UNIVERSITY OF OSLO

Candidate no

Faculty of mathematics and natural sciences

Exam in: STK 4540 — Non-Life Insurance Mathematics

Day of examination: December, 9th, 2015

Examination hours: 09:00 – 13:00

This problem set consists of 15 pages.

Appendices: None

Permitted aids: Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

The Poisson model is a common model for claim frequency.

1a

Under certain circumstances the negative binomial model is preferred before Poisson for claim frequency. What are those circumstances? How can we select one of the two models?

Answer:

- An assumption underlying the pure Poisson model without regression variables is that the underlying claim intensities are equal. This assumption can be checked calculating the dispersion coefficient, esimated by $D = \frac{s^2}{\bar{n}}$, where s^2 is the sample variance and $\bar{n}$ is the sample mean. If the dispersion coefficient, is much larger than 1, this indicates that the underlying claim intensities are unequal.
- If the underlying claim intensities are unequal the negative binomial model performs better than the Poisson model.
- Visual plotting of model against actual frequency can be done.
- QQ plot can be done.

□

(Continued on page 2.)

1b

Let N denote the number of claims for a period T and assume that

$$N|\mu \sim \text{Poisson}(\mu T), \tag{1}$$

so that μ is stochastic.

Specific models for μ are handled through the mixing relationship

$$Pr(N = n) = \int_0^\infty Pr(N = n|\mu)g(\mu)d\mu \tag{2}$$

where $g(\mu)$ is the density function of μ.

Assume that

$$Pr(N = n|\mu) = \frac{(\mu T)^n}{n!}e^{-\mu T} \text{ and } g(\mu) = \frac{(\alpha/\xi)^\alpha}{\Gamma(\alpha)}\mu^{\alpha-1}e^{-\mu\alpha/\xi} \tag{3}$$

where

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx. \tag{4}$$

Prove that

$$Pr(N = n) = \frac{\Gamma(n+\alpha)}{\Gamma(n+1)\Gamma(\alpha)}p^\alpha(1-p)^\alpha \text{ where } p = \frac{\alpha}{\alpha+\xi T}. \tag{5}$$

Answer: By the mixing formula presented in (2) it is obtained that

$$Pr(N = n) = \int_0^\infty \frac{(\mu T)^n}{n!}e^{-\mu T} \times \frac{(\alpha/\xi)^\alpha}{\Gamma(\alpha)}\mu^{\alpha-1}e^{-\mu\alpha/\xi}d\mu, \tag{6}$$

or when reorganised,

$$Pr(N = n) = \frac{T^n(\alpha/\xi)^\alpha}{n!\Gamma(\alpha)}\int_0^\infty \mu^{n+\alpha-1}e^{-\mu(T+\alpha/\xi)}d\mu. \tag{7}$$

Substituting $z = \mu(T + \alpha/\xi)$ in the integrand yields

$$Pr(N = n) = \frac{T^n(\alpha/\xi)^\alpha}{n!\Gamma(\alpha)(T+\alpha/\xi)^{n+\alpha}}\int_0^\infty z^{n+\alpha-1}e^{-z}dz, \tag{8}$$

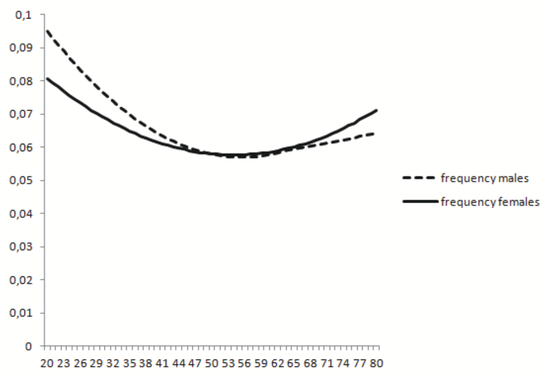
where the integrand is $\Gamma(n + \alpha)$. Hence

$$Pr(N = n) = \frac{\Gamma(n+\alpha)}{\Gamma(n+1)\Gamma(\alpha)}\frac{T^n(\alpha/\xi)^\alpha}{(T+\alpha/\xi)^{n+\alpha}} = \frac{\Gamma(n+\alpha)}{\Gamma(n+1)\Gamma(\alpha)}p^\alpha(1-p)^n, \tag{9}$$

where $p = \alpha/(\alpha + \xi T)$. This is the density function stated in (5). □

(Continued on page 3.)

