11.5.16

STK4600

Solutions to Ex # 9

1a)

$$\hat{e}_x = 0.5 + \frac{1}{I_x} \sum_{t=x+1}^{105} I_t$$

where I_x = number of survivals at age x

$$\Longrightarrow I_x \cdot \hat{e}_x = \frac{I_x}{2} + \sum_{t=x+1}^{105} I_t$$

$$\Longrightarrow I_0 \hat{e}_0 - I_x \cdot \hat{e}_x = \frac{I_0 - I_x}{2} + \sum_{t=1}^{x} I_t$$

and:

$$\hat{e}_x = \frac{I_0}{I_x} \cdot \hat{e}_0 - \frac{1}{I_x}\left(\sum_{t=1}^{x} I_t\right) - \frac{I_0 - I_x}{2 \cdot I_x}$$

$$\hat{e}_{10} = \frac{100000}{99683} \cdot 82.95 - \frac{1}{99683} \times 997273 - \frac{317}{199366}$$

$$= 83.214 - 10.004 - .002 = \underline{73.208}$$

$$\hat{e}_{20} = \underbrace{\frac{100000}{99525} \times 82.95}_{83.3459} - 20.0299 - .0024 = \underline{63.3136}$$

b) $$q(x) = \frac{m(x)}{1 + \frac{1}{2} m(x)} \iff m(x) = q(x) + \frac{1}{2} m(x) q(x)$$

$$\iff m(x)\left[1 - \frac{1}{2} q(x)\right] = q(x)$$

$$\iff m(x) = \frac{q(x)}{1 - \frac{1}{2} q(x)}$$

$$m(20) = \frac{.68 \times 10^{-3}}{1 - .34 \times 10^{-3}} = 68.02 \times 10^{-3}$$

$$m(82) = \frac{.06491}{.967545} = .06709 = \underline{67.09 \text{ per } 1000}$$

$$m(85) = \frac{.08859}{.955705} = .09270 = \underline{92.70 \text{ per } 1000}$$

$$m(88) = \frac{.13171}{.934145} = .14100 = \underline{141.00 \text{ per } 1000}$$

c) For a 20-year old woman:

$$\text{"Chance is"} = \frac{63897}{99525} = .64 = \underline{64\%}$$

For a 20-year old man:

$$\frac{47685}{99246} = .48 = \underline{48\%}$$

2a) $c = 0 \Rightarrow k(t) = k(t-1) + \sigma e(t) \Rightarrow k(t) \approx k$, ind. of t

$\Rightarrow \log m(x,t) = a(x) + b(x) \cdot k$

Since $\sum k(t) = 0 \Rightarrow k = 0$ & $\log m(x,t) \equiv a(x)$

b).

$$m(x,t) = \underbrace{e^{a(x)}}_{\alpha(x)} \cdot \underbrace{e^{b(x)}}_{\beta(x)}{}^{\cdot k(t)} \underbrace{e^{\varepsilon(x,t)}}_{\varepsilon^*}$$

$$= \alpha(x) \cdot [\beta(x)]^{k(t)} \cdot \varepsilon^*(x,t)$$

c)

$$\log m(x,t) = \log m(x, T_1) + b(x)[k(t) - k(T_1)]$$

$$\Rightarrow \widehat{Var}[\log m(x,t)] = b^2(x) \widehat{Var}[k(t)]$$

$$= b^2(x) \cdot Var\left\{ k(T_1) + \hat{c}(t - T_1) - SE(\hat{c}) Z (t - T_1) + \hat{\sigma} \sum_{s=T_1+1}^{t} e(s) \right\}$$

$$= b^2(x) \, Var\left\{ (t - T_1) SE(\hat{c}) \, Z + \hat{\sigma} \sum_{s=T_1+1}^{t} e(s) \right\}$$

$$= b^2(x) \left[(t - T_1)^2 \frac{\hat{\sigma}^2}{T_1 - T_0} Var(Z) + \hat{\sigma}^2 \sum_{s=T_1+}^{t} Var(e(s)) \right]$$

$$= b^2(x) \left[(t - T_1)^2 \frac{\hat{\sigma}^2}{T_1 - T_0} + \hat{\sigma}^2 (t - T_1) \right]$$

$$= b^2(x) \hat{\sigma}^2 \left[(t - T_1) \frac{t - T_1 + (T_1 - T_0)}{T_1 - T_0} \right]$$

$$= b^2(x) \hat{\sigma}^2 \left[\frac{(t - T_1)(t - T_0)}{(T_1 - T_0)} \right]$$