Bevis for teorem 10.3

$$0 = \sum \alpha_{ij} f_i(t_1) g_j(t_2) = \sum \alpha_{ij} (f_i \otimes g_j)(t_1, t_2)$$

Definerer $h_i(t_2) = \sum_j \alpha_{ij} g_j(t_2)$

$$0 = \sum_{\alpha_{ij}} \alpha_{ij} f_i(t_1) g_j(t_2) = \sum_i \left(\sum_j \alpha_{ij} g_j(t_2) \right) f_i(t_1)$$

$$= \sum_i h_i(t_2) f_i(t_1)$$

Siden f_i er lineært uavhengige, så må $h_i(t_2) = 0$ alle i, t_2

$\Rightarrow h_i = 0$ alle i

$\Rightarrow \sum_j \alpha_{ij} g_j = 0$

Siden g_j er lineært uavhengige, så må $\alpha_{ij} = 0$, alle j, i

$\Rightarrow (f_i \otimes g_j)$ er lineært uavhengige.

La oss regne ut projeksjonen fra $V_1 \otimes V_1$ ned i $V_0 \otimes V_0$

$\mathrm{proj}_{V_0 \otimes V_0}(\phi_{1,k_1} \otimes \phi_{1,k_2})$

k_1, k_2 partall, ort. dekomp teorem

$= \sum_{n_1, n_2} \langle \phi_{1,k_1} \otimes \phi_{1,k_2}, \phi_{0,n_1} \otimes \phi_{0,n_2} \rangle (\phi_{0,n_1} \otimes \phi_{0,n_2})$

$\neq 0$ på $\left(\frac{k_1}{2}, \frac{k_1}{2} + \frac{1}{2}\right]$

$= \sum_{n_1 n_2} \langle \phi_{1,k_1}, \phi_{0,n_1} \rangle \langle \phi_{1,k_2}, \phi_{0,n_2} \rangle \phi_{0,n_1} \otimes \phi_{0,n_2}$

kun $n_2 = \frac{k_1}{2}$ som bidrar

$= \underbrace{\langle \phi_{1,k_1}, \phi_{0,\frac{k_1}{2}} \rangle}_{\frac{\sqrt{2}}{2}} \underbrace{\langle \phi_{1,k_2}, \phi_{0,\frac{k_2}{2}} \rangle}_{\frac{\sqrt{2}}{2}} \phi_{0,\frac{k_1}{2}} \otimes \phi_{0,\frac{k_2}{2}}$

$= \underline{\frac{1}{2} \phi_{0,\frac{k_1}{2}} \otimes \phi_{0,\frac{k_2}{2}}}$

proj. ned i ortogonalkomplementet til $V_0 \otimes V_0$ i $V_1 \otimes V_1$:

$= \phi_{1,k_1} \otimes \phi_{1,k_2} - \mathrm{proj}_{V_0 \otimes V_0}(\phi_{1,k_1} \otimes \phi_{1,k_2})$

$= \phi_{1,k_1} \otimes \phi_{1,k_2} - \frac{1}{2} \phi_{0,\frac{k_1}{2}} \otimes \phi_{0,\frac{k_2}{2}}$

$= \left(\frac{1}{\sqrt{2}}(\phi_{0,\frac{k_1}{2}} + \psi_{0,\frac{k_1}{2}})\right) \otimes \left(\frac{1}{\sqrt{2}}(\phi_{0,\frac{k_2}{2}} + \psi_{0,\frac{k_2}{2}})\right) - \frac{1}{2} \phi_{0,\frac{k_1}{2}} \otimes \phi_{0,\frac{k_2}{2}}$

$= \underline{\frac{1}{2} \phi_{0,\frac{k_1}{2}} \otimes \phi_{0,\frac{k_2}{2}}} + \frac{1}{2} \phi_{0,\frac{k_1}{2}} \otimes \psi_{0,\frac{k_2}{2}} + \frac{1}{2} \psi_{0,\frac{k_1}{2}} \otimes \phi_{0,\frac{k_2}{2}} + \frac{1}{2} \psi_{0,\frac{k_2}{2}} \otimes \psi_{0,\frac{k_2}{2}}$

$\underline{- \frac{1}{2} \phi_{0,\frac{k_1}{2}} \otimes \phi_{0,\frac{k_2}{2}}}$

$= \underbrace{\frac{1}{2} \phi_{0,\frac{k_1}{2}} \otimes \psi_{0,\frac{k_2}{2}}}_{W_m^{(0,1)}} + \underbrace{\frac{1}{2} \psi_{0,\frac{k_1}{2}} \otimes \phi_{0,\frac{k_2}{2}}}_{W_m^{(1,0)}} + \underbrace{\frac{1}{2} \psi_{0,\frac{k_1}{2}} \otimes \psi_{0,\frac{k_2}{2}}}_{W_m^{(1,1)}}$

$\boxed{\phi_{1,k_1} = \frac{1}{\sqrt{2}} \phi_{0,\frac{k_1}{2}} + \frac{1}{\sqrt{2}} \psi_{0,\frac{k_1}{2}}}$