1c



Female and male frequencies as a function of policyholder age

Assume that the following plot is presented, showing claim frequency for male and females as a function of policyholder age.

What does the plot tell you about the impact of gender and policyholder age on risk? Please propose a claim frequency regression model that includes these effects.

Answer:

- Young policyholders are most risky regardless of gender, but young males are more risky than young females.
- The risk attains a minimum at approximately 50 years for males and some years later for females.
- The risk rises for both genders as policyholders get older, but old males are less risky than old females. However, the risk never reaches the level of the youngest policyholders.
- The plot suggests an interaction between gender and policyholder age. The most important feature to capture is young males, but also old males can represent a model term, depending on policy exposure.
- A model proposal could be

$$\log(\mu_j) = b_0 + b_1 x_{j1} + b_2 x_{j2} + b_3(x_{j1} x_{j2}), \tag{10}$$

  where b_0 represents the intercept, b_1 is the effect of policyholder age, x_{j1} is the policyholder age of policyholder j, b_2 is the effect of gender, x_{j2} is the gender of policyholder j and (x_{j1}, x_{j2}) is an interaction term which is 1 if a male is below 35 years and 0 otherwise. This model disregards the potential interaction between age and gender for old males.

□

(Continued on page 4.)

1d

Table 4 shows a result from a Poisson regression.

Interpret how the claim frequency varies with the parameters gender and policyholder age in the model in Table 4.

Answer:

- In the model females are less risky than males.
- In the model the young policyholders are most risky. The risk decreases with policyholder age until the policyholder age reaches $56 - 70$ where the minimum is attained. The risk increases slightly for the oldest policyholder age group.

□

Variable	Value	Regression estimate	standard deviation
Intercept		-2.315	0.0065
Gender	Male	0	0
Gender	Female	-0.037	0.027
Policyholder age group	20-25	0	0
Policyholder age group	26-39	-0.501	0.068
Policyholder age group	40-55	-0.541	0.067
Policyholder age group	56-70	-0.711	0.07
Policyholder age group	71-94	-0.637	0.073

Table 1: *Regression estimates with standard deviation for a Poisson regression.*

1e

If the model in Table 4 were to be used in pricing, is it advisable to implement these estimates directly? (*Hint: What happens to the price if a customer changes policyholder age group?*)

Answer:

- The subdivision in Table 4 is far to crude to be implemented directly. If a policyholder changes policyholder age group the price would change considerably, which could provoke quite a few clients.
- A model proposal as presented in (10) would provide a continuous relationship between policyholder age and claim frequency.
- Alternatively, more flexible mathematical formulations could be provided by polynomials of higher order.

□

(Continued on page 5.)

Problem 2

The Solvency II directive establishes a revised set of capital adequacy rules for insurance and reinsurance undertakings in the EEA. The starting point for assessing the available capital of an undertaking is to value its assets and liabilities. The liabilities of insurance undertakings include the technical provisions which constitute a significant proportion of their balance sheets.

2a

What is the purpose of Solvency II?

Answer: The purpose of Solvency II is to

- to protect policy holders across the EU,
- to optimize capital allocation by aligning capital requirements to actual risk,
- to create an equal and consistent regulatory regime across the EU,
- to create regulations that are consistent with the ones in comparable industries (particularly banking),
- to create an improved «platform» for proper regulation and supervision, based on increased transparency, more data and better documentation.
- Solvency II has been developed to improve the weaknesses of Solvency I:

  Based on a number of individual directives from the 1970s – Solvency I was formally established in 2002.

  Solvency I is not a harmonised framework at the EU-level: There are significant differences between the various countries e.g. in the valuation of provisions.

  Solvency I is very basic in terms of risk measurement: Insurance risk is the only type of risk taken into account; and only at a high level.

  Solvency I is often supplemented by other, national regulations. E.g. in Norway insurers were also required to comply with banking regulations (Basel I).

  Solvency I is «good» at preventing insolvencies, but it has not required insurers to maintain a level of capital corresponding to the risk exposure of the entity

□

(Continued on page 6.)

