MAT-INF1310, Spring 2008
Compulsory assignment 1
Deadline: 7 March, 14-30

To pass it is enough to solve 5 problems out of 9.

1. The temperature $y(x)$ at position x along a rod satisfies the differential equation

$$y'' + \frac{\pi^2}{4} y = 0.$$

Find the temperature distribution in the rod if $y(0) = 1$ and $y(1) = \sqrt{3}$.

2. You have put your cat on a diet. His basic metabolism consumes 200 calories per day. His exercise program consumes 100 calories per day per kilogram of body mass. Food intake provides 750 calories per day. Caloric intake that is not consumed by basic metabolism or exercise is converted into fat; calories needed for basic metabolism or exercise in excess of caloric intake are obtained from the fat store. Body fat stores or releases calories at the rate of 5000 calories per kilogram of fat. Denote by $m(t)$ the mass, in kilograms, of your cat on day t of this diet regimen. Write a differential equation for $m(t)$. If your cat sticks to this diet, what is his ultimate mass?

3.(i) An equation of the form

$$y' = A(x)y^2 + B(x)y + C(x)$$

is called the Ricatti equation. Assume y_1 is a particular solution. Show that the substitution $y = y_1 + \frac{1}{v}$ transforms the Ricatti equation into the linear equation

$$\frac{dv}{dx} + (B + 2Ay_1)v = -A.$$

(ii) Find all solutions of the equation

$$xy' + xy^2 - y - x^3 = 0 \quad (\text{for } x > 0).$$

4.(i) Assume f and g are strictly positive differentiable functions on an interval such that $W(f, g) \equiv 0$. Show that f and g are linearly dependent.

(ii) Give examples of differentiable functions f and g on the real line such that $W(f, g) \equiv 0$, but f and g are linearly independent.

5.(i) Consider the initial value problem

$$\begin{cases} y' = f(x, y) \\ y(0) = y_0 \end{cases}$$

on $[0, a)$. Assuming that the function f is continuous and nonincreasing in y, so that $f(x, y_1) \geq f(x, y_2)$ if $y_1 < y_2$, show that there exists at most one solution.

(ii) Exhibit two different solutions of the initial value problem

$$\begin{cases} y' = -|1 - y^2|^{1/2} \\ y(0) = 1 \end{cases}$$

on $[0, +\infty)$.

(iii) Find all solutions in (ii).