Teorem 1.16

y_n for $\chi_{-a,a}$:

$$y_n = \frac{1}{T}\int^{T} \chi_{-a,a}\, e^{-2\pi i n t/T}\, dt$$

$$= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \chi_{-a,a}\, e^{-2\pi i n t/T}\, dt$$

$$= \frac{1}{T}\int_{-a}^{a} e^{-2\pi i n t/T}\, dt = \frac{1}{T}\Big[-\frac{T}{2\pi i n} e^{-2\pi i n t/T}\Big]_{-a}^{a}$$

$$= -\frac{1}{2\pi i n}\left(e^{-2\pi i n a/T} - e^{2\pi i n a/T}\right) \qquad \left(\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)\right)$$

$$= \frac{1}{\pi n \cdot 2i}\left(e^{2\pi i n a/T} - e^{-2\pi i n a/T}\right)$$

$$= \frac{1}{\pi n}\sin(2\pi n a/T)$$

Teorem 1.17

Anta $f \to x_n$, $g \to y_n$

Fourierkoeff. for $af+bg$:

$$\frac{1}{T}\int_0^T (af(t)+bg(t))e^{-2\pi i n t/T}dt = a\left(\frac{1}{T}\int_0^T f(t)e^{-2\pi i n t/T}dt\right)+b\left(\frac{1}{T}\int_0^T g(t)e^{-2\pi i n t/T}dt\right)$$

$$= a x_n + b y_n$$

(1)

$y_n = \overline{y_{-n}}$ når f reell:

$$y_n = \frac{1}{T}\int_0^T f(t)e^{-2\pi i n t/T}dt = \frac{1}{T}\int_0^T \overline{f(t)e^{2\pi i n t/T}}dt$$

$$= \overline{\frac{1}{T}\int_0^T f(t)e^{-2\pi i(-n)t/T}dt} = \overline{y_{-n}}$$

(4)

$$g(t) = f(t-d)$$

$$\frac{1}{T}\int_0^T g(t)e^{-2\pi i n t/T}dt = \frac{1}{T}\int_0^T f(t-d)e^{-2\pi i n t/T}dt$$

$u = t-d \Rightarrow t = u+d$

$$= \frac{1}{T}\int_{-d}^{T-d} f(u)e^{-2\pi i n(u+d)/T}du$$

$u \to t$

$$= \frac{1}{T}\int_0^T f(t)e^{-2\pi i n t/T - 2\pi i n d/T}dt = e^{-2\pi i n d/T}\underbrace{\int_0^T f(t)e^{-2\pi i n t/T}dt}_{y_n}$$

$$= e^{-2\pi i n d/T} y_n$$

Derivere en Fourierrekke:

Lemma 1.16: $(f_N)' = (f')_N$ (hvis f er deriverbar)

<u>Bevis</u>: $y_n = \langle f, e^{2\pi i n t/T}\rangle = \frac{1}{T}\int_0^T f(t) e^{-2\pi i n t/T}\, dt$

$$= \frac{1}{T}\left(\underbrace{\left[f(t)\left(-\frac{T}{2\pi i n} e^{-2\pi i n t/T}\right)\right]_0^T}_{0} - \left(-\frac{T}{2\pi i n}\right)\int_0^T f'(t) e^{-2\pi i n t/T}\, dt\right)$$

$$= \frac{1}{T}\cdot\frac{T}{2\pi i n}\int f'(t) e^{-2\pi i n t/T}\, dt$$

$$= \frac{T}{2\pi i n}\cdot \text{(Fourierkoeffisient n for } f')$$

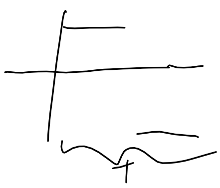
$$= \frac{T}{2\pi i n}\langle f', e^{2\pi i n t/T}\rangle \quad \left(\Rightarrow \frac{2\pi i n}{T}\langle f, e^{2\pi i n t/T}\rangle = \langle f', e^{2\pi i n t/T}\rangle\right)$$

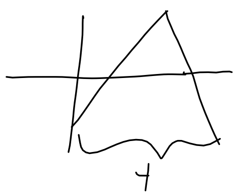
$$(f_N(t))' = \left(\sum_{n=-N}^{N}\langle f, e^{2\pi i n t/T}\rangle e^{2\pi i n t/T}\right)' = \sum_{n=-N}^{N}\langle f, e^{2\pi i n t/T}\rangle \frac{2\pi i n}{T} e^{2\pi i n t/T}$$

$$= \sum_{n=-N}^{N}\langle f', e^{2\pi i n t/T}\rangle e^{2\pi i n t/T}$$

$$= (f'(t))_N$$

Dette kan brukes til å forenkle utregningen av trekantpulsens Fourierrekke, ved å basere seg på firkantpulsens Fourierrekke:





$$f_s = \frac{T}{4}(f_t)'$$

$$(f_s)_N = \frac{T}{4}((f_t)')_N = \frac{T}{4}((f_t)_N)'$$

$$\frac{4}{T}(f_s)_N = ((f_t)_N)'$$

$$\frac{4}{T}\int_0^T (f_s(t))_N\, dt + C = (f_t)_N$$

$$S(\underbrace{e^{2\pi i \nu t}}_{f}) = \int_{-\infty}^{\infty} g(s)\, \underbrace{e^{2\pi i \nu (t-s)}}_{f(t-s)}\, ds$$

$$= \int_{-\infty}^{\infty} g(s)\, \underbrace{e^{2\pi i \nu t}\, e^{-2\pi i \nu s}}\, ds$$

$$= e^{2\pi i \nu t} \underbrace{\int_{-\infty}^{\infty} g(s) e^{-2\pi i \nu s}\, ds}_{\lambda_S(\nu)} = \underline{\lambda_S(\nu) e^{2\pi i \nu t}}$$