2b

What are the most important risk categories in Solvency II (the standard model) and what are their drivers?

Answer:

- The most important risk categories in non-life insurance are market risk and insurance risk.
- The drivers of market risk are interest rate levels, the development of equity prices, the development of property prices and other asset classes the insurance company is invested in. Furthermore, currency could be a risk driver, as well as concentration.
- The drivers of insurance risk are premium risk, reserve risk, lapse risk and Non-life catastrophe risk.

2c

Under Solvency II the projection of run-off triangles is one of the allowed methods for valuing the technical provisions for non-life insurance business.

The simplest of the run-off triangle methods is the chain ladder method.

Introduce

$$C_{ij}, \text{ cumulative claims from accident year } i, \text{ reported through the end of period } j, \tag{11}$$

$$m, \text{ is the last development period that is known,} \tag{12}$$

$$\hat{f}_j = \frac{\sum_{i=1}^{m-j} C_{ij+1}}{\sum_{i=1}^{m-j} C_{ij}}, \text{ is the one period development loss factor.} \tag{13}$$

The run-off triangle in Table 3 shows cumulative payments for the period 2008-2012.

Fill out the triangle using the chain ladder method.

Claim year	Development year				
	0	1	2	3	4
2008	7 008	25 877	31 723	32 718	33 019
2009	30 105	65 758	76 744	79 560	
2010	89 181	171 787	201 381		
2011	109 818	198 015			
2012	97 250				

Table 2: *Run-off triangle for cumulative payments.*

(Continued on page 7.)

Answer:

The formula (13) yields the factors $\hat{f}_1 = \frac{25877+65758+171787+198015}{7008+30105+89181+109818} = 1.958$, $\hat{f}_2 = 1.176$, $\hat{f}_3 = 1.035$ and $\hat{f}_4 = 1.0092$.

Applying these factors to the triangle in the Table above yields

Claim year	Development year				
	0	1	2	3	4
2008	7 008	25 877	31 723	32 718	33 019
2009	30 105	65 758	76 744	79 560	80 282
2010	89 181	171 787	201 381	208 456	210 373
2011	109 818	198 015	233 971	242 192	244 420
2012	97 250	190 428	223 988	231 858	233 991

Table 3: *Run-off triangle for cumulative payments.*

□

2d

Another method for modelling delay is the method invented by Kaminsky (1987). Let q_l be the probability that a claim is settled l periods after the incident took place, where $q_0 + \ldots + q_L = 1$ if L is maximum delay. The process is multinomial if different events are independent. Suppose there are J policies under risk in a period of length T. The number of claims $\mathcal{N}$ is typically Poisson distributed with parameter $\lambda = J\mu T$, but not all are settled at once. If $\mathcal{N}_l$ are those settled l periods later, then $\mathcal{N}_0 + \ldots + \mathcal{N}_L = \mathcal{N}$, and the earlier assumptions make the conditional distribution of $\mathcal{N}_0, \ldots, \mathcal{N}_L$ given $\mathcal{N}$ multinomial with probabilities $q_0, \ldots, q_L$.

This Poisson/multinomial modelling implies

$$\mathcal{N}_l \sim Poisson(J\mu T q_l),\ l = 0, \ldots, L \tag{14}$$

and

$$\mathcal{N}_0, \ldots, \mathcal{N}_L \text{ stochastically independent.} \tag{15}$$

Prove (14) and (15).

Answer: Let $\mathcal{N}$ be the total number of claims during a period T and $\mathcal{N}_l$ those among them settled l periods later for $l = 0, \ldots, L$. Clearly $\mathcal{N} = \mathcal{N}_0 + \ldots + \mathcal{N}_L$ and with $n = n_0 + \ldots + n_L$,

$$Pr(\mathcal{N}_0 = n_0, \ldots, \mathcal{N}_L = n_L) = Pr(\mathcal{N}_0 = n_0, \ldots, \mathcal{N}_L = n_L | \mathcal{N} = n) Pr(\mathcal{N} = n) \tag{16}$$

where by assumption

$$Pr(\mathcal{N}_0 = n_0, \ldots, \mathcal{N}_L = n_L | \mathcal{N} = n) = \frac{n!}{n_0! \cdots n_L!} q_0^{n_0} \cdots q_L^{n_L} \tag{17}$$

