

Solutions theoretical exercise for STK4900/9900.

Exercise 4

- a) A fair guess could be a correlation of about 0.9.
- b) $\overline{MPH} = 27.37$ and $SD_{MPH} = 10.59$, $\overline{MPG} = 17.73$ and $SD_{MPG} = 3.06$ and the empirical covariance between MPH and MPG is $\widehat{\text{cov}}(MPG, MPH) = 29.92$.

The Pearson correlation equals $r = \frac{\widehat{\text{cov}}(MPG, MPH)}{SD_{MPH}SD_{MPG}} = \frac{29.92}{10.59 \cdot 3.06} = 0.923$.

- c) Since $1 \text{ KPL} = (1/0.425) 1 \text{ MPG}$ and $1 \text{ KPH} = (1/1.61) 1 \text{ MPH}$ we get that the standard deviations of KPL and KPH becomes $SD_{KPL} = SD_{MPG}/0.425 = 7.2$ and $SD_{KPH} = SD_{MPH}/1.61 = 6.58$.

We also have that the empirical covariance between KPL and KPH becomes $\widehat{\text{cov}}(KPL, KPH) = \widehat{\text{cov}}(MPG, MPH)/(0.425 \cdot 1.61) = 43.72$

It then turns out that the correlation between KPL and KPH is equal to $\frac{\widehat{\text{cov}}(KPL, KPH)}{SD_{KPL}SD_{KPH}} = \frac{\widehat{\text{cov}}(MPG, MPH)}{SD_{MPH}SD_{MPG}} = r = 0.923$.

The correlation coefficient is in general not affected by linear transformations of the variables.