La $\underbrace{\{t_k\}_{k=0}^{511}}_{S_1}$ være filterkoeffisientene (S_1) som brukes i MP3-standarden.
La oss se på filteret (S_2) med koeffisienter $\underbrace{\cos(k\omega_c)t_k}_{S_2}$

$$\lambda_{S_2}(\omega) = \sum_{k=0}^{511} \cos(k\omega_c) t_k e^{-ik\omega}$$

$$= \sum_{k=0}^{511} \frac{1}{2}\left(e^{ik\omega_c} + e^{-ik\omega_c}\right) t_k e^{-ik\omega}$$

$$= \frac{1}{2}\left(\sum_{k=0}^{511} t_k e^{ik\omega_c} e^{-ik\omega} + \sum_{k=0}^{511} t_k e^{-ik\omega_c} e^{-ik\omega}\right)$$

$$= \frac{1}{2}\left(\sum_{k=0}^{511} t_k e^{-ik(\omega-\omega_c)} + \sum_{k=0}^{511} t_k e^{-ik(\omega+\omega_c)}\right)$$

$$= \underline{\frac{1}{2}\left(\lambda_{S_1}(\omega-\omega_c) + \lambda_{S_1}(\omega+\omega_c)\right)}$$

MP3-standarden velger $\underline{\omega_c = \frac{3\pi}{64}, \frac{5\pi}{64}, \ldots \frac{7\pi}{64}, \ldots, \frac{63\pi}{64}}$

(3.1) : $z_n = (x_{n-1} + 2x_n + x_{n+1})/4$ ($\frac{\text{andre}}{\text{tredje}}$ rad i Pascals trekant)

$S = \{\underline{\tfrac{1}{2}}, \tfrac{1}{2}\}$ $\left(z_n = \tfrac{1}{2}(x_n + x_{n-1})\right)$
første rad i Pascals trekant)

$\lambda_S(\omega) = \frac{1}{2} + \frac{1}{2}e^{-i\omega} = e^{-i\frac{\omega}{2}}\frac{1}{2}\left(e^{i\frac{\omega}{2}} + e^{-i\frac{\omega}{2}}\right) = \underline{e^{-i\frac{\omega}{2}}\cos\frac{\omega}{2}}$

$\lambda_{S^k}(\omega) = (\lambda_S(\omega))^k = \left(e^{-\frac{\omega}{2}}\cos\frac{\omega}{2}\right)^k = \underline{e^{-i\frac{k\omega}{2}}\cos^k\frac{\omega}{2}}$

$= \frac{1}{2}(1 + e^{-i\omega})^k$ → får koeffisienter fra rad k i Pascals trekant.

S^k er et lavpassfilter ($\omega = \pi \Rightarrow \cos\frac{\omega}{2} = 0$)

S^k er veldig flat nær π, siden vi regner ut $\cos^k\left(\frac{\omega}{2}\right)$

($\omega = \pi$ er en null med multiplisitet k)

Enkleste filteret: E_d: $\vec{x} \to \vec{z}$

$z_n = x_{n-d} = \sum t_k x_{n-k}$

$E_d = \{\underline{0}, \ldots, 0, \underset{d}{1}\}$

$$\lambda_{E_d}(\omega) = \underline{e^{-id\omega}} \qquad |\lambda_{E_d}(\omega)| = 1$$

Oppgave 3.17

Er operasjonen som beholder annenhver komponent et filter?

(nuller ut annenhver).

Kall denne for S. S er lineær.

$$Se_0 = e_0 \qquad Se_1 = 0 \qquad Se_2 = e_2. \qquad Se_3 = 0$$

matrisen til S blir dermed

$$\begin{pmatrix} 1 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & 0 & 1 & \\ \vdots & \vdots & 0 & \ddots \\ 0 & 0 & \vdots & \end{pmatrix}$$

På hoveddiagonalen står det $(1, 0, 1, 0, \ldots)$ slik at S ikke er konstant på diagonaler, så den er ikke et filter.

Repetisjon fra del I

$\langle f,g\rangle = \frac{1}{T}\int_0^T f(t)\overline{g(t)}\,dt$

(kont. lyd)

$\underbrace{\cos(2\pi nt/T),\ \sin(2\pi nt/T),\ e^{\pm 2\pi i nt/T}}_{\text{rene toner}}$

$\{e^{2\pi i nt/T}\}$ ortonormale

$V_{N,T} = \text{span}\{e^{2\pi i nt/T}\}_{n=-N}^{N}$

(dim $2N+1$)

$f_N \overset{\text{def}}{=}$ minste kvadraters tilnærming til f fra $V_{N,T}$

$f_N = \sum_{n=-N}^{N} \langle f, e^{2\pi i nt/T}\rangle e^{2\pi i nt/T}$

(ort. dekomp teorem)

$= \sum_{n=-N}^{N} \underbrace{\frac{1}{T}\int_0^T f(t)e^{-2\pi i nt/T}\,dt}_{y_n \text{ (Fourierkoeff)}}\, e^{2\pi i nt/T}$