(Continued on page 8.)

and

$$Pr(\mathcal{N}=n) = \frac{\lambda^n}{n!}e^{-\lambda} = \frac{\lambda^{n_0+\ldots+n_L}}{n!}e^{-\lambda(q_0+\ldots+q_L)} = \frac{1}{n!}(\lambda^{n_0}e^{-q_0\lambda})\cdots(\lambda^{n_L}e^{-q_L\lambda}) \tag{18}$$

since $q_0 + \ldots + q_L = 1$. This yields

$$Pr(\mathcal{N}_0 = n_0, \ldots, \mathcal{N}_L = n_L) = \frac{q_0^{n_0}\cdots q_L^{n_L}}{n_0!\cdots n_L!}(\lambda^{n_0}e^{-q_0\lambda})\cdots(\lambda^{n_L}e^{-q_L\lambda}) = \prod_{l=0}^{L}\frac{(q_l\lambda)^{n_l}}{n_l!}e^{-q_l\lambda} \tag{19}$$

as claimed in (14) and (15). □

Problem 3

The cumulative function and the density function of the exponential distribution is

$$F(x) = 1 - e^{-x/\xi},\ x > 0 \text{ and } f(x) = \frac{1}{\xi}e^{-x/\xi}, \tag{20}$$

where mean and standard deviation are given

$$E(X) = \xi \text{ and } \mathrm{Var}(X) = \xi^2. \tag{21}$$

The Weibull family is related to the exponential through

$$Z = \beta X^{1/\alpha},\ X\text{exponential with mean 1}, \tag{22}$$

where α and β are positive parameters.

3a

Find the cumulative distribution function and the density function of the Weibull distribution.

Propose a way to generate a stochastic variable from the Weibull distribution using the inversion sampler. (*Hint: Utilize the relation $F(x) = U \iff X = F^{-1}(U)$ where U is sampled from a standard uniform distribution.*

Answer: The distribution function of Z is

$$F(z) = Pr(X \leq (Z/\beta)^\alpha) = 1 - e^{-(z/\beta)^\alpha}, \tag{23}$$

since $Z = \beta X^{1/\alpha} \iff X = (Z/\beta)^\alpha$ and X is Exponential with mean 1.

The density function $f(z)$ is found by differentiating $F(z)$ with respect to z:

$$f(z) = \frac{d}{dz}F(z) = --\frac{\alpha}{\beta}(\frac{z}{\beta})^{\alpha-1}e^{-(z/\beta)^\alpha} = \frac{\alpha}{\beta}(\frac{z}{\beta})^{\alpha-1}e^{-(z/\beta)^\alpha}. \tag{24}$$

To generate a stochastic variable from the Weibull distribution using the inversion sampler $F(x) = U \iff X = F^{-1}(U)$:

$$U = 1 - e^{-(Z/\beta)^\alpha} \iff 1 - U = e^{-(Z/\beta)^\alpha} \iff \beta(-\log(U))^{1/\alpha} = Z, \tag{25}$$

since U and $1-U$ have the same uniform distribution on $[0,1]$. □

(Continued on page 9.)

3b

Find the likelihood function of the Weibull given observations $z_1, \ldots, z_n$,

$$\mathcal{L}(\alpha, \beta) = \sum_{i=1}^{n} \log(f_i(\alpha, \beta, z_i)). \tag{26}$$

Find

$$\frac{\partial \mathcal{L}(\alpha, \beta)}{\partial \beta} \tag{27}$$

and find $\hat{\beta}_\alpha$ such that $\frac{\partial \mathcal{L}(\alpha,\beta)}{\partial \beta} = 0$.

