UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in MAT-BIO2100 — Mathematical Biology

Day of examination: Wednesday, June 1, 2011.

Examination hours: 09:00 – 13:00.

This problem set consists of 3 pages.

Appendices: None.

Permitted aids: All handwritten and printed aids in addition to approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 SIRS diseases.

Consider a population consisting of susceptibles (S), infected (I) and recovered and immune (R) individuals. Assume that the recovered gradually loose their immunity.

1a

Explain the following model

$$\begin{aligned} S' &= \delta R - \beta IS \\ I' &= \beta IS - \gamma I \\ R' &= \gamma I - \delta R, \end{aligned} \tag{1}$$

where β, δ and γ are positive constants.

1b

Show that the total population $N = S + I + R$ is constant.

1c

Show that the disease can spread in a healthy population if $R_0 = \beta N/\gamma > 1$.

1d

Find all fixpoints for (1), and determine their stability.

1e

Assume that a vaccine which gave permanent immunity to the disease was available. What percentage of the total population must be vaccinated to eradicate the disease?

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Problem 2

In humans, sex is determined genetically. Females have two X chromosomes, while males have only one. Males inherit their X chromosome from their mother, and their Y chromosome from their father. A gene appearing only on the X chromosome is said to be X *linked.*
An X linked recessive gene produces a red-green color blindness in humans. A woman with normal vision whose father was color blind has children with a color blind man.

2a

What is the genotype of the woman? What is the probability that the first child from this mating will be a color blind boy?

2b

Assume now that the females of genotype xx are sterile, but that the presence of the recessive x has no effect on fitness in the xX or the xY genotypes. Assume random mating and show that the difference equations for the frequencies of x in males (p) and in females (r) of reproductive age are

$$p' = \frac{1}{2}r, \quad r' = \frac{\frac{1}{2}r + p(1-r)}{1 - \frac{1}{2}pr}. \tag{2}$$

What do you think will happen with p and r as the generations pass?

Problem 3

In a lake, in the absence of predatory birds, the fish population evolves according to the logistic equation with a fixed carrying capacity K_1. Let $x(t)$ denote the number of fish, and $y(t)$ the number of predatory birds near the lake. Assume that the number of fish caught by the birds per time unit is proportional to xy, and that y evolves according to the logistic equation with a carrying capacity proportional to $1 + x$.

3a

Write a system of differential equations describing the evolution of x and y. State clearly what your parameters mean.

3b

Set $u = x/x^*$, $v = y/y^*$ and $\tau = t/t^*$ and non-dimensionalize your equations to obtain

$$\begin{aligned} \frac{du}{d\tau} &= \alpha u(1-u) - uv \\ \frac{dv}{d\tau} &= \beta v\left(1 - \frac{v}{1 + K_1 u}\right). \end{aligned} \tag{3}$$

What are the parameters x^*, y^*, t^*, α and β in terms of your parameters from **3a)**?

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3c

Find all fixpoints of (3) and determine their stability.

THE END