FASIT til eksamen i STK4540, 03.01.2012

Problem 1

a) The pure premium is the expected value of the portfolio pay-out $\mathcal{X}$ during a given period; i.e. $\pi^{\text{pu}} = E(\mathcal{X})$. It is a break-even price that excatly balances cost when overhead and profit are not taken into account. The market premium is $\pi = (1+\gamma)\pi^{\text{pu}}$ where the loading γ fluctuates with the state of the market.

b) If $X_1, +\ldots, X_J$ are independent claims from J policy holders and $\mathcal{X} = X_1 + \ldots + X_J$), then

$$\frac{\text{sd}(\mathcal{X})}{E(\mathcal{X})} = \frac{\bar{\sigma}}{\sqrt{J}\bar{\xi}}$$

where $\bar{\xi}$ and $\bar{\sigma}^2$ are the average mean claim and average variance of claims for the entire population. The ratio goes to zero as $J \to \infty$.

c) The rule of double expectations yield for identical risks

$$E(\mathcal{X}) = J\xi_\mu\xi_z \qquad \text{and} \qquad \text{var}(\mathcal{X}) = J^2\xi_z^2\sigma_\mu^2 + J\xi_\mu(\xi_z^2 + \sigma_z^2)$$

where ξ_μ and σ_μ are mean and standard deviation for μ ξ_z and σ_z for claims. It flollows that

$$\frac{\text{sd}(\mathcal{X})}{E(\mathcal{X})} \to \frac{\sigma_\mu}{\xi_\mu} \qquad \text{as} \qquad J \to \infty,$$

ansd the standard deviation does not vanish in relative terms unless $\sigma_\mu = 0$.

d) The reserve q_ϵ is the solution of the equation $\Pr(\mathcal{X} > q_\epsilon) = \epsilon$ for a given small number ϵ. Let $\lambda = J\mu$ be the *fixed* Poisson parameter for the portfolio. Monte Carlo produces approximate solutions through the commands

1 $X^* \leftarrow 0$ 2 Draw $N^* \sim \text{Poisson}(J\mu)$
3 Repeat N^* times
    4 Draw Z^* and $X^* \leftarrow X^* + Z^*$
5 Return X^*

Repeat this m times and sort the simulated portfolio claims in descening order as $X^*_{(1)} \geq, \ldots, \geq X^*_{(m)}$ which yields $X^*_{(m\epsilon)}$ as the approximate reserve. When μ is random, add the initial command

Draw μ^*

before going through the others with $\mu = \mu^*$.

e) The first row where the variation is much larger corresponds to the random μ. The 95% and 99% percent reserve under the two scenarios are 104.3 and 138.3 under the random μ regime and 74.1 and 91.4 under the fixed μ regime.

Problem 2

a)

$$\pi^{\rm re} = J\mu \int_b^\infty (z-b)f(z)dz = J\mu \int_b^\infty \{1-F(z)\}\,dz$$

after integration by parts.

b)$\pi^{\rm re} = J\mu\xi^{\rm re}$ where $\xi^{\rm re} = \beta(1+b/\beta)^{-\alpha-1}/(\alpha-1)$.

c) $\mathcal{X}^{\rm re} = \min(Z_1, b) + \ldots + \min(Z_N, b)$ and replace row 4 in 1d) by

Draw Z^* and $X^* \leftarrow X^* + \min(Z^*, b)$

d) The pure re-insurance premium balances re-insurance payments on average, and the exceeding amount is an average loss.

e) A b between 5 and 10 is optimal.

f) Replace β by $\beta(1+I)$.

g) The premium with inflation becomes $\pi^{\rm re} = J\mu\xi^{\rm re}$ where now $\xi^{\rm re} = \beta(1+I)(1+b/(1+I)\beta)^{-\alpha-1}/(\alpha-1)$, and the denominator is more sensitively dependent on I when α increses.

Problem 3

a) By Bayes formula

$$f(\mu|n_1,\ldots,n_K) = cf(n_1,\ldots,n_K|\mu)f(\mu) = c'\mu^{n_1+\ldots+n_K+\alpha-1}e^{-\mu(K+\alpha/\xi_\mu)}$$

after inserting for the density functions. This is a Gamma density function as stated.

b) Obvious.

c) $E(\bar{n}|\mu) = \mu$ which implies that $E(\hat{\pi}-\pi|\mu) = 0$ and the result follows by the rule of double expectation.

d) The first part follows from the Gamma distribution identified in a). Now by the rule of double variance

$$\mathrm{var}(\hat{\pi}-\pi) = E\{\mathrm{var}(\hat{\pi}-\pi)|n_1,\ldots,n_K\} + \mathrm{var}\{E(\hat{\pi}-\pi)|n_1,\ldots,n_K)\}$$

or

$$\mathrm{var}(\hat{\pi}-\pi) = E\{\xi_z^2\hat{\mu}^2/(\alpha+K\bar{n})\} + 0$$

which leads to the result given after inserting for $\hat{\mu}$.

e) When K is infinite, everything about the client is known and if α is infinite, there is no variation among customers. In either case estimates of π haven't any uncertainty. If $\alpha = 0$, the prior has infinte variance and is valueless so that $\hat{\pi} = \xi_z\bar{n}$ with variance as stated.