Answer: The likelihood function of the Weibull given observations $z_1, \ldots, z_n$,

$$\begin{aligned}
\mathcal{L}(\alpha, \beta) &= \sum_{i=1}^{n} \log(f_i(\alpha, \beta, z_i)) = \sum_{i=1}^{n} \log \frac{\alpha}{\beta}(\frac{z_i}{\beta})^{\alpha-1} e^{-(z_i/\beta)^\alpha} \\
&= \sum_{i=1}^{n} \{\log(\alpha) - \log(\beta) + (\alpha - 1)\log(z_i) - (\alpha - 1)\log(\beta) - (z/\beta)^\alpha\} \\
&= n\log(\alpha) + (\alpha - 1)\sum_{i=1}^{n} \log(z_i) - n\alpha\log(\beta) - \frac{1}{\beta^\alpha}\sum_{i=1} z_i^\alpha.
\end{aligned}$$

Furthermore,

$$\frac{\partial \mathcal{L}(\alpha, \beta)}{\partial \beta} = -n\alpha\frac{1}{\beta} - (-\alpha)\beta^{-\alpha-1}\sum_{i=1}^{n} z_i^\alpha,$$

and

$$\frac{\partial \mathcal{L}(\alpha, \beta)}{\partial \beta} = 0 \iff \frac{n\alpha}{\beta} = \frac{\alpha}{\beta^{\alpha+1}}\sum_{i=1}^{n} z_i^\alpha,$$

which yields that

$$\hat{\beta}_\alpha = \{\sum_{i=1}^{n} z_i^\alpha\}^{1/\alpha}.$$

□

3c

Find the cumulative distributive function of the *over-threshold* distribution for the Weibull distribution defined as the distribution of $Z_b = Z - b$ given $Z > b$.

Answer:

$$\begin{aligned}
Pr(Z - b > z | Z > b) &= \frac{Pr(Z > z + b, Z > b)}{Pr(Z > b)} = \frac{Pr(Z > z + b)}{Pr(Z > b)} = \frac{1 - F(z + b)}{1 - F(b)} \\
&= \frac{1 - (1 - e^{-((z+b)/\beta)^\alpha}}{1 - (1 - e^{-(b/\beta)^\alpha})} = e^{-((z+b)/\beta)^\alpha + (b/\beta)^\alpha}.
\end{aligned}$$

□

(Continued on page 10.)

3d

What does a known result say about the distribution of over-threshold distributions in general?

Answer: The general result formulated by Pickands says that there exists a parameter α (not depending on b and possibly infinite) and some sequence β_b such that

$$\bar{F}_b(\beta_b y) \to \bar{P}(y/\alpha) \text{ as } b \to \infty \text{ where } \bar{P}(y/\alpha) = \begin{cases} (1+y)^{-\alpha}, & \text{if } 0 < \alpha < \infty, \\ e^{-y} & \text{if } \alpha = \infty. \end{cases} \tag{28}$$

Here the limit $\bar{P}(y/\alpha)$ is the tail distribution of the Pareto model, and $Z_b = Z - b|Z > b$ becomes Pareto(α, β) as $b \to \infty$. □

3e

In practice the data set at hand may not contain extreme enough observations so that the result in part d) can be utilized.

To view this function the *sample mean excess plot* is constructed.
The *sample mean excess function* is defined as

$$e_n(b) = \frac{\sum_{i=1}^{n}(X_i - b)\mathcal{I}(X_i > b)}{\sum_{i=1}^{n}\mathcal{I}(X_i > b)}, \tag{29}$$

where $\mathcal{I}(X_i > b)$ is the indicator function such that $\mathcal{I}(X_i > b) = 1$ if $(X_i > b)$ and 0 otherwise.

How can sample mean excess plots be of assistance when a model for the extreme right tail is selected?

Answer: The mean of the over-threshold distribution (if it exists) is known as the *the mean excess* function and becomes for the Pareto distribution

$$E(Z_b|Z > b) = \frac{\beta + b}{\alpha - 1} = \xi + \frac{b}{\alpha - 1} \text{ (requires } \alpha > 1) \tag{30}$$

where $\xi = E(Z)$. From the equation above it is clear that for the Pareto distribution the mean excess function is linear in b.

The plot below shows that the mean excess function has different characteristics for different parametric claim size distributions. If the sample mean excess plot resembles some of the shapes in the plot below, this may often be used as a help when modelling the tail distribution.

To select extreme right tail distribution simply plot the sample mean excess plot in R. if the plot presented resembles some of the shapes in the plot below, this might indicate that the resembling shape is a suitable candidate for the extreme right tail.

(Continued on page 11.)

