MAT-INF1310, Spring 2009
Mandatory Assignment 1
Deadline: March 6, 14:30.

In order to pass, it is enough to solve 4 of the 7 problems. Remember that all claims have to be argued for. A correct answer is not considered without sufficient arguments.

1. Find a solution of the differential equation

$$\frac{dy}{dx} = 2xy + 3x^3 e^{x^2}$$

which satisfies $y(\ln 2) = 0$.

2. Suppose that the fish population $P(t)$ in a lake is attacked by a disease at time $t = 0$, with the result that the fish cease to reproduce (so that the birth rate is $\beta = 0$) and the death rate δ (deaths per week per fish) is thereafter proportional to $1/\sqrt{P}$. If there were initially 900 fish in the lake and 441 were left after 6 weeks, how long did it take all the fish in the lake to die?

3. Find *all* solutions of the differential equation $y'' - 4y' + 9y = xe^x$.

4. Let y_1 and y_2 denote two linearly independent solutions of the differential equation

$$y'' + p(x)y' + q(x)y = 0,$$

where p and q are continuous functions on an open (non-empty) interval I containing the point a. Suppose that Y is a third solution of the equation. This means that there are unique numbers $c_1, c_2 \in \mathbb{R}$ for which

$$Y(x) = c_1 y_1(x) + c_2 y_2(x), \qquad x \in I.$$

Express the values of the constants c_1 and c_2 in terms of the values of the functions y_1, y_2, and Y (and their derivatives) at the point a.

5. Verify that if c is a constant, then the function defined by

$$y(x) = \begin{cases} 1 & \text{if } x \leq c, \\ \cos(x - c) & \text{if } c < x < c + \pi, \\ -1 & \text{if } x \geq c + \pi \end{cases}$$

satisfies the differential equation $y' = -\sqrt{1 - y^2}$ for all x. Then determine (in terms of a and b) how many different solutions the initial value problem $y' = -\sqrt{1 - y^2}$, $y(a) = b$ has.

6. Suppose that the mass in a mass–spring–dashpot system with $m = 10$, $c = 9$ and $k = 2$ is set in motion with $x(0) = 0$ and $x'(0) = 5$ (see Figure 2.4.1 on page 135 in Edwards & Penney). Find how far the mass moves to the right before starting back toward the origin.

7. Let p, q and r be continuous functions on some open (non-empty) interval I. Prove that the equation

$$y^{(3)} + p(x)y'' + q(x)y' + r(x)y = 0$$

has three solutions on the interval I which are linearly independent.