# Fasit til STK4540, Høst 2011

**Oppgave 1**
a) If $\xi = E(X_j)$ and $\sigma = \text{sd}(X_j)$, then $\text{sd}(\mathcal{X})/E(\mathcal{X}) = \sigma/(\xi\sqrt{J}) \to 0$ as $J \to \infty$.

b) If $\xi(\omega) = E(X_j|\omega)$ and $\sigma(\omega) = \text{sd}(X_j|\omega)$, then $E(\mathcal{X}|\omega) = J\xi(\omega)$ and $\text{var}(\mathcal{X}|\omega) = J\sigma^2(\omega)$ so that $E(\mathcal{X}) = JE\{\xi(\omega)\}$ and $\text{var}(\mathcal{X}) = JE\{\sigma^2(\omega)\} + J^2E\{\xi(\omega)\}$. It follows that $\text{sd}(\mathcal{X})/E(\mathcal{X}) = \sigma/(\xi\sqrt{J}) \to \text{sd}\{\xi(\omega)\}/E\{\xi(\omega)\} > 0$ of $\text{sd}\{\xi(\omega)\} > 0$.

c) Since $E(X_j|\mu) = \mu\xi_z$ and $\text{var}(X_j|\mu)) = \mu(\xi_z^2 + \sigma_z^2)$, it follows by the rule of double expectation that

$$E(X_j) = E(\mu)\xi_z \qquad \text{and} \qquad \text{var}(X_j) = E(\mu)(\xi_z^2 + \sigma_z^2) + \text{var}(\mu)\xi_z^2,$$

but there is still independence bewteen policy holders so that a) holds.

d) But with a common $\mu$

$$E(\mathcal{X}) = JE(\mu)\xi_z \qquad \text{and} \qquad \text{var}(\mathcal{X}) = JE(\mu)(\xi_z^2 + \sigma_z^2) + J^2\text{var}(\mu)\xi_z^2,$$

and not all risk is diversified away by letting $J \to \infty$.

**Oppgave 2**
**a)** If the exchange rate is $c$, $\beta$ is changed to $\beta/c$ in the new currency.

**b)** The expression becomes

$$\frac{1 + \alpha e^{b/\beta}}{1 + \alpha e^{(b+z)/\beta}} \to e^{-z/\beta} \qquad \text{as} \qquad b \to \infty,$$

and the over threshold distribution is exponential with scale parameter $\beta$.

**c)** Over threshold distributions when there is no upper limit on the parent distribution becomes either exponential or Pareto.

**d)** $Z = \beta \log\{(1 + U/\alpha)/(1 - U)\}$ der $U$ er uniform.

**Oppgave 3**
a) I R blir kommandoene X=rep(0,m)
N=rpois(m,$\lambda$)
for ( i in 1:m){
U=runif([N[i])
Z=$\beta$*log((1+U/$\alpha$)/(1-U))
X[i]=sum(Z)}

b) Reservene er 42.0 og 48.9 og normalfordelingsantalsen gir 41.1 og 46.7, ikke mye mindre.

c) Hvis kontakten er ved totalen bruk R-kommandoen

mean(pmax(pmin(X-a,0),b)
der X er fra a).
Om kontrakten er per hendelse bruk R-kommandoene
U=runif(m)
Z=$\beta$*log((1+U/$\alpha$)/(1-U)
$\lambda$*mean(pmax(pmin(Z-a,0),b)

d) For den første kontrakten
X-pmax(pmin(X-a,0),b).
For den andre gjør den siste kommandoen i programmet i a) om til
X[i]=sum(Z-pmax(pmin(Z-a,0),b)).

e) 30, 40 og 42.9 for a=30, 40 og 50.

**Oppgave 4**
a) Ruin problemet og ruinligningen er å bestemme initiell kapital $v_0 = v_{0\epsilon}$ slik at

$$\Pr(\min(Y_1, \ldots, Y_K) < 0 | v_{0\epsilon}) = \epsilon$$

b) With mK simulations of $\mathcal{X}$ stored as X by means of the program in Exercise 3a) use R-commands
X=matrix(X,K,m)
d=1/(1+r)**(1:K)
S=apply(d*(X-J*$\pi$),2,cumsum)
Smin=apply(S,2,max)
$v_{0\epsilon}$=sort(S)[m*$\epsilon$]

c) By simulating the cedent $\mathcal{X}$ mK times as in 3c) and follow the steps in b) with $J\pi$ replaced by $J\pi - \Pi^{\text{re}}$ where $\Pi^{\text{re}}$ is the re-insurance premium computed as as in 3c).

d) If Q is a $K \times m$ matrix of simulations of price levels $Q_1, \ldots, Q_K$ use the commands in b) with the third one modified to
S=apply(d*Q*(X-J*$\pi$),2,cumsum)

e)R-program for Q is
I=matrix(1,K,m)
$X = \log\{1 + i_0/(1 + \theta)\}$
for (k in 1:K){
X=a*X+$\sigma$*rnorm(m)
I(k,]=(1+$\theta$)*exp(X)-1}
Q=apply(1+I,2,cumprod)