$$\lambda_{s,n} = \sum_{k=0}^{N-1} s_k e^{-2\pi i k n/N} \qquad \vec{\lambda}_s = DFT_N \vec{s}$$

Bevis for teorem 3.14

$$\lambda_{s,n} = \sum_{k=0}^{N-1} s_k e^{-2\pi i k n/N} = \sum_{0\le k<\frac{N}{2}} s_k e^{-2\pi i k n/N} + \sum_{\frac{N}{2}\le k<N} s_k e^{-2\pi i k n/N}$$

$$= \sum_{0\le k<\frac{N}{2}} t_k e^{-2\pi i k n/N} + \sum_{\frac{N}{2}\le k<N} t_{k-N} e^{-2\pi i k n/N} \qquad \boxed{u=k-N}$$

$$= \sum_{0\le k<\frac{N}{2}} t_k e^{-2\pi i k n/N} + \sum_{-\frac{N}{2}\le u<0} t_u e^{-2\pi i (u+\underline{N})n/N}$$

$$= \sum_{0\le k<\frac{N}{2}} t_k e^{-2\pi i k n/N} + \sum_{-\frac{N}{2}\le k<0} t_k e^{-2\pi i k n/N}$$

$$= \underline{\sum_{-\frac{N}{2}\le k<\frac{N}{2}} t_k e^{-2\pi i k n/N}}$$

$$\lambda_s(2\pi n/N) = \sum_k t_k e^{-ik\omega} = \underline{\sum_k t_k e^{-2\pi i k n/N}} \qquad (\omega = 2\pi n/N)$$

$\lambda_s(\omega)$: Plottes på $[0, 2\pi]$, eller $[-\pi, \pi]$

$\omega = 0$: laveste frekvens $\quad (n=0 \Rightarrow \omega = 0,\ n = N-1 \Rightarrow \omega$ nær 2π

$\omega = \pm\pi$ høyeste frekvens $\quad (n = \frac{N}{2} \Rightarrow \omega = \frac{2\pi n}{N} = \pi$

Hvordan plotte $\lambda_s(\omega)$? Plott verdiene $\lambda_s(2\pi n/N)$ (feks $N = 100$)

$$\lambda_s(2\pi n/N) = \lambda_{s,n}$$

$$\vec{\lambda}_s = \boxed{DFT_N \vec{s}.}$$

$$S = \left\{\tfrac{1}{4}, \underline{\tfrac{1}{2}}, \tfrac{1}{4}\right\}$$

$$\vec{s} = \left(\tfrac{1}{2}, \tfrac{1}{4}, 0, \dots, \tfrac{1}{4}\right)$$

Teorem 3.18, siste punkt:

$$\lambda_{S_1 S_2}(2\pi n/N) = \lambda_{S_1 S_2, n} = \lambda_{S_1,n}\lambda_{S_2,n}$$
$$= \lambda_{S_1}(2\pi n/N)\,\lambda_{S_2}(2\pi n/N)$$

$\Rightarrow \lambda_{S_1 S_2}(\omega) = \lambda_{S_1}(\omega)\lambda_{S_2}(\omega)$ for alle ω på formen $\omega = \frac{2\pi n}{N}$

Eksempel 3.18

$$S = \{ \overset{t_{-1}}{\tfrac{1}{4}}, \underline{\tfrac{1}{2}}, \tfrac{1}{4} \}$$

$$\lambda_S(\omega) = \frac{1}{4}e^{i\omega} + \frac{1}{2} + \frac{1}{4}e^{-i\omega} = \frac{1}{2} + \frac{1}{2}\cos\omega$$

3.19 Anta at $\lambda_{S_1}(\omega) = \cos(2\omega)$, $\lambda_{S_2}(\omega) = 1 + 3\cos\omega$

Hva blir filterkoeffisientene til $S_1 S_2$?

$$\lambda_{S_1 S_2}(\omega) = \lambda_{S_1}(\omega)\lambda_{S_2}(\omega) = \cos(2\omega)(1+3\cos\omega)$$

$$= \frac{1}{2}\left(e^{2i\omega} + e^{-2i\omega}\right)\left(1 + \frac{3}{2}\left(e^{i\omega} + e^{-i\omega}\right)\right)$$

$$= \frac{1}{2}\left(\underline{e^{2i\omega}} + e^{-2i\omega}\right)\left(\underline{\frac{3}{2}e^{i\omega}} + \underline{\underline{1}} + \underline{\underline{\underline{\frac{3}{2}e^{-i\omega}}}}\right)$$

$$= \underset{k=-3}{\frac{3}{4}e^{3i\omega}} + \underset{k=-2}{\frac{1}{2}e^{2i\omega}} + \underset{k=-1}{\frac{3}{4}e^{i\omega}} + \underset{\substack{k=1\\ t_1}}{\frac{3}{4}e^{-i\omega}} + \underset{\substack{k=2\\ t_2}}{\frac{1}{2}e^{-2i\omega}} + \underset{\substack{k=3\\ t_3}}{\frac{3}{4}e^{-3i\omega}}$$

$$= \sum t_k e^{-ik\omega}$$

$$\Rightarrow \underline{\underline{S_1 S_2 = \left\{\frac{3}{4}, \frac{1}{2}, \frac{3}{4}, \underline{0}, \frac{3}{4}, \frac{1}{2}, \frac{3}{4}\right\}}}$$

