② Exerc. 7

→ technical provision for 2018:

$(143337-139153)+(155918-148893)+\ldots+(219192-198906)$
$=44055$

technical provision for 2021:

$254344-246920=7424$

③

Exerc. 7

Prob. 2 : We know from L. 5.26 that

$$\mathrm{Var}[\hat{f}_j^{(m)}] = \sigma_j^2 E[(\sum_{i=1}^{m-j-1} N_{i,i+j})^{-1}] \qquad (*)$$

On the other hand,

$$\sum_{i=1}^{m-j-1} N_{i,i+j} \geq (m-j-1) \min(N_{1,1+j}, \dots, N_{m-j-1,m-1})$$

$$\Rightarrow \mathrm{Var}[\hat{f}_j^{(m)}] \leq \sigma_j^2 \frac{1}{m-j-1} E[\frac{1}{\min(N_{1,1+j}, \dots, N_{m-j-1,m-1})}]$$

$$\overset{E[X] = \int_0^\infty P(X>y)dy}{=} \sigma_j^2 \frac{1}{m-j-1} \int_0^\infty P(\frac{1}{\min(N_{1,1+j}, \dots, N_{m-j-1,m-1})} > y) dy$$

$$= \sigma_j^2 \frac{1}{m-j-1} \int_0^\infty P(\min(N_{1,1+j}, \dots, N_{m-j-1,m-1}) < \frac{1}{y}) dy$$

$$\overset{\text{substitution for } u = \frac{1}{y}}{=} \sigma_j^2 \int_0^\infty \frac{1}{y^2} P(\underbrace{\min(N_{1,1+j}, \dots, N_{m-j-1,m-1})}_{\in \{1,2,3,\dots\}} < y) dy$$

$$\overset{N_{i,i+j} \geq 1}{=} \sigma_j^2 \frac{1}{m-j-1} \int_1^\infty \frac{1}{y^2} \underbrace{P(\min(N_{1,1+j}, \dots, N_{m-j-1,m-1}) < y)}_{= 1 - P(\min(N_{1,1+j}, \dots, N_{m-j-1,m-1}) \geq y)} dy$$

$$= \sigma_j^2 \frac{1}{m-j-1} \Big\{ \underbrace{\int_1^\infty \frac{1}{y^2} dy}_{=1} - \int_1^\infty \frac{1}{y^2} \underbrace{P(\min(N_{1,1+j}, \dots, N_{m-j-1,m-1}) \geq y)}_{\substack{= P(N_{1,1+j} \geq y, \dots, N_{m-j-1,m-1} \geq y) \\ \overset{\text{i.i.d property}}{=} P(N_{1,1+j} \geq y) \cdots P(N_{m-j-1,m-1} \geq y) \\ = (P(N_{1,1+j} \geq y))^{m-j-1}}} dy$$

$$= \sigma_j^2 \frac{1}{m-j-1} \Big\{ 1 - \int_1^\infty \frac{1}{y^2} (P(N_{1,1+j} \geq y))^{m-j-1} dy \Big\}$$