Exercise 5.8

Let $\boldsymbol{\phi}_1 = \{\phi_{1,0}, \phi_{1,1}, \phi_{1,2}\}$ and $\mathcal{C}_1 = \{\phi_{0,0}, \psi_{0,0}, \phi_{0,1}\}$ be a bases for V_1. We will consider the change of basis from $\boldsymbol{\phi}_1$ to $\mathcal{C}_1$. We have the following relations (recall that $\phi_{1,k} = \sqrt{2}\phi(2x-k)$)

$$\begin{aligned}
\phi_{1,0} &= \frac{\sqrt{2}\phi_{0,0} + \sqrt{2}\psi_{0,0}}{2} = \frac{\phi_{0,0} + \psi_{0,0}}{\sqrt{2}} \\
\phi_{1,1} &= \frac{\sqrt{2}\phi_{0,0} - \sqrt{2}\psi_{0,0}}{2} = \frac{\phi_{0,0} - \psi_{0,0}}{\sqrt{2}} \\
\phi_{1,2} &= \sqrt{2}\phi_{0,1} - \phi_{1,0} = \sqrt{2}\phi_{0,1} - \frac{\phi_{0,0} + \psi_{0,0}}{\sqrt{2}} = \frac{1}{\sqrt{2}}(-\phi_{0,0} - \psi_{0,0} + 2\phi_{0,1})
\end{aligned}$$

The last equality follows, since we have assumed that all basis functions have period 3/2. This means that

$$\phi_{0,1}(t) = \begin{cases} 1 & \text{if } t \in [0, 1/2) \cup [1, 3/2] \\ 0 & \text{otherwise} \end{cases}$$

Now we can write the change of basis matrix from $\boldsymbol{\phi}_1$ to $\mathcal{C}_1$ (i.e. the DWT matrix) as

$$\underset{\mathcal{C}_1 \leftarrow \boldsymbol{\phi}_1}{P} = \begin{bmatrix} [\phi_{1,0}]_{\mathcal{C}_1} & [\phi_{1,1}]_{\mathcal{C}_1} & [\phi_{1,2}]_{\mathcal{C}_1} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$