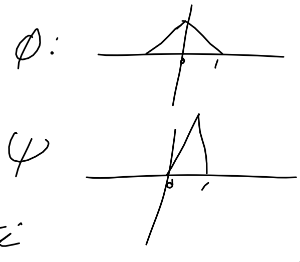
Sek. 5.5

Stykkevis lineære wavelets.



Der definerte vi $\psi(t) = \frac{1}{\sqrt{2}} \phi_{1,1}(t)$

Vil endre definisjonen av denne slik at

vi får forsvinnende momenter, det vil si

$\int_0^N t^k \psi(t)\,dt \qquad k=0,\dots,s$ (s forsvinnende momenter)

$\int_0^N \psi(t)=0 \qquad \Rightarrow \int_0^N P_s(t)\psi(t)\,dt=0 \Rightarrow \int_0^N P_s(t)\psi_{m,n}(t)\,dt=0 \qquad u=2^m t-n$

Hvorfor er forsvinnende momenter bra? (seksjon 7.2)

Anta at C_m, Φ_m er ortonormale basiser.

Vi kan skrive $f\in V_m$ som $f = \underbrace{\sum_n \langle f,\phi_{0,n}\rangle\phi_{0,n}}_{V_0} + \underbrace{\sum_{m'=0}^{m-1}\sum_n \langle f,\psi_{m',n}\rangle\psi_{m',n}}_{W_0,W_1,\dots,W_{m-1}}$

Anta at f er s ganger deriverbar.

Da kan vi skrive $f = P_s + Q_s$

polynom av grad s (Taylorpolynomet) — liten, begrenset av restleddet i Taylors formel $\le \frac{f^{(s+1)}(c)}{(n+1)!}x^{n+1}$

Derfor:

$$f = \sum_n \langle f,\phi_{0,n}\rangle\phi_{0,n} + \sum_{m'=0}^{m-1}\sum_n \langle P_s+Q_s,\psi_{m',n}\rangle\psi_{m',n}$$

$$= \underbrace{\sum_n \langle f,\phi_{0,n}\rangle\phi_{0,n}}_{\in V_0} + \underbrace{\sum_{m'=0}^{m-1}\sum_n \langle P_s,\psi_{m',n}\rangle\psi_{m',n}}_{0\text{, siden }\psi\text{ har forsvinnende momenter}} + \underbrace{\sum_{m'=0}^{m-1}\sum_n \langle Q_s,\psi_{m',n}\rangle\psi_{m',n}}_{\text{Liten, siden }Q_s\text{ er liten}}$$

$\Rightarrow$ lavresolusjonstilnærmingen $\sum_n \langle f,\phi_{0,n}\rangle\phi_{0,n}$ er en veldig god tilnærming til f

La oss endre vår stykkevis lineære ψ (0 forsv. mom.) til $\hat{\psi}$ (2 forsv. mom.)

Endrer ved å sette $\hat{\psi} := \psi - \alpha \phi_{0,0} - \beta \phi_{0,1}$

Vil velge α, β s.a. $\int_0^N \hat{\psi}(t)dt = \int_0^N t\hat{\psi}(t)dt = 0$

Oppgave 5.38: $\boxed{\hat{\psi}(t) = \psi(t) - \frac{1}{4}(\phi_{0,0}(t) + \phi_{0,1}(t))}$ har 2 forsv. mom.

$\alpha = \beta = \frac{1}{4}$

$$\int \hat{\psi}(t)dt = \int(\psi - \alpha\phi_{0,0} - \beta\phi_{0,1})dt = \int \psi dt - \underline{\alpha}\int \phi_{0,0}(t)dt - \underline{\beta}\int \phi_{0,1}(t)dt = 0$$

$$\int t\hat{\psi}(t)dt = \int t\psi(t)dt - \alpha\int t\phi_{0,0}(t)dt - \beta\int t\phi_{0,1}(t)dt = 0$$

Dette betyr:

fra sist mandag.

$$\hat{\psi}_{0,n} = \psi_{0,n} - \frac{1}{4}(\underline{\phi_{0,n}} + \underline{\phi_{0,n+1}})$$

$$= \frac{1}{\sqrt{2}}\phi_{1,2n+1} - \frac{1}{4}\frac{1}{\sqrt{2}}\left(\frac{1}{2}\phi_{1,2n-1} + \phi_{1,2n} + \frac{1}{2}\phi_{1,2n+1}\right) - \frac{1}{4}\frac{1}{\sqrt{2}}\left(\frac{1}{2}\phi_{1,2n+1} + \phi_{1,2n+2} + \frac{1}{2}\phi_{1,2n+3}\right)$$

$$= \frac{1}{\sqrt{2}}\left(-\frac{1}{8}\phi_{1,2n-1} - \frac{1}{4}\phi_{1,2n} + \frac{3}{4}\phi_{1,2n+1} - \frac{1}{4}\phi_{1,2n+2} - \frac{1}{8}\phi_{1,2n+3}\right)$$

