Eksempel 5.9

Regn ut en DWT av $\vec{x} = (\underbrace{\underbrace{1,\dots,1}_{512},\underbrace{0,\dots,0}_{512}}_{1024=2^{10}})$ over 10 nivåer

Tenker oss $\vec{x}$ som koordinatene til $f \in V_{10}$ (dim $V_{10} = 1024$)

(dim $V_0 = 1$)

vil få $(\underbrace{\phi_0}_{1}, \underbrace{\psi_0}_{1}, \underbrace{\psi_1}_{2}, \dots, \underbrace{\psi_9}_{512})$ $\quad (\phi_{m,n} = 2^{m/2}\phi(2^m t-n))$

$\vec{x}$ svarer til funksjonen $\sum_{n=0}^{511} 1 \cdot \underbrace{\phi_{10,n}}_{=2^{10/2}=2^5=32 \text{ på } [n2^{-10},(n+1)2^{-10})}$ i V_{10} $\quad [n2^{-m},(n+1)2^{-m})$

$\underbrace{[0, 1\cdot 2^{-10}), [1\cdot 2^{-10}, 2\cdot 2^{-10}), \dots, [511\cdot 2^{-10}, \underbrace{512\cdot 2^{-10}}_{\frac{1}{2}})}_{[0,\frac{1}{2})}$

$\Rightarrow \vec{x}$ er koordinatene til funksjonen f som er 32 på $[0,\frac{1}{2})$, 0 ellers.

$\phi_{1,0}$ er $\neq 0$ på $[0,\frac{1}{2})$ $\quad (n=0, m=1 \;;\; [n2^{-m},(n+1)2^{-m})$

$\underbrace{\sqrt{2}}$ på $[0,\frac{1}{2})$

Ser da at $f = 32 \overbrace{\frac{1}{\sqrt{2}}\phi_{1,0}}^{\text{går opp til 1}} \Rightarrow f = \frac{32}{\sqrt{2}}\phi_{1,0}$

etter en DWT over 10 nivåer skal f uttrykkes ved: $\phi_{0,n}, \psi_{1,n}, \psi_{2,n}, \dots, \psi_{9,n}$

$f = \frac{32}{\sqrt{2}}\phi_{1,0} = \frac{32}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\phi_{0,0} + \frac{1}{\sqrt{2}}\psi_{0,0}\right) = \underline{16}\,\phi_{0,0} + \underline{\underline{16}}\,\psi_{0,0}$

$\Rightarrow$ DWT $\vec{x} = (\underline{16}, \underline{\underline{16}}, 0, \dots, 0)$ ■

$$\underset{\text{for } T_2}{\left[\begin{array}{c|c} T_1 & 0 \\ \hline 0 & T_2 \end{array}\right]\begin{bmatrix} \overset{\text{egenv. for } T_1}{\vec{v}_1} \\ \vec{v}_2 \end{bmatrix}} = \begin{bmatrix} T_1\vec{v}_1 + 0\,\vec{v}_2 \\ 0\cdot\vec{v}_1 + T_2\vec{v}_2 \end{bmatrix} = \begin{bmatrix} T_1\vec{v}_1 \\ T_2\vec{v}_2 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda\vec{v}_1 \\ \lambda\vec{v}_2 \end{bmatrix} = \lambda\begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \end{bmatrix}$ egenvektor, samme egenverdi.

Eksempel 5.12

La oss regne ut koordinatene til $\overset{\text{proj}_{V_m}}{\downarrow} f(t) = 1 - \frac{t}{N}$ i basisene $\underbrace{\psi_m}_{w_{m,n}}$

$$\text{proj}_{V_m} f \underset{\text{(ort. dekomp teorem)}}{=} \sum_n \langle f, \phi_{0,n} \rangle \phi_{0,n} + \sum_{m=1} \overbrace{\sum_n \langle f, \psi_{m,n} \rangle}^{\psi_m}_{\underbrace{}_{w_{m,n}}} \psi_{m,n}$$

$$w_{m,n} = \langle f, \psi_{m,n} \rangle = \int_0^N \left(1 - \frac{t}{N}\right) \psi_{m,n}(t)\,dt$$

$$= \underbrace{\int_0^N \psi_{m,n}(t)}_{=0} - \frac{1}{N} \int_0^N t\, \psi_{m,n}(t)\,dt = -\frac{1}{N} \int_0^N t\psi_{m,n}(t)\,dt$$

$$= -\frac{1}{N} \int_{2^{-m}n}^{2^{-m}(n+1)} t\, \psi_{m,n}(t)\,dt = -\frac{2^{m/2}}{N} \left(\int_{2^{-m}n}^{2^{-m}(n+\frac{1}{2})} t\,dt - \int_{2^{-m}(n+\frac{1}{2})}^{2^{-m}(n+1)} t\,dt \right)$$

$$= -\frac{2^{m/2}}{N} \left(\left[\tfrac{1}{2}t^2\right]_{2^{-m}n}^{2^{-m}(n+\frac{1}{2})} - \left[\tfrac{1}{2}t^2\right]_{2^{-m}(n+\frac{1}{2})}^{2^{-m}(n+1)} \right)$$

$$= -\frac{2^{m/2-1}}{N} \left(2^{-2m} \left((n+\tfrac{1}{2})^2 - n^2 - (n+1)^2 + (n+\tfrac{1}{2})^2 \right) \right)$$

$$= -\frac{1}{N 2^{3m/2+1}} \left(n^2 + n + \tfrac{1}{4} - n^2 - n^2 - 2n - 1 + n^2 + n + \tfrac{1}{4} \right)$$

$$= \frac{1}{N 2^{3m/2+2}}$$