$= \sum_{n=-N}^{N} y_n e^{2\pi i nt/T}$

$\langle \vec{x},\vec{y}\rangle = \sum_{k=0}^{N-1} x_k\overline{y_k}$

(digital lyd; $\mathbb{R}^N$)

$\frac{1}{\sqrt{N}} e^{2\pi i kn/N} = \phi_n$ (Fourierbasisvektor)

$\langle \phi_n, \phi_m\rangle = 0$ hvis $n \neq m$

$\|\phi_n\| = 1$

$\langle \phi_n, \phi_m\rangle = \frac{1}{N}\sum_{k=0}^{N-1} e^{2\pi i k(n-m)/N}$

$= \frac{1}{N}\frac{1-\left(e^{2\pi i(n-m)/N}\right)^N}{1-e^{2\pi i(n-m)/N}} = 0$

Ikke alltid vi trenger å regne ut Fourierintegralene:

$f(t) = \cos^2(2\pi nt/T)$

$= \left(\frac{1}{2}\left(e^{2\pi i nt/T} + e^{-2\pi i nt/T}\right)\right)^2 = \frac{1}{4}\left(e^{2\pi i 2nt/T} + 2 + e^{-2\pi i 2nt/T}\right)$

$\Rightarrow y_{\underline{2n}} = y_{\underline{-2n}} = \frac{1}{4},\ y_{\underline{0}} = \frac{1}{2}$

for $N \geq 2n$: $f_N(t) = f(t)$

for $N \geq 0,\ N < 2n \Rightarrow \underline{f_N(t) = \frac{1}{2}}$

sammenhengen mellom frekvens og DFT-indeks:

$e^{2\pi i \nu t}$ | $e^{2\pi i k n/N}$

$\nu t_k = kn/N$ $\boxed{t = k T_s = \frac{k}{f_s}}$

$\nu \frac{k}{f_s} = \frac{kn}{N}$

$\frac{\nu}{f_s} = \frac{n}{N}$

$\boxed{\nu = \frac{n f_s}{N}}$ $n = \frac{N}{2} \Rightarrow \underline{\nu = \frac{f_s}{2}}$

DFT til $\vec{x}$ kalles $\vec{y}$: $y_n = \sum_{k=0}^{N-1} x_k e^{-2\pi i k n/N}$

IDFT: $x_k = \frac{1}{N} \sum y_n e^{2\pi i k n/N}$

F_N, Fouriermatrisen: $F_N = \frac{1}{\sqrt{N}} DFT_N$, $F_N^H = F_N^{-1}$

---

frekvensene i $e^{2\pi i n t/T}$ er $\nu = \frac{n}{T}$

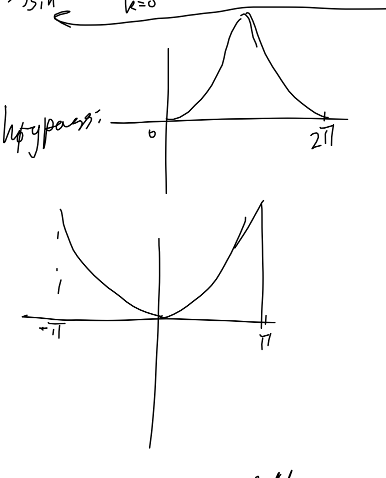
høyeste frekvens i $V_{N,T}$ er $\frac{N}{T}$

$\vec{\lambda}_S = DFT_N \vec{s} \Rightarrow \vec{s} = IDFT_N \vec{\lambda}_S$

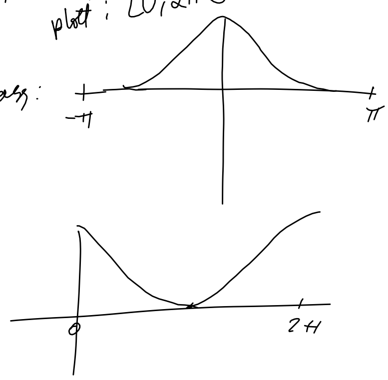
$\lambda_{S,n} = \sum_{k=0}^{N-1} s_k e^{2\pi i k n/N}$ $\lambda_S(\omega) = \sum_k t_k e^{-ik\omega}$

$\omega = 2\pi n/N$ plott: $[0, 2\pi)$ eller $[-\pi, \pi]$

høypass:



lavpass:



Ideelt lavpassfilter:

$\lambda_S = (\underbrace{1, \dots, 1}_{L+1}, 0, \dots, 0, \underbrace{1, \dots, 1}_{L})$

Ideelt høypassfilter:

$\lambda_S = (0, \dots, 0, \underbrace{1, \dots, 1}_{2L+1}, 0, \dots, 0)$

impl. av ideelt lavpassfilter: siden F_N kan brukes til å diagonalisere S:

1. kjør F_N
2. null ut høye frek.
3. kjør F_N^H

$\underbrace{S = F_N^H D F_N}_{S \text{ filter}}$