$f \in V_{M,T}$ $\quad f(t) = f_M(t) = \sum_{n=-M}^{M} z_n e^{2\pi i n t/T}$ $\quad \nu = \frac{n}{T}$

$y_n \to z_n$

$N \to M$

$N > 2M \to \underbrace{\frac{N}{T}}_{f_s} > 2\underbrace{\frac{M}{T}}_{\nu}$

Bevis for teoremet 2.9

$$x_k = f(kT/N) = \sum_{n=-M}^{M} z_n e^{2\pi i n t/T} \quad t \to kT/N$$

$$= \sum_{n=-M}^{M} z_n e^{2\pi i nk/N} \qquad +N \qquad 2\pi i Nk/N = 2\pi i k$$

$$= \sum_{n=-M}^{-1} z_n e^{2\pi i nk/N} + \sum_{n=0}^{M} z_n e^{2\pi i nk/N}$$

$n = u-N$

$u = N+n$

$$= \sum_{u=N-M}^{N-1} z_{u-N}\, e^{2\pi i uk/N} + \sum_{n=0}^{M} z_n e^{2\pi i nk/N}$$

$$= \sum_{n=0}^{M} z_n e^{2\pi i nk/N} + \sum_{n=M+1}^{N-M-1} 0 \cdot e^{2\pi i nk/N} + \sum_{n=N-M}^{N-1} z_{n-N}\, e^{2\pi i nk/N}$$

$$\rightarrow N\,\mathrm{IDFT}(\overbrace{z_0, z_1, \dots, z_M}, \overbrace{\underbrace{0, \dots, 0}_{N-(2M+1)}}, z_{-M}, z_{-M+1}, \dots, z_{-1})$$

$$= \vec{x}$$

$$\Rightarrow (z_0, \dots, z_M, 0, \dots, 0, z_{-M}, \dots, z_{-1}) = \underline{\underline{\frac{1}{N}\mathrm{DFT}\,\vec{x}}}$$

Eksempel 2.16 – 2.18

Kan endre en lyd ved å nulle ut noen frekvenser / DFT-koeffisienter, og bevare andre frek.

audioread → $\vec{x}$ → DFT → nuller ut y_n → IDFT → $\underbrace{\text{spille av}}_{\text{play}}$
filter
play blocking.

eks. 2.16: nuller ut alt bortsett fra de høyeste eller laveste L frekvensene.
beholdes

lower = 1 / True beholder laveste L: $z_{-L}, \dots, z_0, \dots, z_L$ → $\underbrace{\overbrace{y_0, \dots, y_L, \quad y_{N-L}, \dots, y_{N-1}}}$

lower = 0 / False beholder høyeste L frek: z_{-M}, z_M → $y_{\frac{N}{2}-L}, \dots, y_{\frac{N}{2}}, \dots, y_{\frac{N}{2}+L}$

forw_comp_rev_DFT(L , lower,

eks 2.17: parameter threshold til forw_comp_rev_DFT.
nullstiller y_n som slik at $|y_n| \leq$ threshold.

eks 2.18: parameter n til forw_comp_rev_DFT
$y_m = \cdots d_2 d_1 d_0 \bullet d_{-1} d_{-2} d_{-3} \cdots \rightarrow \cdots d_{n+2} d_{n+1} d_n 00.0000 \cdots$

Bevis teorem 2.15

Bevis: $0 \leq n < \frac{N}{2}$ (antar at N er partall)

$$y_n = \sum_{k=0}^{N-1} x_k e^{-2\pi i k n/N} = \sum_{k=0}^{\frac{N}{2}-1} x_{2k} e^{-2\pi i (2k) n/N} + \sum_{k=0}^{\frac{N}{2}-1} x_{2k+1} e^{-2\pi i (2k+1) n/N}$$

$$= \sum_{k=0}^{\frac{N}{2}-1} \underbrace{x_{2k}}_{x^{(e)}} e^{-2\pi i k n/(\frac{N}{2})} + e^{-2\pi i n/N} \sum_{k=0}^{\frac{N}{2}-1} \underbrace{x_{2k+1}}_{x^{(o)}} e^{-2\pi i k n/(N/2)}$$

$$= (\mathrm{DFT}\, x^{(e)})_n + \underbrace{e^{-2\pi i n/N}}_{D_{N/2}} (\mathrm{DFT}\, x^{(o)})_n$$

$$(y_0, \dots, y_{\frac{N}{2}-1}) = \mathrm{DFT}_{\frac{N}{2}} x^{(e)} + D_{N/2}\, \mathrm{DFT}_{\frac{N}{2}} x^{(o)}$$

Tilsvarende vises

$$(y_{\frac{N}{2}}, \dots, y_{N-1}) = \mathrm{DFT}_{\frac{N}{2}} x^{(e)} - D_{\frac{N}{2}}\, \mathrm{DFT}_{\frac{N}{2}} x^{(o)}$$

DFT

$$= \underbrace{\begin{pmatrix} I_{\frac{N}{2}} & D_{\frac{N}{2}} \end{pmatrix} \begin{bmatrix} F_{\frac{N}{2}} & 0 \\ 0 & F_{\frac{N}{2}} \end{bmatrix}} \begin{bmatrix} x^{(e)} \\ x^{(o)} \end{bmatrix}$$

$$= \begin{pmatrix} I_{\frac{N}{2}} & D_{\frac{N}{2}} \end{pmatrix} \begin{bmatrix} F_{\frac{N}{2}} & 0 \\ 0 & F_{\frac{N}{2}} \end{bmatrix}$$

$$= \left[F_{\frac{N}{2}} + D_{\frac{N}{2}} 0 \quad I_{\frac{N}{2}} 0 + D_{\frac{N}{2}} F_{\frac{N}{2}} \right]$$

$$= \left[F_{\frac{N}{2}} \quad D_{\frac{N}{2}} F_{\frac{N}{2}} \right]$$

$$\left[F_{\frac{N}{2}} \quad D_{\frac{N}{2}} F_{\frac{N}{2}} \right] \begin{bmatrix} x^{(e)} \\ x^{(o)} \end{bmatrix} = F_{\frac{N}{2}} x^{(e)} + D_{\frac{N}{2}} F_{\frac{N}{2}} x^{(o)}$$

Blokkmatriser (elementene i matrisene er igjen matriser

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} E & F \\ G & H \end{pmatrix} = \begin{pmatrix} AE + BG & AF + BH \\ \cdots & \cdots \end{pmatrix}$$