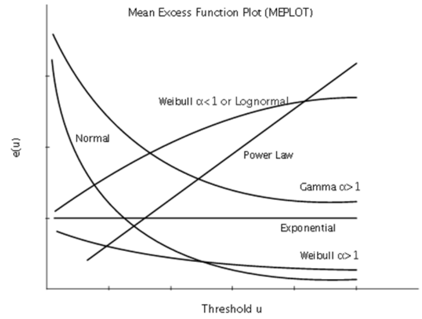


Figure 3: Shape of the mean excess function $e(u)$ for some classical distributions as a function of the threshold u.

Mean excess function for different parametric claim size distributions

□

Problem 4

Assume that you are presented some natural disaster data for Norway for the period 1980-2014. Based on the data you are asked to estimate the next year's premium for natural disasters for Norway. An incident like a hurricane, a storm or a flood can lead to many small claims. Up to December 2014 there are 59 such incidents including in total over 160 000 small claims. In this task small claims occurring on the same day are joined together into one large claim to reduce the amount of data.

Therefore, the claim frequency of interest here is the number of claim days per year. Imagine that the yearly claim frequency, i.e., the number of claim days per year shows an increasing trend in the period 1980-2014.

Assume that the claim frequency of the number of claim days per year follows a negative binomial distribution with a time trend. Let N_i be the number of days with natural disasters occurring in year i and assume that

$$N_i \sim \textit{Negative Binomial} \text{ with parameters } a, b \text{ and } p$$

and

$$E(N_i) = \frac{1-p}{p} \times (ai+b), i = 0, 1, ...$$

and

$$Var(N_i) = \frac{1-p}{p^2} \times (ai+b), i = 0, 1, ...$$

4a

If $a = 0$, what does this tell you about the development of the claim frequency from 1980 until today? Answer the same question assuming that $a > 0$.

Answer: If the parameter $a = 0$ the model predicts that there is no trend in the claim frequency from 1980 up to today. If $a > 0$ the modelled claim frequency increases linearly from year to year. □

(Continued on page 12.)

4b

Assume that the maximum likelihood principle has been applied to estimate a, b and p and assume that $a = 1.227, b = 129.6$ and $p = 0.3643$.

How many days with natural disasters per year does the model predict for 1980? How many days with natural disasters per year does the model predict for 2015?

Answer: In 1980 the modelled number of claims is 226 while in 2015 the modelled number of claims is 301. □

4c

On average the claim size per claim day has been more than 1.7 MNOK, measured with the value of the NOK today. The distribution of the claims is severely skewed to the right. This means that most claims are small and that a few are really large. When the claim size is modelled, this property is important to capture.

Average	Standard deviation	Skewness	99% quantile	99.5% quantile
1 732 214	25 982 012	46.9	7 954 324	21 665 234

Table 4: *Annual claim intensities broken down on gear type and driving limit.*

Propose an algorithm that models claim size bearing in mind that you want a tail in the claim size distribution that is adequately heavy. You may write in pseudo code or use the R language.

Answer: To obtain a tail that is adequately heavy a mixture distribution is proposed, where the non-parametric distribution is used up to a threshold b. Above the threshold b the Pareto distribution is used, bearing in mind that all over-threshold distributions become Pareto when b is large enough.

A claim Z may be written

$$Z = (1 - I_b)Z_{\leq b} + I_b Z_{>b} \tag{31}$$

where

$$Z_{\leq b} = Z|Z \leq b,\ Z_{>b} = Z|Z > b\ and I_b = 0 if Z \leq b, 1 otherwise. \tag{32}$$

The threshold b may be selected inspecting the percentiles or using the sample mean excess plot on a subset of the original dataset. Using the latter technique the threshold b should then be selected where the sample mean excess peaks in the plot. Assume in the following that b is selected as the 99th percentile in the original claim size distribution. The procedure to sample a random claim size is then given:

(Continued on page 13.)

1. Sample a random number U between 0 and 1
2. If U is less than 0.99 than sample a claim size at random amongst the 99% smallest observed claims.
3. If U is greater than 0.99 the claim size is calculated as the sum of the observed 99% percentile and an addition sampled from the Pareto distribution with the estimated parameters α and β.

