Solutions theoretical exercises for STK4900/9900.

Exercise 1 Group 1 (Steroid) $N(\mu_1, \sigma^2), n_1 = 8$ observations.

Group 2 (Control) $N(\mu_2, \sigma^2), n_2 = 10$ observations.

90% confidence interval for $\mu_1 - \mu_2$ is given by $\bar{x}_1 - \bar{x}_2 \pm c \cdot se(\bar{x}_1 - \bar{x}_2)$ where $c = 1.746 = 95$ percentile of Student t-distribution with $n_1 - 1 + n_2 - 1 = 16$ degrees of freedom (df) and $se(\bar{x}_1 - \bar{x}_2) = s_p\sqrt{1/n_1 + 1/n_2} \approx 1.21$ since the pooled standard deviation is calculated as

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{7 \cdot 2.6^2 + 9 \cdot 2.5^2}{16}} \approx 2.54.$$

This gives 90% confidence interval $(-9.8, -5.6)$ (with one decimals accuracy).

Exercise 2 Significant difference in blood pressure between Calcium and control groups, same model as in Exercise 1. Here $n_1 = 10$ are the number of observations in calcium group and $n_2 = 11$ the number of individuals in the control group. Observed values are reduction in blood pressure.

Want to test whether calcium intake *reduce* blood pressure, thus a one sided test problem, i.e.

$$\text{H}_0 : \mu_1 = \mu_2 \qquad \text{vs.} \qquad \text{H}_1 : \mu_1 > \mu_2$$

The test statistic is given as

$$t = \frac{\bar{x}_1 - \bar{x}_2}{se(\bar{x}_1 - \bar{x}_2)} = \frac{5 - (-0.27)}{7.38\sqrt{1/10 + 1/11}} = 1.64.$$

If the null is true this value is drawn from a t-distribution with 19 degrees of freedom. From the t-table we then get a p-value between 0.05 and 0.10, i.e. not significant (exact p-value becomes 0.059).

Exercise 3

a) We have two measurements on the same fabric, one with abraded and one with unabraded. It could then well be a dependence between the two measurements. In particular we here get a correlation between the measurements of 0.90. Hence we do not have two independent samples and an independent two-sample t-test does not apply.

   It would still be reasonable to check differences between the measurements by computing differences, finding the one-sample confidence interval of the (theoretical) mean of the difference and testing whether they are different from zero.

b) With D_i = difference between strengths with abraded and unabraded we assume $D_i \sim \text{N}(\mu, \sigma^2)$ and independent. We get the average $\hat{\mu} = \bar{D} = 3.29$ and the empirical standard deviation of the D_i equal to $s = 4.18$. A 95% confidence interval for the difference in strength μ is given as $\bar{D} \pm c \cdot se(\bar{D}) = \bar{D} \pm c \cdot s/\sqrt{n}$ where $n = 7$ observations and $c = 2.45$ is the 97.5 percentile of the t-distribution with 6 degrees of freedom.

The 95% confidence interval then equals (-0.58,7.16), containing the value zero, hence a significant difference has not been demonstrated.