Bevis for lemma 5.11

Anta n partall: $\phi_{0,n} \neq 0$ på $[n, n+1)$ — når treffer de hverandre?

$\phi_{1,n} \neq 0$ på $[n2^{-1}, (n+1)2^{-1})$

$$\left[\frac{n}{2}, \frac{n+1}{2}\right] \cap [n_1, n_1+1) \neq 0 \quad \text{hvis } n_1 = \frac{n}{2}$$

$$\mathrm{proj}_{V_0}\, \phi_{1,n} = \sum_{k=0}^{N-1} \langle \phi_{1,n}, \phi_{0,k} \rangle \phi_{0,k}$$

kun bidrag for $k = \frac{n}{2}$

$$= \underbrace{\langle \phi_{1,n}, \phi_{0,\frac{n}{2}} \rangle}_{} \phi_{0,\frac{n}{2}} = \frac{\sqrt{2}}{2} \phi_{0,\frac{n}{2}} = \frac{1}{\sqrt{2}} \phi_{0,\frac{n}{2}}$$

$$\int_0^N \phi_{1,n} \phi_{0,\frac{n}{2}}\, dt = \int_{\frac{n}{2}}^{\frac{n+1}{2}} \sqrt{2} \cdot 1\, dt = \frac{\sqrt{2}}{2}$$

$$\mathrm{proj}_{W_0}\, \phi_{1,n} = \phi_{1,n} - \mathrm{proj}_{V_0}\, \phi_{1,n} = \phi_{1,n} - \frac{1}{\sqrt{2}} \phi_{0,\frac{n}{2}}$$

$$= \phi_{1,n} - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} \phi_{1,n} + \frac{1}{\sqrt{2}} \phi_{1,n+1}\right)$$

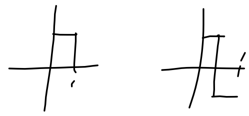
$$= \phi_{1,n} - \frac{1}{2}\phi_{1,n} - \frac{1}{2}\phi_{1,n+1}$$

$$= \frac{1}{2}\phi_{1,n} - \frac{1}{2}\phi_{1,n+1}$$

$$= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \underbrace{(\phi_{1,n} - \phi_{1,n+1})}_{\psi_{0,n}}$$

$$= \frac{1}{\sqrt{2}} \psi_{0,\frac{n}{2}}$$

def: $\psi_{0,n} = \frac{1}{\sqrt{2}}(\phi_{1,2n} - \phi_{1,2n+1})$



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Tilsvarende for n oddetall.

Mer generelt:

$$\mathrm{proj}_{V_0}(g_1) = \mathrm{proj}_{V_0}\left(\sum_{n=0}^{2N-1} c_{1,n} \phi_{1,n}\right)$$

$$= \mathrm{proj}_{V_0}\left(\sum_{n=0}^{N-1} c_{1,2n}\phi_{1,2n} + \sum_{n=0}^{N-1} c_{1,2n+1}\phi_{1,2n+1}\right)$$

$$= \sum_{n=0}^{N-1} c_{1,2n}\, \mathrm{proj}_{V_0}\, \phi_{1,2n} + \sum_{n=0}^{N-1} c_{1,2n+1}\, \mathrm{proj}_{V_0}\, \phi_{1,2n+1}$$

$$= \sum_{n=0}^{N-1} c_{1,2n} \frac{1}{\sqrt{2}} \phi_{0,n} + \sum_{n=0}^{N-1} c_{1,2n+1} \frac{1}{\sqrt{2}} \phi_{0,n}$$

$$= \sum_{n=0}^{N-1} \underbrace{\frac{c_{1,2n} + c_{1,2n+1}}{\sqrt{2}}}_{c_{0,n}} \phi_{0,n}$$

Beviş for prop. 5.12

$f \in V_1$ $\quad f_{n,1}$ på $[n, n+\frac{1}{2})$ $\rightarrow$ $= \underbrace{\left[2n \cdot \frac{1}{2} + (2n+1)\frac{1}{2}\right)}_{\phi_{1,2n} \neq 0}$

$f_{n,2}$ på $[n+\frac{1}{2}, n+1)$ $\qquad \phi_{m,n}(t) = 2^{m/2}\phi(2^m t - 1)$

$$f = \sum_{n=0}^{N-1}\left(f_{n,1}\, \phi_{1,2n}\frac{1}{\sqrt{2}} + f_{n,2}\,\phi_{1,2n+1}\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}} \sum_{n=0}^{N-1}\left(f_{n,1}\,\phi_{1,2n} + f_{n,2}\,\phi_{1,2n+1}\right)$$

$$\mathrm{proj}_{V_0} f = \frac{1}{\sqrt{2}}\sum_{n=0}^{N-1}\left(f_{n,1}\,\mathrm{proj}_{V_0}\,\phi_{1,2n} + f_{n,2}\,\mathrm{proj}_{V_0}\,\phi_{1,2n+1}\right)$$

$$= \frac{1}{\sqrt{2}}\sum_{n=0}^{N-1}\left(f_{n,1}\frac{\phi_{0,n}}{\sqrt{2}} + f_{n,2}\frac{\phi_{0,n}}{\sqrt{2}}\right) = \frac{1}{2}\sum_{n=0}^{N-1}\left(f_{n,1} + f_{n,2}\right)\phi_{0,n}$$

$$= \underbrace{\sum_{n=0}^{N-1} \frac{f_{n,1}+f_{n,2}}{2}\,\phi_{0,n}}_{\text{verdien denne har på } [n, n+1) \text{ er } \frac{f_{n,1}+f_{n,2}}{2}}$$

Oppgave 5.1

Vis at koordinatvektoren for en $f \in V_0$ i Φ_0 er $(f(0), f(1), \dots, f(N-1))$ — koords i Φ_0

Vi har at $f(t) = \sum_{n=0}^{N-1} c_{0,n} \phi_n(t)$

$\phi_n \neq 0$ kun på $[n, n+1)$

regn ut i heltall, $t=k$: $f(k) = \sum_{n=0}^{N-1} c_{0,n} \phi_n(k)$

kun $n=k$ bidrar

$= c_{0,k} \phi_k(k) = \underline{c_{0,k}}$

$\Rightarrow$ koordinatene i Φ_0 er $\underline{\underline{(f(0), f(1), \dots, f(N-1))}}$