An R code for this procedure is:

```
z=all natural catastrophes;
p=0.01;
alpha=1.478;
beta=57 895 000;
m=100000;
n1=(1-p)*length(z);
z=sort(z);
U=runif(m);
ind = floor(1+U*n1);
Y=z[n1]+(U**(-1/alpha)-1)*beta;
L=runif(m)<1-p;
Z=L*z[ind]+(1-L)*Y
```

□

4d

The agency managing the natural disasters of Norway is considering a reinsurance program to cover really large natural disasters. The contract of interest is the $a \times b$ contract for single events.

Propose an algorithm that models portfolio liability for natural disasters using the claim frequency model of part a) and b) and the claim size distribution you developed in part c). When this is done, modify the algorithm so that the $a \times b$ contract on single events is used on single events.

Answer: When a model for the claim frequency based on the history 1980-2014 was used, the negative binomial with a trend term was best. However, going one year forward, the Poisson distribution may be used in the simulations, calibrating λ from the expected number of catastrophe days in 2015 obtained from the negative binomial distribution. in a) and b)

Without reinsurance.

```
TotalClaims_upto99 <- TotalClaims[TotalClaims<=percentile_99_tot]

Number_of_simulations <- 100000
```

(Continued on page 14.)

```
simulations_total <- c()
TotalClaims_sorted <- sort(TotalClaims)
number_of_event_days<-301

for (i in 1:Number_of_simulations)
  {
  number_of_cat_events <- rpois(1,number_of_event_days); #Model for how many sma

  uniform_vector <- runif(number_of_cat_events)  ;
  ind=floor(1+uniform_vector*8987) ; # indices for non parametric sampling for t
  Y=percentile_99_tot+(uniform_vector**(-1/alpha_ex)-1)*beta_ex; # extreme catas
  L=runif(number_of_cat_events)<p;
  Z=L*TotalClaims_sorted[ind]+(1-L)*Y;

  simulations_total[i] <- sum(Z)
}
```

With reinsurance.

```
a<-600 000 000
b<- 1 200 000 000
TotalClaims_upto99 <- TotalClaims[TotalClaims<=percentile_99_tot]
number_of_event_days<-301

Number_of_simulations <- 100000
simulations_total <- c()
TotalClaims_sorted <- sort(TotalClaims)

for (i in 1:Number_of_simulations)
  {
  number_of_cat_events <- rpois(1,number_of_event_days); #Model for how many sma

  uniform_vector <- runif(number_of_cat_events)  ;
  ind=floor(1+uniform_vector*8987) ; # indices for non parametric sampling for t
  Y=percentile_99_tot+(uniform_vector**(-1/alpha_ex)-1)*beta_ex; # extreme catas
  L=runif(number_of_cat_events)<p;
  Z=L*TotalClaims_sorted[ind]+(1-L)*Y;
  Z_ce=pmax(pmin(Z,a),Z-b);


  simulations_total[i] <- sum(Z_ce)
}
```

4e

The preferred contract is an $a \times b$ where $a = 600MNOK$ and $b = 1200MNOK$. The reinsurance yields a reduction of required reserve, as

(Continued on page 15.)

displayed in Table 5. The same table also displays that some claims are saved. Before reinsurance was considered the natural disaster pool obtained a return on capital of 10%. A return on capital is here defined as operating profit divided by required capital (or required reserve).

	Average portfolio liability in MNOK	Required reserve in MNOK
Without reinsurance	600	3 123
With reinsurance	550	2 139

Table 5: *Natural catastrophe liabilities with and without reinsurance.*

How much should the pool be willing to pay for the reinsurance to maintain a similar return on capital when the reinsurance is taken into account? (*Hint: The claims saved by the reinsurance is the minimum to pay for the reinsurance. In addition it is expected that the reinsurance company charges a loading on top of that. How much can this loading be so that the return on capital is acceptable?*).

Answer: Before reinsurance the operating profit of the pool is approximately 312 MNOK (10% of required capital which is 3123 MNOK). When reinsurance is taken into account an operating profit of 214 MNOK would generate the same return on capital since the required capital has decreased to 2139 MNOK. Since the average portfolio liability is decreased with 50 MNOK using reinsurance the pool can spend up to approximately 150 MNOK on reinsurance and still maintain a similar return on capital. □