Fra dette får vi halvparten av søylene i koordinatskiftematrisen

$G = P_{\phi_1 \leftarrow \mathcal{C}_1}$ (IDWT) $\mathcal{C}_1 = \{\phi_{0,0}, \underset{1}{\hat{\psi}_{0,0}}, \phi_{0,1}, \underset{3}{\hat{\psi}_{0,1}}, \dots\}$ 5

$$\Rightarrow G_1 = \frac{1}{\sqrt{2}}\left\{-\frac{1}{8}, -\frac{1}{4}, \underline{\frac{3}{4}}, -\frac{1}{4}, -\frac{1}{8}\right\}$$, siden fremdeles $\phi_{0,n} = \frac{1}{\sqrt{2}}\left(\frac{1}{2}\phi_{1,2n-1} + \phi_{1,2n} + \frac{1}{2}\phi_{1,2n+1}\right)$

G_0 er fremdeles $\frac{1}{\sqrt{2}}\{\frac{1}{2}, 1, \frac{1}{2}\}$,

Ser spesielt at $\hat{\psi} = \hat{\psi}_{0,0} = \frac{1}{\sqrt{2}}\left(-\frac{1}{8}\phi_{1,-1} - \frac{1}{4}\phi_{1,0} + \frac{3}{4}\phi_{1,1} - \frac{1}{4}\phi_{1,2} - \frac{1}{8}\phi_{1,3}\right)$

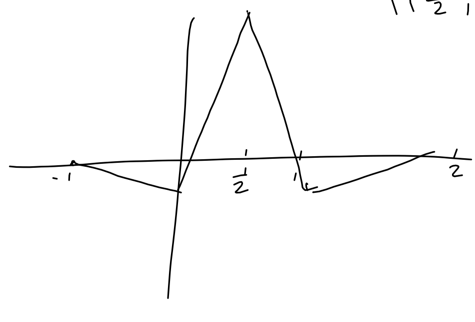
$\phi_{1,0}$: $\sqrt{2}$ for $t = 0$
0 for $t = \pm 1, \pm 2, \dots$

$\phi_{1,1}$: $\sqrt{2}$ for $t = \frac{1}{2}$
0 for $t = \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \dots$

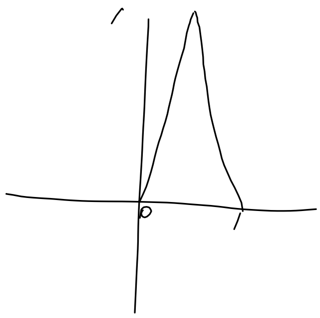
$\hat{\psi}$ går gjennom $\left(-\frac{1}{2}, -\frac{1}{8}\right)$, $\left(0, -\frac{1}{4}\right)$, $\left(\frac{1}{2}, \frac{3}{4}\right)$, $\left(1, -\frac{1}{4}\right)$, $\left(\frac{3}{2}, -\frac{1}{8}\right)$

0 i alle andre "halvtall"

$\hat{\psi}$:



ψ:



Vi har at $G = P_{\Phi_1 \leftarrow \hat{C}_1} = \underbrace{P_{\Phi_1 \leftarrow C_1}} P_{C_1 \leftarrow \hat{C}_1}$

$C_1 = \{\phi_{0,0}, \psi_{0,0}, \phi_{0,1}, \psi_{0,1}, \ldots\}$

$\hat{C}_1 = \{\phi_{0,0}, \hat{\psi}_{0,0}, \phi_{0,1}, \hat{\psi}_{0,1}, \ldots\}$

har vi fra før

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & \\ \frac12 & 1 & \frac12 & \\ 0 & 0 & 1 & \\ \vdots & \vdots & \frac12 & \cdots \\ & & 0 & \\ 0 & & \vdots & \\ \frac12 & \cdot & 0 & \end{pmatrix}$$

$\frac{1}{\sqrt{2}} B_{\frac12}$

$\hat{\psi}_{0,n} = \psi_{0,n} - \frac14(\phi_{0,n} - \phi_{0,n+1})$

gir halvparten av søylene i $P_{C_1 \leftarrow \hat{C}_1}$

$\phi_{0,n} = \phi_{0,n}$

$$\lambda \rightarrow \begin{pmatrix} 1 & -\frac14 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ \vdots & -\frac14 & 1 & -\frac14 & \cdots \\ \vdots & 0 & 0 & 1 & \\ \vdots & \vdots & \vdots & -\frac14 & \cdots \\ & & & 0 & \\ 0 & 0 & \cdot & \vdots & \end{pmatrix}$$

→ elementær liftingmatrise av like type (partallstype).

skriver $A_{-\frac14}$ for denne (mer generelt A_λ)

$$\Rightarrow \boxed{\overset{G}{P_{\Phi_1 \leftarrow \hat{C}_1}} = \frac{1}{\sqrt{2}} * B_{\frac12} * A_{-\frac14}}$$

$$\boxed{\overset{H}{P_{\hat{C}_1 \leftarrow \Phi_1}} = \sqrt{2} * A_{\frac14} * B_{-\frac12}}$$