Bevis for teorem 9.9 | S_1 : filterkoeffs t_1
| S_2 : filterkoeffs t_2

1. La X være bildet, og anvend først S_1 på søylene i X. Dette gir et nytt bilde Y med komponenter

søyle j

$$Y_{i,j} = \sum_{k_1} (t_1)_{k_1} X_{i-k_1, j}$$

2. La Z være bildet vi får ved å anvende S_2 på radene i Y:

$$Z_{i,j} = \sum_{k_2} (t_2)_{k_2} Y_{i, j-k_2}$$

$$= \sum_{k_1, k_2} (t_1)_{k_1} (t_2)_{k_2} X_{i-k_1, j-k_2} = \sum_{k_1, k_2} a_{k_1,k_2} X_{i-k_1} X_{j-k_2}$$

der A er molekylet med elementer

$$a_{k_1,k_2} = (t_1)_{k_1} (t_2)_{k_2}$$

$$(S_1 \otimes S_2)(\vec{x} \otimes \vec{y}) = (S_1 \otimes S_2)\left((\sum x_i \vec{e}_i) \otimes (\sum y_j e_j)\right)$$

$$= (\sum x_i e_i)(\sum y_j e_j)^T$$

$$= (S_1 \otimes S_2)\left(\sum_{i,j} x_i y_j \vec{e}_1 \otimes \vec{e}_j\right)$$

$$\sum x_i y_j e_i e_j^T = \sum x_i y_j e_i \otimes e_j$$

$$= \sum_{i,j} x_i y_j (S_1 \otimes S_2)(\vec{e}_i \otimes \vec{e}_j)$$

$$= \sum_{i,j} x_i y_j (S_1 \vec{e}_i) \otimes (S_2 \vec{e}_j)$$

$$= \sum_{i,j} x_i y_j S_1 \vec{e}_i \vec{e}_j^T S_2^T$$

$$= S_1 (\sum x_i \vec{e}_i)(\sum_j y_j \vec{e}_j)^T S_2^T$$

$$= S_1 x y^T S_2^T = \underline{S_1 (x \otimes y) S_2^T}$$

$$= \underline{(S_1 x) \otimes (S_2 y)}$$

Teorem 9.14

Bevis: $(S_1 \otimes S_2)(\vec{e}_i \otimes \vec{e}_j) = (S_1\vec{e}_i) \otimes (S_2\vec{e}_j)$

$= S_1\vec{e}_i (S_2\vec{e}_j)^T$

$= S_1 \vec{e}_i \vec{e}_j S_2^T$

$= S_1 (\vec{e}_i \otimes \vec{e}_j) S_2^T$

$\Rightarrow (S_1 \otimes S_2)X = S_1 X S_2^T$ for alle X

Korollar 9.5

$(S_1 \otimes T_1)(S_2 \otimes T_2)X = (S_1 \otimes T_1)(S_2 X T_2^T)$

$= S_1 (S_2 X T_2^T) T_1^T = (S_1 S_2) X (T_1 T_2)^T$

$\Rightarrow (S_1 \otimes T_1)(S_2 \otimes T_2) = (S_1 S_2) \otimes (T_1 T_2)$ $((S_1 S_2) \otimes (T_1 T_2))X$