Problem 1.

Let B be an $n \times n$ matrix and set $A = kI + B$, $k \in \mathbb{R}$. Show that $e^{tA} = e^{kt}e^{tB}$.

Problem 2.

Find and classify all critical points of the system

$$\begin{cases} x' = y - x \\ y' = (x - 1)(y - 2). \end{cases}$$

Problem 3.

Find a function $y(x)$ defined on $[0, +\infty)$ which solves the initial value problem

$$y' = y/x^2 + e^{-1/x}, \qquad y(1) = 0.$$

Problem 4.

a) Find a general solution $y(x)$ of the differential equation

$$y^{(3)} - 2y'' - y' + 2y = 0.$$

b) Find a general solution $y(x)$ of the differential equation

$$y^{(3)} - 2y'' - y' + 2y = \cos x.$$

Problem 5.

Consider the matrix

$$A = \begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix}.$$

a) Find a fundamental matrix of the system $x' = Ax$.

b) Calculate e^{tA}.

c) Solve the initial value problem

$$x' = Ax + \begin{pmatrix} e^t \\ 0 \end{pmatrix}, \qquad x(0) = 0.$$