Seksjon 1.2
eksempel 1.12

$$f_s(t) = \begin{cases} 1 & 0 \le t < \frac{T}{2} \\ -1 & \frac{T}{2} \le t < T \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T f_s(t)\,dt = \frac{1}{T}\left(\int_0^{\frac{T}{2}} 1\,dt - \int_{\frac{T}{2}}^{T} 1\,dt\right) = \frac{1}{T}\left(\frac{T}{2}-\frac{T}{2}\right) = \underline{0}$$

$$a_n = \frac{2}{T}\int_0^T f_s(t)\cos(2\pi nt/T)\,dt$$

$$= \frac{2}{T}\int_0^{\frac{T}{2}}\cos(2\pi nt/T)\,dt - \frac{2}{T}\int_{\frac{T}{2}}^{T}\cos(2\pi nt/T)\,dt$$

$$= \frac{2}{T}\left(\frac{T}{2\pi n}\Big[\sin(\overbrace{2\pi nt/T}^{\pi n})\Big]_0^{\frac{T}{2}} - \frac{T}{2\pi n}\Big[\sin(\overbrace{2\pi nt/T}^{2\pi n})\Big]_{\frac{T}{2}}^{T}\right)$$

$$= \underline{0}$$

$$b_n = \frac{2}{T}\left(\int_0^{\frac{T}{2}}\sin(2\pi nt/T)\,dt - \int_{\frac{T}{2}}^{T}\sin(2\pi nt/T)\,dt\right)$$

$$= \frac{2}{T}\frac{T}{2\pi n}\left(\Big[-\cos \overbrace{2\pi nt/T}^{\pi n}\Big]_0^{\frac{T}{2}} + \Big[\cos(\overbrace{2\pi nt/T}^{2\pi n})\Big]_{\frac{T}{2}}^{T}\right)$$

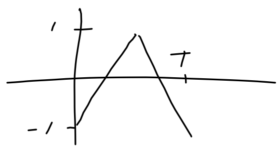
$$= \frac{1}{\pi n}\left(-\cos\pi n + 1 + 1 - \cos\pi n\right) = \frac{2(1-\cos(\pi n))}{\pi n} = \begin{cases} 0 & n \text{ partall} \\ \frac{4}{\pi n} & n \text{ oddetall} \end{cases}$$

$$f_N(t) = \sum_{n=1}^{N} b_n \sin 2\pi nt/T = \underline{\sum_{\substack{n \text{ oddetall} \\ 1 \le n \le N}} \frac{4}{\pi n}\sin(2\pi nt/T)}$$

f_N er en sinus-rekke. ren tone

første leddet: $\frac{4}{\pi}\sin(2\pi t/T)$

Eksempel 1.13 For trekantpulsen:



$b_n = 0$

$$a_n = \begin{cases} 0 & n \text{ partall} \\ -\frac{8}{n^2\pi^2} & n \text{ oddetall} \end{cases}$$

Bevis: Anta at f er symmetrisk.

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(2\pi n t/T)\,dt$$

$$= \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin(2\pi n t/T)\,dt$$

$du = -dt$
$u = -t$

$$= \frac{2}{T}\left(\int_{-\frac{T}{2}}^{0} f(t)\sin(2\pi n t/T)\,dt + \int_{0}^{\frac{T}{2}} f(t)\sin(2\pi n t/T)\,dt\right)$$

$$= \frac{2}{T}\left(\int_{-\frac{T}{2}}^{0} f(t)\sin(2\pi n t/T) - \int_{0}^{-\frac{T}{2}} f(-u)\sin(-2\pi n u/T)\,du\right)$$

$$= \frac{2}{T}\left(\int_{-\frac{T}{2}}^{0} f(t)\sin(2\pi n t/T) + \int_{0}^{-\frac{T}{2}} f(u)\sin(2\pi n u/T)\,du\right)$$

$$= \frac{2}{T}\left(\int_{-\frac{T}{2}}^{0} f(t)\sin(2\pi n t/T) - \int_{-\frac{T}{2}}^{0} f(t)\sin(2\pi n t/T)\,dt\right) = \underline{0}$$

Vi har at

$$\cos(2\pi nt/T) = \frac{1}{2}\left(e^{2\pi i nt/T} + e^{-2\pi i nt/T}\right)$$

$$\sin(2\pi nt/T) = \frac{1}{2i}\left(e^{2\pi i nt/T} - e^{-2\pi i nt/T}\right)$$

$$\left(e^{ix} = \cos x + i \sin x \right)$$

$$e^{-ix} = \cos x - i \sin x$$

Komplekst indreprodukt: Samme aksiomer som for et reelt indreprodukt (feks $\langle cf, g\rangle = \underline{c\langle f,g\rangle}$), men med et par forskjeller, feks at $\langle f,g\rangle = \overline{\langle g,f\rangle}$

Konsekvens av dette: $\underline{\langle f, cg\rangle} = \overline{\langle cg, f\rangle} = \overline{c\langle g,f\rangle} = \bar{c}\,\overline{\langle g,f\rangle} = \underline{\bar{c}\langle f,g\rangle}$

Siden $F_{N,T}$ er en ortonormal basis:

$$\|f_N\|^2 = \Big\langle \underbrace{\sum_{n=-N}^{N} y_n e^{2\pi i n t/T}}_{f_N}, \sum_{n=-N}^{N} y_n e^{2\pi i n t/T} \Big\rangle$$

$$= \sum_{n=-N}^{N} \sum_{m=-N}^{N} y_n \overline{y_m} \langle e^{2\pi i n t/T}, e^{2\pi i m t/T} \rangle$$

$$= \sum_{n=-N}^{N} |y_n|^2 \cdot 1 = \sum_{n=-N}^{N} |y_n|^2 \quad \text{(Parsevals teorem)}$$

Eksempel 1.24

$$f(t) = \underbrace{e^{2\pi i t/T_2}}_{\text{ren tone}}$$ def. på $[0, T]$, $T < T_2$

$$y_n = \frac{1}{T}\int_0^T f(t) e^{-2\pi i n t/T} dt = \frac{1}{T}\int_0^T e^{2\pi i t/T_2} e^{-2\pi i n t/T} dt$$

$$= \frac{1}{T}\int_0^T e^{2\pi i t\left(1/T_2 - n/T\right)} dt = \frac{1}{T} \frac{1}{2\pi i\left(\frac{1}{T_2} - \frac{n}{T}\right)} \left[e^{2\pi i t\left(\frac{1}{T_2} - \frac{n}{T}\right)}\right]_0^T$$

$$= \frac{1}{2\pi i\left(\frac{T}{T_2} - n\right)} \left[e^{2\pi i T\left(\frac{1}{T_2} - \frac{n}{T}\right)} - 1\right] \rightarrow e^{2\pi i \frac{T}{T_2} - 2\pi i n}$$

$$= \frac{e^{2\pi i \frac{T}{T_2}} - 1}{2\pi i\left(\frac{T}{T_2} - n\right)} \qquad " \; e^{2\pi i \frac{T}{T_2}}$$

$\searrow$ La $\frac{T}{T_2} \rightarrow 1$ $\quad \rightarrow 0 \quad n \neq 1$

$T/T_2 = 0.9$

$T/T_2 = 0.5$

$$x \to 1 \quad \frac{e^{2\pi i x} - 1}{2\pi i (x - n)}$$

$$y_1 \quad x \to 1 \quad \frac{e^{2\pi i x} - 1}{2\pi i (x-1)} \rightarrow \frac{\cancel{2\pi i} e^{2\pi i x}}{\cancel{2\pi i}} \rightarrow 1$$