Bevis for observasjon 3.22

La S_1 ha filterkoeffs t_k, S_2 ha filterkoeffs $(-1)^k t_k$

$$\lambda_{S_2}(\omega) = \sum_k (-1)^k t_k e^{-ik\omega} = \sum_k t_k (e^{-i\pi})^k e^{-ik\omega}$$
$$= \sum_k t_k e^{-ik\pi} e^{-ik\omega}$$
$$= \sum_k t_k e^{-ik(\omega+\pi)} = \lambda_{S_1}(\omega+\pi)$$

$\Rightarrow$ rollene til $0, \pi$ blir byttet om

$\Rightarrow$ S_2 er lavpass hvis S_1 er høypass, og omvendt.

Legge til ekko:

$$\sum_{k=d} t_k x_{n-k}$$

$$z_n = \underset{t_0=1}{x_n} + \underbrace{c x_{n-d}}_{t_d = c}$$

$$\Rightarrow S = \{\underline{1}, \dots, 0, \underset{\substack{\uparrow \\ \text{plass } d.}}{c}\}$$

$$\lambda_S(\omega) = \sum t_k e^{-ik\omega} = \underline{\underline{1 + c e^{-id\omega}}}$$

(ser at $1-c \leq |\lambda_S(\omega)| \leq 1+c$)

eks 3.31

$$z_n = \frac{1}{3}(x_{n-1} + x_n + x_{n+1}) \qquad \text{(glatter ut verdien)}$$

$$S = \{\tfrac{1}{3}, \underline{\tfrac{1}{3}}, \tfrac{1}{3}\}$$

$$\lambda_S(\omega) = \frac{1}{3}e^{i\omega} + \frac{1}{3} + \frac{1}{3}e^{-i\omega} = \frac{1}{3} + \frac{2}{3}\cos\omega$$

$$z_n = \frac{1}{2L+1}(x_{n-L} + \dots + x_0 + \dots + x_{n+L})$$

$$S = \frac{1}{2L+1}\{\underset{-L}{1}, \dots, \underline{1}, \dots, \underset{L}{1}\} \qquad \vec{s} = \frac{1}{2L+1}(\underbrace{1, \dots, 1}_{L+1}, 0, \dots, 0, \underbrace{1, \dots, 1}_{L})$$

$$\lambda_S = DFT_N \vec{s}$$

I eks. 2.2 regnet vi ut at

$$DFT_N(1, \dots, 1, 0, \dots, 0, 1, \dots, 1) = \vec{y}, \text{ der } y_n = \frac{\sin(\pi n(2L+1)/N)}{\sin(\pi n/N)}$$

$$\Rightarrow DFT\,\vec{s} = \vec{y} \text{ der } y_n = \boxed{\frac{1}{2L+1}\frac{\sin(\pi n(2L+1)/N)}{\sin(\pi n/N)} = \lambda_{S,n}} \rightarrow \omega = 2\pi n/N, \quad \pi n/N = \frac{\omega}{2}$$

$$\lambda_{S,n} = \lambda_S(2\pi n/N) \qquad \Rightarrow \lambda_S(\omega) = \frac{1}{2L+1}\frac{\sin((2L+1)\omega/2)}{\sin(\omega/2)}$$

Eks. 3.32 Ideelle lavpassfilter.

$$\vec{\lambda}_s = (\underbrace{1,\dots,1}_{\text{lave}}, \underbrace{0,\dots,0}_{\text{høye}}, \underbrace{1,\dots,1}_{\text{lave}})$$

$$\vec{s} = \text{IDFT}_N \vec{\lambda}_s = \frac{1}{N}\overline{\text{DFT}_N \vec{\lambda}_s} = \frac{1}{N}\overline{\text{DFT}_N \overline{\vec{\lambda}_s}} = \overline{\frac{1}{N}\text{DFT}_N(\vec{\lambda}_s)}$$

$$= \overline{\frac{1}{N}\text{DFT}_N(1,\dots,1,0,\dots,0,1,\dots,1)}$$

$$= \overline{\left\{\frac{1}{N}\frac{\sin(\pi k(2L+1)/N)}{\sin \pi k/N}\right\}_{k=0}^{N-1}} = \left\{\frac{1}{N}\frac{\sin(\pi k(2L+1)/N)}{\sin \pi k/N}\right\}_{k=0}^{N-1}$$

Eks. 3.33: For å forenkle dette filteret, så kan vi prøve å nullstille noen av filterkoeffisientene.

$$\longrightarrow t_k = \left\{\frac{1}{N}\frac{\sin(\pi k(2L+1)/N)}{\sin \pi k/N}\right\}_{-N_0}^{N_0}$$