MAT1001, Fall 2011

Oblig 2

Deadline: Thursday, November 10, 1430 pm

You may work on these exercises together with your fellow students, but each of you has to hand in his/her own version of the solution to the assignment for approvement. Please put your name and course code (MAT 1001) on your answer sheet. The assignments must be handed in to the Mathematics Department's reception by the given date and time. The time limit is absolute and assignments will not be accepted after the deadline unless an agreement has been made with the administration beforehand.

Each exercise counts 0-5 points, all together a maximum of 50 points. Your Oblig 2 is approved if your score is 25 points or more and if you have made an attempt to solve all exercises.

Exercise 1

A sound wave is modeled by the function

$$f(t) = A\cos\left(\omega(t - t_0)\right)$$

where A is the amplitude and ω is the angular frequency. The frequency of the sound is given by $\frac{\omega}{2\pi}$. The acrophase t_0 is the least positive value of t which gives a maximum for $f(t)$.

a) Use the summation formula for cosinus to write $f(t)$ as a linear combination of $\sin(\omega t)$ and $\cos(\omega t)$.

b) We add two sound waves of the same frequency. For simplicity we assume $A = 1$ og $\omega = 1$. Write the sum

$$\cos(t - t_0) + \cos(t - t_1)$$

as $C\cos(t - t_2)$. Show that $C = \sqrt{2 + 2\cos(t_0 - t_1)}$.

c) Consider two special cases, i) $t_0 = t_1$ and ii) $t_1 - t_0 = \pi$. Explain what happens to the two sound waves in the two cases.

Exercise 2

Given a separable differential equation

$$\frac{dy}{dt} = h(y)g(t)$$

We are going to vary the two functions $h(y)$ and $g(t)$ to enlighten different situations that can occur in the theory of separable differential equations. Remember that the solution to the equation is a function $y = y(t)$. The variable t should be thought of as time, and y something we measure.

a) First we put $h(y) = y$ and $g(t) = \lambda$ (a constant). Find a general solution in this case. The constant λ can be either positive, negative or 0, and these cases correspond to quite different characteristics of the solution. Explain what happens to the solutions in the three cases when $t \to \infty$.

b) We now keep $h(y) = y$, but put $g(t) = \sin t$, i.e a harmonic function. Thus we have an example of a differential equation where the derivative is forced to vary periodically. Solve this equation and show that the solutions are periodic.

c) Another variation. Let $h(y) = \sqrt{y}$ and $g(t) = e^{-t}$, and let $y(0) = 1$. Find the solution of the equation in this case.

Exercise 3

In this exercise the task is to solve a separable differential equation by the method of partial fractions. Consider the differential equation

$$\frac{dy}{dt} = ay(A - y)$$

where $y > 0$ and A and a are positive constants.

a) Find new constants α and β such that

$$\frac{1}{y(A-y)} = \frac{\alpha}{y} + \frac{\beta}{A-y}$$

b) Use the solution of exercise a) to find a general solution of the equation given above, when we let $y(0) = \frac{A}{2}$. What happens to y when $n \to \infty$?

c) The differential equation describes some physical process. When $t \to \infty$ we find an equilibrium for y. Thus the value of y in the long run will stabilize on this value. Explain how we can deduce the equilibrium from the equation (without computing as we did in b) and c).)

d) Let $y = y(t)$ be a solution of the equation. By using the equation only, you should decide when the change rate of y reaches its maximum.

THE END